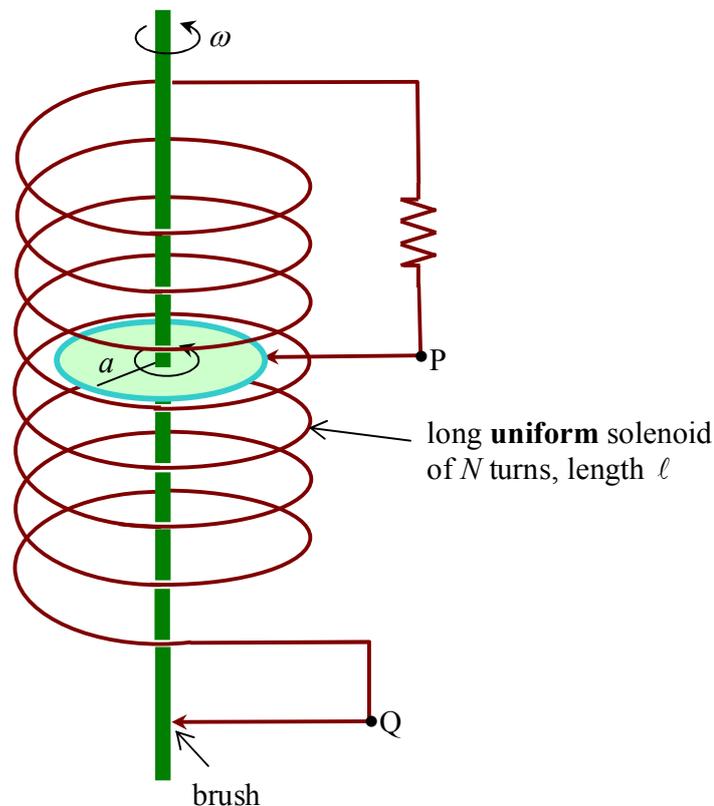


A Self-excited Magnetic Dynamo

A metallic disc of radius a mounted on a slender axle is rotating with a constant angular velocity ω inside a long solenoid of inductance L whose two ends are connected to the rotating disc by two brush contacts as shown. The total resistance of the whole circuit is R . A small magnetic disturbance can initiate the growth of an induced electromotive force across the terminals P, Q.



- 2.1) Write down the differential equation for $i(t)$, the current through the circuit. Express your answer in terms of L, R , and the induced e.m.f. (\mathcal{E}) across the terminals P and Q. (1.0 point)
- 2.2) What is the value of the magnetic flux density (B) in terms of i, N, ℓ , and the permeability of free space μ_0 ? Ignore the magnetic field generated by the disc and the axle. (1.5 points)



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- 2.3) What is the expression for the induced e.m.f. (\mathcal{E}) in terms of μ_0, N, a, ℓ, i , and the angular velocity ω ? (2.0 points)
- 2.4) Solve the equation in question 2.1 for current at any time t in terms of the initial current $i(0)$, and other parameters. (1.5 points)
- 2.5) What is the minimum value of the angular velocity that will permit the current to grow? Give your answers in terms of R, μ_0, N, a , and ℓ . (2.0 points)
- 2.6) In order to maintain a certain steady angular velocity ω , what must be the value of torque applied to the axle at the instant t ? (2.0 points)
