Geometry of Water Fountain

Introduction

Water sprayers are found at various places such as agriculture farms, green parks or urban areas for functional purposes or for aesthetic art installation. Consider a hemispherical shaped fountain sprayer of radius r at the height h from the ground as shown in a cross-sectional diagram in **Fig. 1**. Let the radius of the hemisphere to be small compared to the range R, hence can be treated as a point source. However there are $\rho(\theta)$ number of holes per unit area at angle θ. The water spurts in all directions at the same initial velocity v_o.

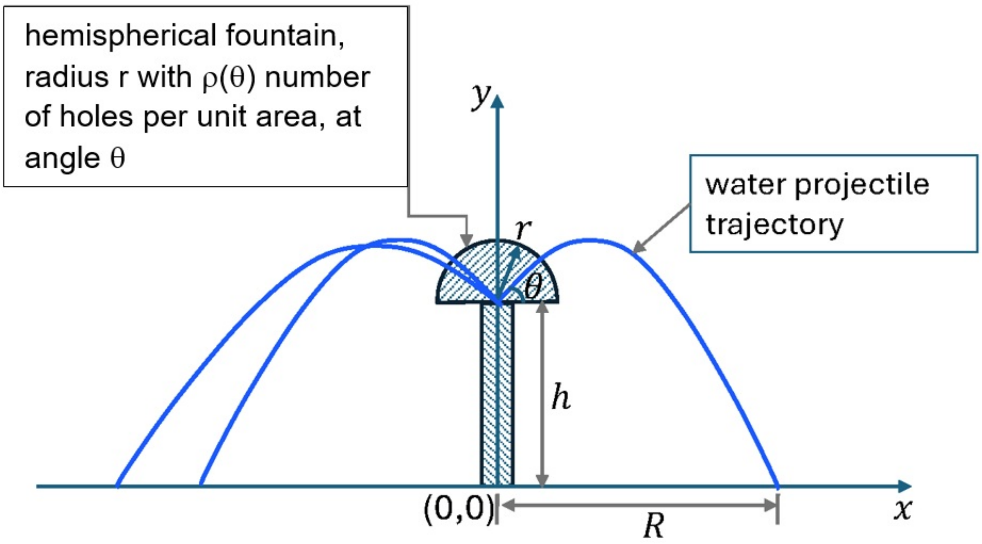

Fig. 1: Schematic cross section of a hemispherical shaped fountain sprayer.

Part A: Uniformly Distributed Holes on the Surface of the Hemisphere (6.0 points)

First consider the holes distribution on the surface of the hemisphere to be uniform (i.e. $\rho(\theta)$ is a constant).

A.1	Express the instantaneous position $x(t)$ and $y(t)$ of a water element (can be treated as a stream of particles) launched at velocity v_o at angle θ.	**0.5pt**

A.2	Derive the water trajectory relation $y = y(x, \theta)$ and express in a same type of trigonometric function (e.g. in terms of sine, cosine or tangent ONLY). Note that $\theta > 0$.	**1.0pt**

A.3	The envelope is the minimal boundary which encloses all water trajectories (i.e. no water will be found beyond this boundary). Derive the equation for the envelope for this system. Express your answer as y(x). Sketch the envelope over the trajectories.	**3.0pt**

A.4	Calculate the range $R = R(\theta)$ as a function of angle θ.	**1.0pt**

A.5	Letting $h \to 0$ in the range obtained in **A.4**, show that $R = \frac{v_o^2}{g} \sin\ 2\theta$.	**0.5pt**

Part B: Non-Uniformly Distributed Holes on the Surface of the Hemisphere (4.0 points)

Now, consider the area of holes per unit area $\rho(\theta)$ to be non-uniformly distributed and depended on the angle θ.

B.1	Using the results from part **A.4** above, calculate the elemental area dA_W of an annulus (ring) of radius R and width dR of the water hitting the ground. Express your answer in R, R' and dθ. [hint: consider R'=$\frac{dR}{d\theta}$]	**1.0pt**

B.2	Determine the relation for $\rho(\theta)$ that gives a uniform spray coverage pattern on the ground by considering elemental area on the hemisphere in proportional to the elemental area of the water spray on the ground. Express your answer in terms of R and its derivative. [hint: consider condition where $R\prime(\theta) > 0$].	**1.0pt**

B.3	Letting $h = 0$, calculate explicitly the relation for $\rho(\theta)$ that gives a uniform spray coverage pattern for $\theta < 45^\circ$.	**2.0pt**