III.1. **[1.0 mark]** Let the phase difference between two rays making an angle θ with z direction be δ. Clearly

$$\delta = \omega \Delta t = \omega d \sin\theta / c \tag{1}$$

Then the intensity is given by

$$I\left(\theta\right) = \beta \overline{\left|\mathbf{E}_1 + \mathbf{E}_2\right|^2}$$

Here

$$\begin{aligned}
\left|\mathbf{E}_1 + \mathbf{E}_2\right|^2 &= \left|E_0 \cos(\omega t) + E_0 \cos(\omega t + \delta)\right|^2 \\
&= \left|E_0 \cos(\omega t)\left(1 + \cos\delta\right) - E_0 \sin(\omega t) \sin\delta\right|^2 \\
&= E_0^2 \left[\cos^2\left(\omega t\right)\left(1 + 2\cos\delta + \cos^2\delta\right) + \sin^2\left(\omega t\right)\sin^2\delta \right. \\
&\quad \left. -2\cos(\omega t)\left(1 + \cos\delta\right)\sin(\omega t)\sin\delta\right]
\end{aligned}$$

Since $\overline{\cos^2 \omega t} = \overline{\sin^2 \omega t} = 1/2$ and $\overline{\sin(\omega t)\cos(\omega t)} = 0$, we get the intesity to be

$$I(\theta) = \beta E_0^2 \left(1 + \cos\delta\right)$$

where δ is given by Eq. (1).

(a) Alternate:

$$\begin{aligned}
\left|\mathbf{E}_1 + \mathbf{E}_2\right|^2 &= \left|E_0 \cos(\omega t) + E_0 \cos(\omega t + \delta)\right|^2 \\
&= \left|E_0 \cos(\omega t)\left(1 + \cos\delta\right) + E_0 \sin(\omega t)\sin\delta\right|^2 \\
&= \left|E_0 A \cos(\omega t - \phi)\right|^2
\end{aligned}$$

where

$$\begin{aligned}
A^2 &= \left(1 + \cos\delta\right)^2 + \sin^2\delta \\
&= 2\left(1 + \cos\delta\right)
\end{aligned}$$

Since $\overline{\cos^2(\omega t - \phi)} = 1/2$, we have

$$I(\theta) = \frac{\beta}{2} E_0^2 A^2 = \beta E_0^2 \left(1 + \cos\delta\right)$$

where δ is given by Eq. (1).

III.2. **[1.0 marks]** The beam 1 has travelled extra optical path $= \left(\mu - 1\right) w$. Thus the net phase difference between two beams when they emerge at the angle θ is

$$\delta = \frac{\omega}{c}\left(d \sin\theta - \left(\mu - 1\right) w\right)$$

and $I(\theta) = \beta E_0^2 \left(1 + \cos\delta\right)$.

III.3. **[2.0 marks]** The two beams have travelled exactly same paths upto the slits. Thus when they emerge at an angle θ, they have a net phase difference of $\delta = \omega d \sin\theta / c$. Then, notice

$$
\begin{aligned}
\left|\mathbf{E}_1 + \mathbf{E}_2\right|^2 &= \left|\mathbf{i}\left[\frac{E_0}{\sqrt{2}}\cos(\omega t) + E_0\cos(\omega t + \delta)\right] + \mathbf{j}\left[\frac{E_0}{\sqrt{2}}\sin(\omega t)\right]\right|^2 \\
&= \left|E_0\cos(\omega t)\left(\frac{1}{\sqrt{2}} + \cos\delta\right) + E_0\sin(\omega t)\sin\delta\right|^2 + \frac{E_0^2}{2}\sin^2(\omega t) \\
&= E_0^2\left[\cos^2(\omega t)\left(\frac{1}{2} + \sqrt{2}\cos\delta + \cos^2\delta\right) + \sin^2(\omega t)\sin^2\delta\right. \\
&\quad \left. + 2\cos(\omega t)\left(\frac{1}{\sqrt{2}} + \cos\delta\right)\sin(\omega t)\sin\delta\right] + \frac{E_0^2}{2}\sin^2(\omega t)
\end{aligned}
$$

Thus, after taking time averages, the intesity will be

$$
\begin{aligned}
I(\theta) &= \beta E_0^2\left[\frac{1}{2}\left(\frac{1}{2} + \sqrt{2}\cos\delta + \cos^2\delta\right) + \frac{1}{2}\sin^2\delta\right] + \beta\frac{E_0^2}{4} \\
&= \beta E_0^2\left[1 + \frac{1}{\sqrt{2}}\cos\delta\right] \\
&= \beta E_0^2\left[1 + \frac{1}{\sqrt{2}}\cos\left(\frac{\omega d\sin\theta}{c}\right)\right]
\end{aligned}
$$

The maximum value of the intensity is $\beta E_0^2\left[1 + \frac{1}{\sqrt{2}}\right]$.

The minimum value of the intensity is $\beta E_0^2\left[1 - \frac{1}{\sqrt{2}}\right]$.

III.4. **[2.0 marks]** The electric field at $z = b$ is given by

$$
\begin{aligned}
\mathbf{E}_1(z = b) &= \left(\frac{E_0}{\sqrt{2}}\left(\cos(\omega t - kb)\,\mathbf{i}\cdot\mathbf{i}' + \sin(\omega t - kb)\,\mathbf{j}\cdot\mathbf{i}'\right)\right)\mathbf{i}' \\
&= \frac{1}{\sqrt{2}}\left[E_0\cos\gamma\cos(\omega t - kb) + E_0\sin\gamma\sin(\omega t - kb)\right]\mathbf{i}' \\
&= \frac{1}{\sqrt{2}}\left[E_0\cos(\omega t - kb - \gamma)\right]\mathbf{i}'
\end{aligned}
$$

and at $z = c$

$$
\begin{aligned}
\mathbf{E}_1(z = c) &= \frac{E_0}{\sqrt{2}}\left(\cos(\omega t - kc - \gamma)\,\mathbf{i}'\cdot\mathbf{i}\right)\mathbf{i} \\
&= \frac{1}{\sqrt{2}}\cos\gamma E_0\cos(\omega t - kc - \gamma)\,\mathbf{i}
\end{aligned}
$$

Where as

$$\mathbf{E}_2\,(z = c) = E_0\cos(\omega t - kc)\,\mathbf{i}$$

So, the net phase difference between beam 1 and beam 2 is now

$$\text{Phase difference } \alpha = \gamma.$$

III.5. **[0.5 marks]**

(a) At equator $e = 0$. Then

$$\mathbf{E} = \mathbf{E}_{\mathrm{Eq}} = \mathbf{i}' E_0 \cos(\omega t)\,.$$

(b) Clearly the beam is linearly polarised along $\mathbf{i}'$.

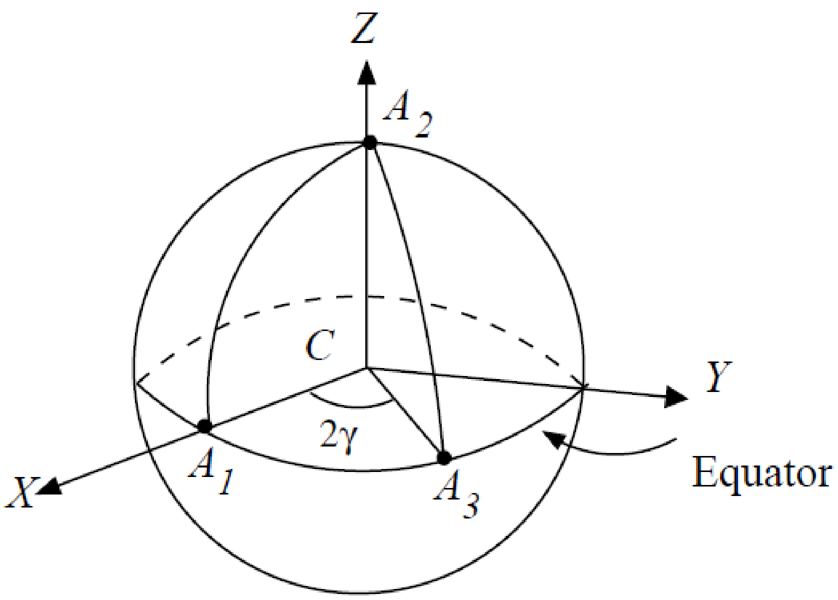
III.6. **[0.5 marks]**

(a) At north pole $e = \pi/4$ and γ can be taken to be 0. Then $\mathbf{i}' = \mathbf{i}$ and $\mathbf{j}' = \mathbf{j}$. The electric field

$$\mathbf{E}_{\mathrm{NP}} = \mathbf{i}\frac{E_0}{\sqrt{2}}\cos(\omega t) + \mathbf{j}\frac{E_0}{\sqrt{2}}\sin(\omega t)\,,$$

(b) Which represents circular polarisation.

III.7. **[1.5 marks]**



III.8. **[1.5 mark]** From figure it is clear that the area of the spherical triangle $A_1A_2A_3$ is $2\pi \times \left(\frac{2\gamma}{2\pi}\right) = 2\gamma$ and the phase difference was γ. Thus the phase difference is half the area of the spherical triangle $A_1A_2A_3$ on Poincare sphere.
Thus $S = 2\alpha$.
Here, the beam 1 passes through various states of polarization and returns to its original state. Though there has been no additional path difference, the beam has picked up a phase γ with respect to the beam 2. This is called as Pancharatnam phase.