The Stern-Gerlach Experiment[1]

The Stern-Gerlach experiment was performed in 1922 and it eventually led to the determination of the magnetic moment of the electron. In this experiment a beam of silver atoms each of mass $m = 1.80 \times 10^{-25}$ kg emerges from an oven kept at temperature T=1.20 $\times$ 10^3 K (see Figure 1). Assume that on emerging from the oven all the atoms have the same momentum along the direction of the beam (z-direction). Ignore gravity.

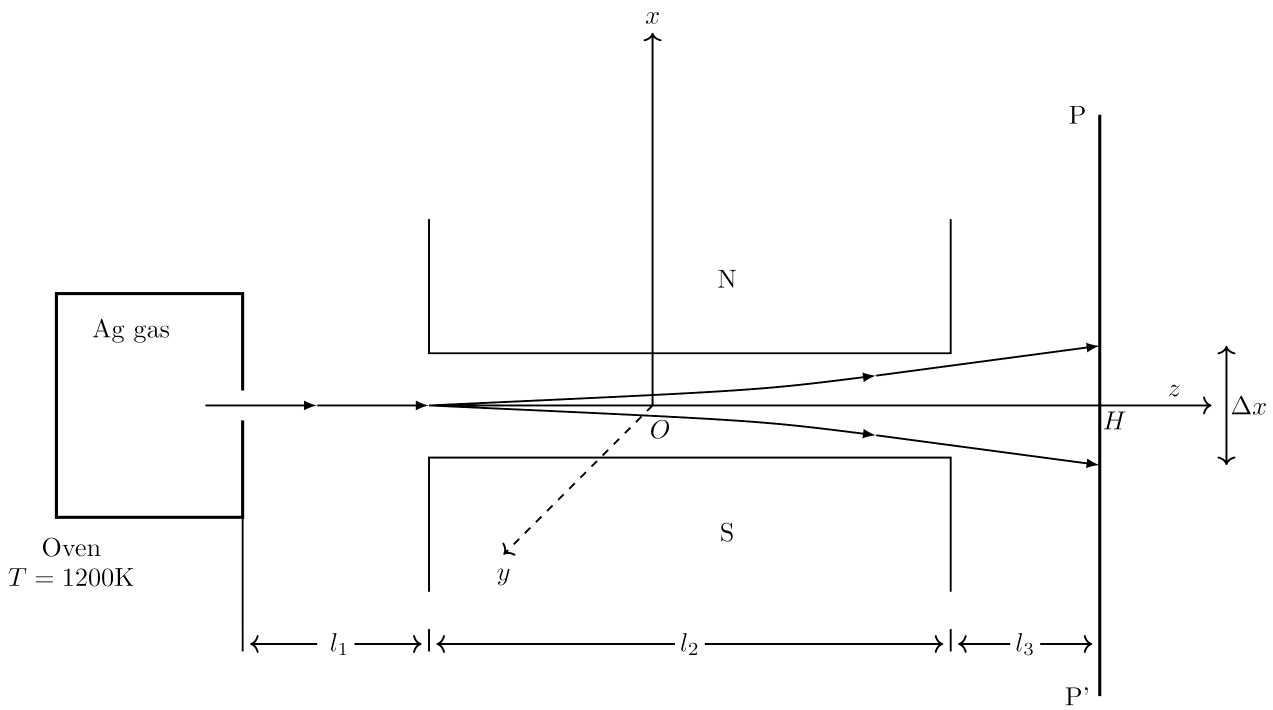

Figure 1: Schematic diagram of the Stern-Gerlach setup.

A.1	**Speed of the Silver Atoms:** The speed v_z of the silver atoms emerging from the oven can be estimated to be $\sqrt{3k_BT/m}$ using the equipartition theorem. Calculate this value.	0.5pt

[1] H. S. Mani (former Director, HRI, Prayagraj) and Gautam Datta (DAIICT, Gandhinagar) were the principal authors of this problem. The contributions of the Academic Committee, Academic Development Group, and the International Board are gratefully acknowledged.

B.1 **The Basic Expression:** After emerging from the oven the silver atoms move along the z- direction over a distance $l_1 = 0.25$ m. Next, the silver atoms pass between two magnets over a distance $l_2 = 0.5$ m. The magnets produce inhomogeneous magnetic field B in the x-direction with constant gradient dB/dx. Assume that the silver atom has a magnetic moment pointing either in + x-direction or - x-direction, i.e. $\vec{\mu}_s = \pm\mu_s\hat{\imath}$. After passing through the magnets the silver atoms pass through a further distance $l_3 = 0.25$ m before striking the screen PP'. The distance between the two striking beams on the screen is Δx. Derive the expression for the splitting distance Δx on the screen. 2pt

The Inhomogeneous Magnetic Field:

This part is concerned with the setup to create the inhomogenous magnetic field ($dB/dx \neq 0$). It consists of a number of sub-parts. Two very long wires parallel to the z-axis carry currents of magnitude I_0 and are located at $A_1\ (0, -a, z)$ and $A_2\ (0, a, z)$ (see figure below). The direction of the current passing through $y = -a$ is $-\hat{k}$ and for the one passing through $y = a$ is $\hat{k}$. The entire system is inside a medium of high relative magnetic permeability μ_r. We take μ = $\mu_0\mu_r$. The wires are insulated and no current leaks into the medium.

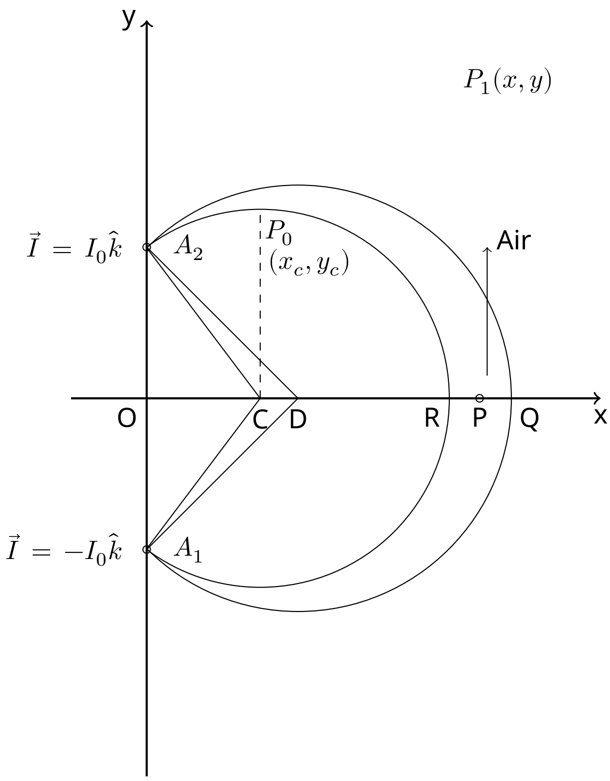

Figure 2: The arrangement for the inhomogeneous magnetic field

C.1	Derive the expression for the magnetic field vector at a point $P_1\,(x,y,0)$ in the x-y plane (see Fig. 2).	1.5pt

C.2	Consider a circle with its centre C on the x-axis ($x_c, 0$) and with radius equal to the distance AC (see Figure 2). Obtain the direction of the magnetic field at circumferential points R (on the x axis) and at P_0 (CP_0 is parallel to the y axis).	0.5pt

C.3	Now a small slice of the high magnetic permeability medium between the circles with radii AC and AD is removed and replaced with air at very low pressure (see Fig. 2). One can show from continuity considerations that the magnetic field in this gap region is given by the same expression as if the magnetic medium were not removed. (We shall assume this and you are not required to prove this). Hence state the expression for the magnetic field at the point $(x,0)$ in the air gap region.	0.5pt

D.1	**The Force:** As mentioned earlier the silver atoms are travelling in the $(x,0,z)$ plane with their velocities parallel to the z-axis and given by $\vec{v} = v_z\hat{k}$. Recall also that the magnetic dipole of the silver atom is $\vec{\mu_s} = \pm\mu_s\hat{\imath}$. Obtain the expression for the magnitude of the force F_x acting on a silver atom along the x-direction in terms of μ_s , I_0 , a μ and relevant coordinates.	0.5pt

E.1	**The Magnetic Field and its Gradient:** We assume that this same force F_x acts over a small distance l_2 along the z axis (Figure 1). Assume also that the silver atoms pass through the mid-point P of RQ (Figure 2). The following experimental values are given: $\dfrac{\mu}{\mu_0} = 10^4\,; \quad a = 0.60\,cm\,; \quad OC = 0.60cm\,; \quad OD = 0.80\,cm\,; I_0 = 2.00A$ Here μ_o is the magnetic permeability of free space. Obtain the numerical value of the magnitudes of the magnetic field B_P and its gradient dB_P/dx at this mid-point in S.I. units.	2.0pt

F.1	**The Magnetic Moment of the Silver Atom:** If $v_z = 500\,\mathrm{m\cdot s^{-1}}$ and magnetic field is calculates as above, the Stern-Gerlach experiment yields a split of $\Delta x = 0.20$ cm. Obtain the value of the magnetic moment of the silver atom μ_s in S.I. units.	1.5pt

G.1	**The spread of the line:** The silver atoms may not all have the same speed. Let there be a spread of 20 % in the beam speed. What would be the consequent spread of the dot δx on the screen?	0.5pt

H.1	**The error in the magnetic moment:** What is the consequent error bar on the evaluation of the magnetic moment $\delta\mu_s$?	0.5pt