SOLUTIONS & MARK SCHEME

BPhO PAPER 2 2009

ANY QUERIES REGARDING SOLUTIONS CONTACT
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MARKS

Q1

(a) Let specific heat capacity be s. As final rate of heat loss is 10 W,

$$10 = (0.25s + 50)\,15\times10^{-3}$$ 2

$$s = 4\left(\frac{10}{15\times10^{-3}} - 50\right)$$

$$s = 2.5\ \mathrm{kJ\,K^{-1}\,kg^{-1}}$$ 1

[3]

(b) (i) Planets move with constant speed not constant velocity 1

(ii) Energy is not a vector, so does not have components 1

(iii) Yes this is correct. The spot is not a physical object so can move with a speed greater than c. 1

(iv) The magnetic north pole is incorrectly named it is a 'south' pole attracting the north pole of the compass needle. 1

(v) To maintain the gas at constant pressure it must expand, so additional energy must be supplied compared, with that at constant volume, to raise its temperature by 1°C. 1

(vi) On expansion, the P.E. of atoms increases, so the K.E. must decrease for conservation of energy. Thus the temperature decreases 1

[6]

(c) (i) $\lambda = 1.00$ m, $f = 440$ Hz, "$f = \frac{1}{2L}\sqrt{\frac{T}{m}}$", length of string $L = \frac{\lambda}{2} = 0.50$

Vel^y, $v = f\lambda = 440\ \mathrm{m\,s^{-1}}$

At 550 Hz, $440 = 550\,(2L')$

length of string $L' = 0.40$ m 1

Thus string must be reduced by 10 cm 1

Q1

(c) (ii) $440 = \frac{1}{1.0}\sqrt{\frac{T}{m}}$ ①

$435.6 = \frac{1}{1.0}\sqrt{\frac{T+\Delta T}{m}}$ ②

Dividing ② by ①, $\frac{435.6}{440} = \sqrt{\frac{T+\Delta T}{T}} = \sqrt{1+\frac{\Delta T}{T}}$ 1

$1 + \frac{\Delta T}{T} = 0.9801$

$\frac{\Delta T}{T} = -0.0199 \approx -2.0\%$ 1

Thus tension must be increased by + 2.0% 1

[5]

(d) Let distance from home to work be D
Time taken to work $= D/8.0$
Time taken to return $= D/4.0$ 1

Average speed $= \frac{\text{Total distance}}{\text{Total time}}$ 1

$= \frac{2D}{D/8.0 + D/4.0} = \frac{16}{3}\ ms^{-1}$

$= 5.3\ ms^{-1}$ 1

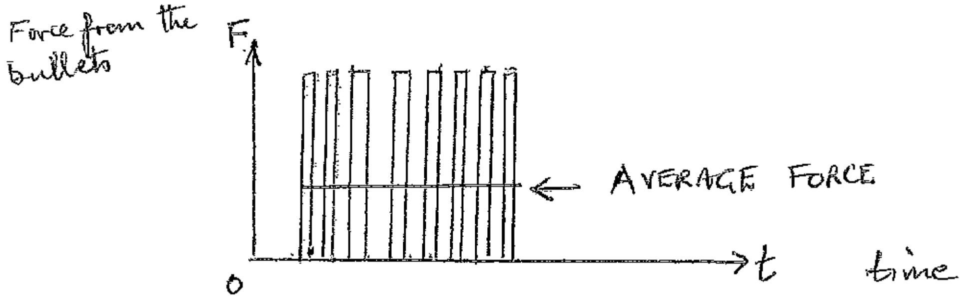
[3]

(e) Momentum of one bullet $= (10\times10^{-3})(12\times10^{3})$ Ns $= 120$ Ns 1

If there are n bullets per minute, $\frac{n}{60}$ bullets per sec, then rate of change of momentum, ie force, given by
$80 = 120\left(\frac{n}{60}\right)$ 1

$n = 40$ bullets per minute 1

(e)



Area under average line = sum areas under rectangles

2

[5]

(f)(i) Weight W balances electrical force:

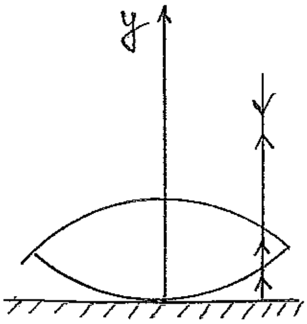
$$W = (0{\cdot}30 \times 10^6)e = (0{\cdot}30 \times 10^6)(1{\cdot}602 \times 10^{-19})$$ 1

$\underline{W = 4{\cdot}8 \times 10^{-14}\ N}$ 1

(ii) If it is independent of pd, it has no charge and is falling under gravity. Thus the viscous force of the air balances the weight, mg, of the droplet when the terminal velocity is reached i.e. $F_{VISCOUS} = mg$. 2

[4]

(g)



Monochromatic light incident normally on a convex lens, wavelength λ, sitting on an optical plate is partially reflected by the surface of the optical plate and the lower surface of the lens (see figure). These rays have a path difference and interfere. There is an extra phase difference of π produced by the optical plate. The circular symmetry about the y-axis gives rise to interference rings.
For constructive interference : $2t = (n+\frac{1}{2})\lambda$ n is an integer
For destructive interference : $2t = n\lambda$

4

[4]

(h) When the pan is moving upwards, with a retardation, approaching its amplitude, if the retarding acceleration is less than $(-g)$ the mass will leave the pan.

For displacement x, the retarding acceleration is

$$-\frac{kx}{m}, \qquad k \text{ is the spring constant}$$

So when $-\dfrac{kx}{m} < -g$ ie $\dfrac{4\pi^2 x}{T^2} > g \quad \left(T = 2\pi\sqrt{\dfrac{m}{k}}\right)$

the mass leaves the pan.

As the amplitude A is varied this will occur first when

$$\frac{4\pi^2 A}{T^2} = g \qquad \text{ie.}\quad A = \frac{T^2 g}{4\pi^2}$$

4

Substituting the data, $A = \dfrac{T^2 g}{4\pi^2} = \dfrac{(0.5)^2\, 9.81}{4\pi^2}$

$$\underline{A = 6.2 \text{ cm}}$$

2

[6]

(i) Let P be atmospheric pressure, which gas experiences when tube horizontal. Let density of mercury be ρ

When tube vertical pressure of gas is $(P + h\rho g)$ where h is length of mercury column; $h = 85$mm

Applying Boyle's Law

$$(50\times10^{-3})P = (45\times10^{-3})(P + h\rho g) \qquad 2$$

$$= (45\times10^{-3})(P + 85\times10^{-3}(14\times10^{3})9.81) \qquad 1$$

$$P = 9\times14\times9.81\times85$$

$$\underline{P = 1.05\times10^{5}\ \text{Pa}} \qquad 1$$

[4]

(j) Time to travel to object and return $= \dfrac{8}{10}\ \dfrac{1}{1250}$ s 1

as trace is 5cm long on oscilloscope screen; period 10cm

Thus if D is distance of object

$$2D = \frac{4}{5}\ \frac{1}{1250}\ (3\times10^{8})\ \text{m} \qquad 1$$

$$\underline{D = 9.60\times10^{4}\ \text{m}} \qquad 1$$

[3]

Q1

(k) (i) Yes, a body rotating in a circle is travelling at constant speed with an acceleration towards its centre. 1

(ii) No, constant velocity requires constant speed 1

(iii) Yes, a SHO has zero velocity when 'spring' fully extended and maximum retardation ie negative acceleration 1

(iv) Yes, when an oscillating mass is approaching its maximum extension, in +ve 'x-direction', it will have a +ve velocity and a negative acceleration; in the -ve 'x-direction' 1

[4]

(l) For constant y:

$$\omega t - kx = \text{constant}$$

So for constant y a small change in x, Δx, and a small change in t, Δt, gives

$$\omega \Delta t - k\Delta x = 0$$

Thus velocity v given by

$$v = \frac{\Delta x}{\Delta t} = \frac{\omega}{k}$$

The speed of wave is $v = \frac{\omega}{k}$ in +ve x-direction 2

(i) $$v = \frac{6.6\times10^3}{20} = 3.3\times10^2\ \text{ms}^{-1}$$ 1

(ii) $$\dot{y} = A\omega\cos(\omega t - kx)$$

$$\text{Max } \dot{y} = A\omega = (1.0\times10^{-7})(6.6\times10^3)$$

$$(\dot{y})_{max} = 6.6\times10^{-4}\ \text{ms}^{-1}$$ 2

[5]

(m) (i) $$\text{Current} = \frac{24}{240} = \frac{1}{10} = 0.10\ \text{A}$$ 1

$$\text{Resistance of each lamp} = \frac{12}{(1/10)} = 120\ \Omega$$ 1

(ii) 19 lamps, each 120 Ω resistance, total resistance = 2280 Ω 1

$$\text{Total power consumed} = \frac{(240)^2}{2280} = 25.3\ \text{W}$$ 1

Q1

m) (iii) When in normal use the temperature of the lamp's resistor is of the order of 1000K, but when tested at 0·10V it is at room temperature; the resistance is appreciably lower at room temperature that at the working temperature, giving a reduced current of 10 mA 2

[6]

(n) $\lambda = 248 \times 10^{-9}$ m

Energy conservation gives, where W is the work function,

$$\text{"}KE = h\nu - We\text{"} \quad 2$$

ie
$$8{\cdot}6 \times 10^{-20} = \frac{hc}{\lambda} - We \quad 1$$

$$= \frac{(6{\cdot}62 \times 10^{-34})(3{\cdot}00 \times 10^{8})}{(2{\cdot}48 \times 10^{-7})} - We \quad 1$$

$$We = 7{\cdot}15 \times 10^{-19} \text{ J}$$

$$\underline{W = 4{\cdot}46 \text{ eV}} \qquad (e = 1{\cdot}602 \times 10^{-19}\text{C}) \quad 1$$

[5]

(o) Conservation of energy requires

$$2m_ec^2 = 2h\nu \quad 2$$

$$h\nu = m_ec^2 = (9{\cdot}11 \times 10^{-31})(3{\cdot}00 \times 10^{8})^2 \text{ J}$$

$$\underline{h\nu = 8{\cdot}2 \times 10^{-14} \text{ J}} \quad 1$$

[3]

(p) (i)

When an observer is in relative motion to that of the source of waves, the frequency and wavelength can differ from those detected when there is no relative motion between source and observer. These changes are known as the Doppler effect. 2

(ii) Beats occur when two waves of slightly different frequencies travel in the same direction. The wave profile is modulated by a frequency equal to the difference of the two frequencies; one hears the modulated frequency in the case of sound waves. 2

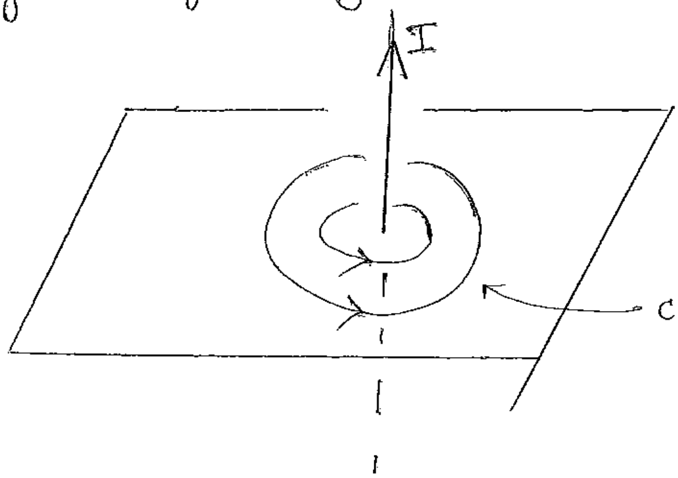
Q1

(p) (iii) A stationary wave is produce by two identical plane waves travelling in opposite directions. They produce a disturbance that does not propagate and gives rise to an alternating series of nodes and antinodes; the motion of the medium varies from zero at the nodes to a maximum at the antinodes. 2

[6]

(q) A magnetic field line is tangential to the magnetic field at each point in space along the line; this is the path a magnetic pole would take under the action of the field. The magnitude of the magnetic field varies along a magnetic field line. 2

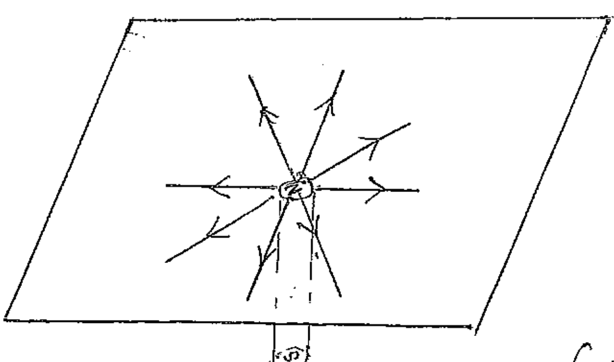
(i)



DIRECTION OF CURRENT AND CIRCLES — 1

CIRCLES — 1

(ii)



RADIAL FIELD LINES 1

CORRECT DIRECTION W.R.T. N POLE (OPP. DIRECTION FOR S POLE) 1

[6]

Q2

(i) If at time $t=0$ stone released and falls a distance s, before colliding with upward stone, at time T_c,

$$s = \frac{1}{2} g T_c^2 \qquad (1)$$

For stone project upwards

$$(h-s) = uT_c - \frac{1}{2} g T_c^2 \qquad (2)$$

1

Substituting (1) into (2)

$$h - \frac{1}{2} g T_c^2 = uT_c - \frac{1}{2} g T_c^2$$

$$\underline{T_c = \frac{h}{u}} \qquad (3)$$

1

(ii) If the stones collide with a common speed v:

Falling stone: $v = gT_c$ (4)

Upward stone: $v = u - gT_c$ (5)

1

From (5) and (4) $u = 2gT_c$ (6)

1

Substituting (6) into (3) $\underline{T_c = \sqrt{\frac{h}{2g}}}$ & $\underline{u = \sqrt{2gh}}$

1

(iii) If the speeds of stones after collision are v' and v'', in opposite directions, conservation of momentum gives

$$mv - mv = mv' - mv''$$

$$v' = v'' \qquad (7)$$

Energy conservation gives,

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = \frac{1}{2}mv'^2 + \frac{1}{2}mv''^2$$

From (7) $mv^2 = mv'^2$

ie $\underline{v = v' = v''}$ from (7)

After the collision the upper stone will rise to its initial starting height. This will take a time T_c, by principle of reversibility. It will the fall to the ground, taking T_c to reach the previous collision height and then, by reversibility of lower stone, a time T_c to reach the ground; that is a time $3T_c$ from the collision time.

Q2...

(iii) The lower stone, by principle of reversibility, will take T_c from collision time to reach the ground.
Thus time between the two stone impacting on the ground is $2T_c$ i.e. $2\sqrt{\frac{h}{2g}} = \sqrt{\frac{2h}{g}}$

4
9

b) MARK SCHEME

(i) Correct labels for "y-axis" in multiples of $\sqrt{\frac{gh}{2}}$ 1
Correct labels for "x-axis" in multiples of $T_c = \sqrt{\frac{h}{2g}}$ 1

$\frac{1}{2}$ mark for each section of "full" graph (upward stone) $1\frac{1}{2}$
$\frac{1}{2}$ mark for each section of "broken" graph (downward stone) $1\frac{1}{2}$

Correct points at which stones reach the ground 1

(ii) Axes correct (from ① + ③ collision occurs after falling $\frac{h}{4}$) 2

Full curve 1

Broken curve

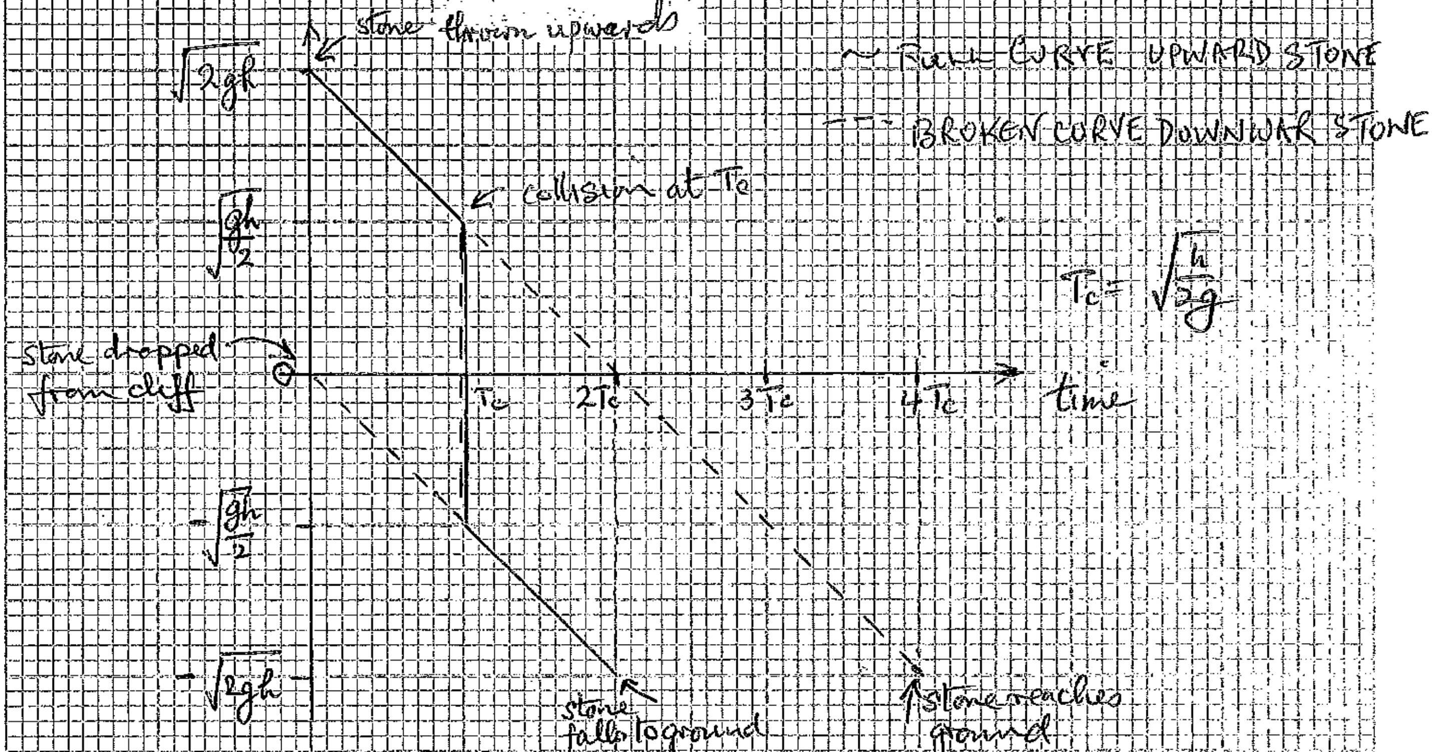
2

11

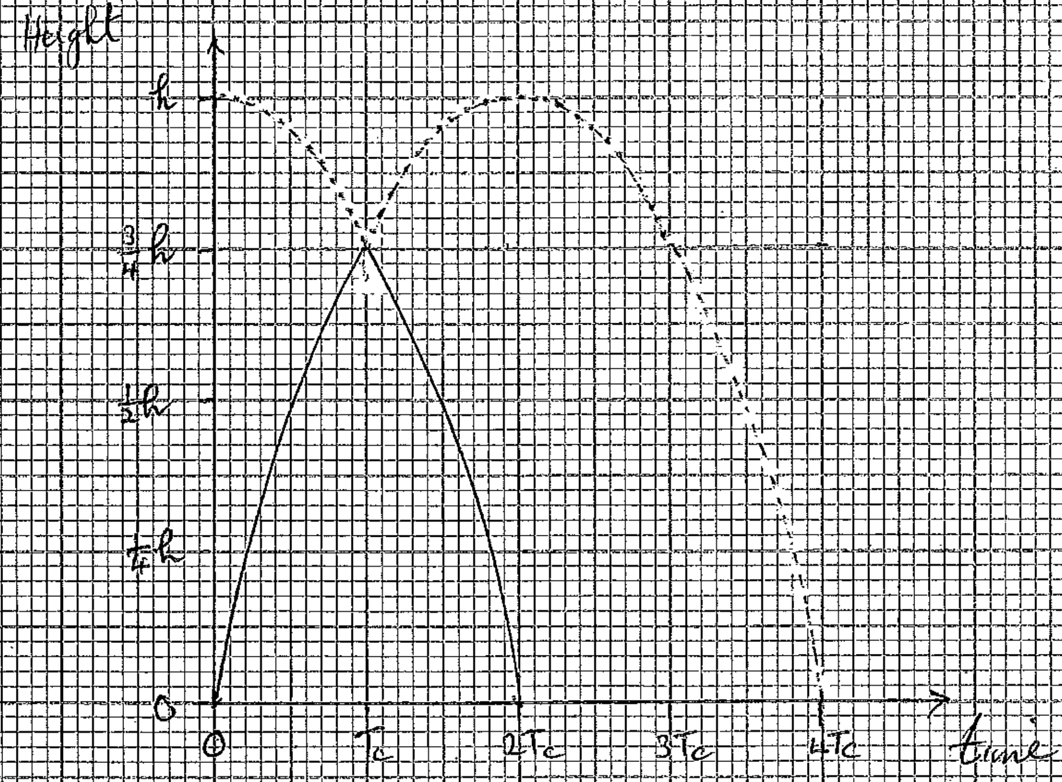
TOTAL 20

Q2 (b)

VELOCITY – TIME GRAPH



HEIGHT – TIME GRAPH



Q3

(i) Man's c. of g. falls 25m 1

Rope length at lowest point is $(25-2)\,m = 23m$ 1

Energy equation at lowest point

$\Delta(PE) =$ ENERGY OF STRETCHED ROPE 1

If man has mass m,

$$mg\,25 = \tfrac{1}{2}k(23-l_0)^2 \qquad (1)$$ 1

(ii) At equilibrium

$$mg = k(25-2-8-l_0)$$ 1

$$mg = k(15-l_0) \qquad (2)$$ 1

(iii) Substituting (2) into (1), eliminating m,

$$l_0^2 + 4l_0 - 221 = 0$$ 1

$$(l_0-13)(l_0+17) = 0$$ 1

$$l_0 = 13\,m \quad \text{or} \quad l_0 = -17m$$ 1

Only acceptable solution, the positive solution,

$$l_0 = 13\,m \qquad (3)$$ 1

[10]

(b) (i) At maximum speed, acceleration zero as motion SHM.

This occurs at the "equilibrium position"

Length of rope is $(25-8-2)m = 15m$ (see (2))

As $l_0 = 13$ extension is 2m 1

Energy equation at "equilibrium" position if man has speed v:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}k(2)^2 = mg(15+2) \qquad (4)$$ 2

From (2) and (3)

$$mg = 2k \qquad (5)$$ 1

Sub^g. (5) into (4)

$$v^2 = -\frac{4k}{m} + 2g(17)$$ 1

$$= -2g + 34g$$

$$= 32g$$

$$v = \sqrt{32g} = 17{\cdot}7\ ms^{-1}$$ 1

Q3

(b) (ii) Maximum acceleration occurs at the greatest extension of the rope

Greatest extension $= [(25-2) - 13]\,m = 10m$ 1

This is 5 times the equilibrium extension. As "$F = kx$" 1

this corresponds to a tension of $T = 5mg$

So net force on mass, mass m, is $4mg$ (ie $T - mg$) 1

Thus the greatest acceleration is $\underline{4g = 39{\cdot}2\ ms^{-2}}$ 1

[10]

Q4

a) (i) Using the usual notation $N = N_0 e^{-\lambda t}$ (1) 1

For the half life τ,

$$\frac{N_0}{2} = N_0 e^{-\lambda\tau}$$ 1

$$\frac{1}{2} = e^{-\lambda\tau}$$

Taking ln, $\lambda\tau = \ln 2$ 1

$$\tau = \frac{\ln 2}{\lambda}$$ (2) 1

(ii) If 87.5% = $\frac{7}{8}$ of the atoms decay only $\frac{1}{8}$ are left

From (1) $\frac{N}{N_0} = \frac{1}{8} = e^{-\lambda t}$ (3) 1

where $\lambda = \frac{\ln 2}{\tau} = \frac{0.6931}{3.3}$ if time measured in hours 1

Thus from (3) $t = \frac{3.3}{0.6931} \ln 8$ hours 1

$$t = 9.9 \text{ hours}$$ 1

[8]

(b) If there are V ccs of blood, there will be 0.5V disintegrations per min after 30 hours. Substituting into (1), measuring time in hours,

$$0.5V = 12 \times 10^3 \exp\left[-30\left(\frac{0.6931}{15}\right)\right]$$ 3

$$V = 24 \times 10^3 \exp[-1.3862]$$ 1

$$= 6.00 \times 10^3 \text{ ccs}$$

$$V = 6.00 \text{ litres}$$ 1

[5]

(c) If A is area of Geiger counter face, the initial total $\frac{4\pi(2.0)^2}{A}$ 360 s^{-1} 1

At distance x it detects the fraction of the counts per sec, $= \frac{4\pi x^2}{A}$ 1

The total 'count rate' after 90 mins. is $\frac{4\pi x^2}{A}(5)$ 1

Using the exp. decay of count rates

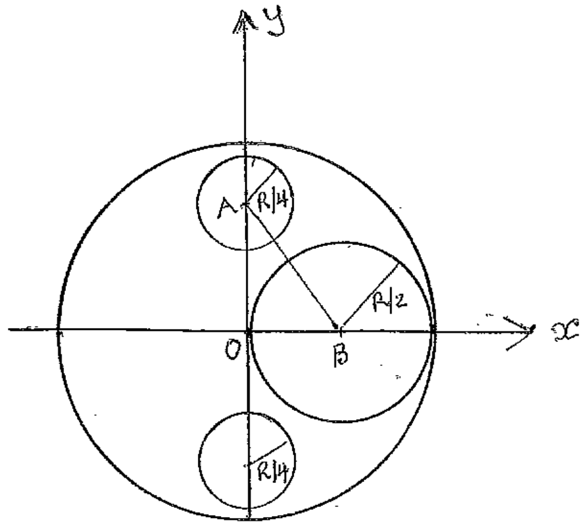
$$\frac{4\pi x^2}{A}(5) = \frac{4\pi(2.0)^2}{A}(360) \exp\left[-90\left(\frac{0.6931}{30}\right)\right]$$ 2

$$x = 6.0 \text{ m}$$ 2

[7]

Q5

(i)



O, A and B are the centres of the circles.
In ΔAOB the disks will not overlap if

$$AB > \tfrac{1}{4}R + \tfrac{1}{2}R = \tfrac{3}{4}R \qquad 1$$

or $\quad (AB)^2 > \frac{9}{16}R^2 = \frac{36}{64}R^2$

Using Pythagoras's Theorem in ΔOAB

$$(AB)^2 = \left(\tfrac{1}{2}R\right)^2 + \left(\tfrac{5}{8}R\right)^2 = \tfrac{41}{64}R^2 \qquad 2$$

As $\quad \frac{36}{64}R^2 < \frac{41}{64}R^2 \quad$ the circles will not overlap $\qquad 1$

(ii) By symmetry C. of G. lies along the x-axis $\qquad 1$

Taking moments about the y-axis, $\bar{x}$ being distance of C. of G,

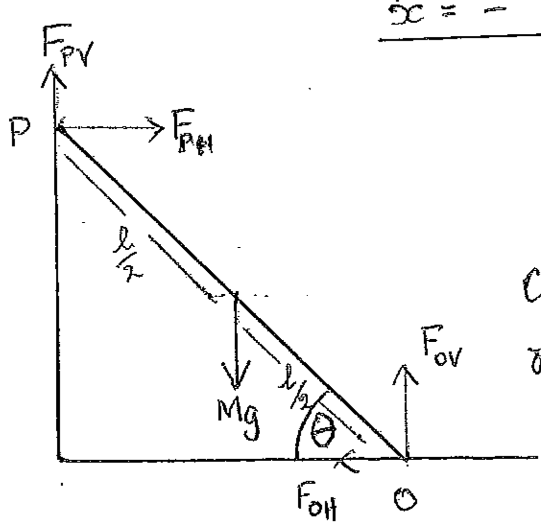
$$\left[\pi R^2 - \pi\left(\tfrac{1}{2}R\right)^2 - 2\pi\left(\tfrac{1}{4}\right)^2 R^2\right]\rho\,\bar{x} = -\tfrac{1}{2}R\left(\rho\pi\left(\tfrac{R}{2}\right)^2\right) \qquad 2$$

$$\tfrac{5}{8}\,\bar{x} = -\tfrac{1}{8}R$$

$$\bar{x} = -\tfrac{1}{5}R \qquad 1$$

[8]

b (i)



Correct diagram of forces — 2

(ii) Resolving:

Horizontally $\qquad F_{PH} = F_{OH} \qquad (1) \qquad 1$

Vertically $\qquad Mg = F_{OV} + F_{PV} \qquad (2) \qquad 1$

Q5

(iii) Taking moments about P, setting $F_{PV} = 0$ for smooth surface,

$$F_{OV}\, l\cos\theta = \tfrac{1}{2} l\, Mg\cos\theta + F_{OH}\, l\sin\theta \qquad 1$$

$$\frac{F_{OV}}{F_{OH}} = \frac{1}{2}\frac{Mg}{F_{OH}} + \tan\theta \qquad 1$$

$$\tan\theta = \frac{F_{OV}}{F_{OH}} - \frac{1}{2}\frac{Mg}{F_{OH}} = \frac{1}{F_{OH}}\left(F_{OV} - \tfrac{1}{2}Mg\right)$$

(iv) From ② with $F_{PV}=0$

$$\tan\theta = \frac{1}{2}\frac{Mg}{F_{OH}} \qquad ③ \qquad 1$$

F_{OV} can always balance Mg, but the largest value of F_{OH} is given by 1

$$F_{OH} = \mu F_{OV} = \mu Mg \qquad \text{eqn } ② \qquad 1$$

This gives the smallest value of $\tan\theta$ and hence θ, θ_{min}, 1

From ③

$$\theta_{min} = \tan^{-1}\left[\frac{1}{\mu Mg}\left(Mg - \tfrac{1}{2}Mg\right)\right]$$

$$= \tan^{-1}\left[\tfrac{1}{2}\left(\frac{1}{0.35}\right)\right] = \tan^{-1}\left(\frac{1}{0.7}\right) \qquad 1$$

$$\underline{\theta_{min} = 55^\circ} \qquad 1$$

[12]

Q6

a) (i) The image S' of S in the mirror is the point where the extension of PA meets the vertical through S. 1

So the path lengths SAP and S'AP are equal, thus reflected ray can be considered to come from S'; the optical path lengths are identical.

(ii) Young's double slit experiment involves a double slit coherent source. Here also S and S' form an effective 'double slit' source originating from S and S'. 2

(iii) The zero order fringe occurs when the optical path difference between SP and SAP, or S'AP, are identical. Here the reflection from the mirror produces an additional phase difference of π. Consequently the zero order fringe will be a destructive interference fringe. (In Young's double slit experiment there is no mirror and the zero order fringe is a constructive interference fringe.) 2

[6]

(b) (i) Using Pythagoras's theorem, where $l_1 = SP$ and $l_2 = S'P$;

$$l_1^2 = (SP)^2 = D^2 + (y-a)^2 \quad 1$$

$$l_2^2 = (S'P)^2 = D^2 + (y+a)^2 \quad 1$$

Path difference, p, given by

$$p = l_2 - l_1 = \left[D^2 + (y+a)^2\right]^{\frac{1}{2}} - \left[D^2 + (y-a)^2\right]^{\frac{1}{2}} \quad 1$$

(ii) Substituting the approximation for $(y \pm a) \ll D$, 1

$$p = l_2 - l_1 = D\left[\left\{1 + \tfrac{1}{2}\left(\frac{y+a}{D}\right)^2 + \dots\right\} - \left\{1 + \tfrac{1}{2}\left(\frac{y-a}{D}\right)^2 + \dots\right\}\right] \quad 1$$

$$= \frac{1}{2D}\left[(y^2 + 2ay + a^2) - (y^2 - 2ay + a^2)\right] \quad 1$$

$$p = l_2 - l_1 = \frac{2ay}{D} \quad 1$$

Q6

b) (iii) ... s'p has a phase change of π at the mirror. This is equivalent to a path increment of $\frac{\lambda}{2}$. 1

For constructive interference : $\phi = \frac{2ay}{D} = (n+\frac{1}{2})\lambda \quad n = 0, 1, 2, \ldots$ 1

For destructive interference $\phi = \frac{2ay}{D} = n\lambda$ 1

(iv) white light fringes occur when all wavelengths interfere constructively. This will occur when $n=0$; for larger values of n some wavelengths will produce constructive, or partial constructive, interference and others destructive, or partial destructive, interference. As n increases from zero one obtains whitish fringes with some colouring. This rapidly 'smudges' out the fringe system as n increases and the wavelengths becomes increasingly out of phase. giving a uniform white display. 4

[14]

Q7

(a)(i) $V_{AB} = \dfrac{1000}{6000}\, 9.0\ V$

$V_{AB} = 1.500\ V$ 1

It is a potential divider, dividing the source potential of 9.0V, in the ratio of the resistors. 1

(ii) The total resistance across AB $= \left(\dfrac{1}{1000} + \dfrac{1}{500}\right)^{-1} = \dfrac{1000}{3} = 333\frac{1}{3}\ \Omega$ 1

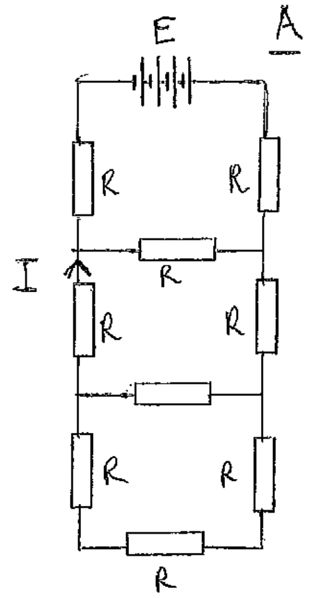
$$V_{AB} = \frac{333\frac{1}{3}}{5000 + 333\frac{1}{3}}\,(9) = \frac{1000}{3}\left(\frac{3}{15000 + 1000}\right) 9\ V$$

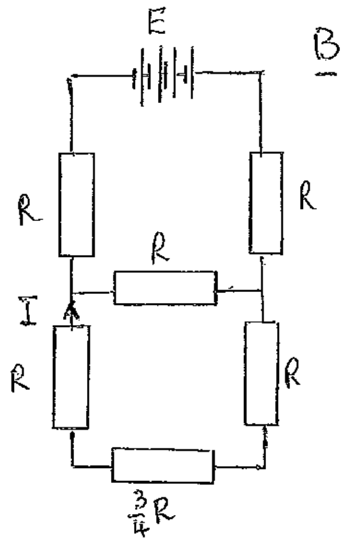
$V_{AB} = 5\frac{5}{8}\ V$ 1

(iii) The capacitor would charge up to the voltage given in (i) ie 1.500V 1

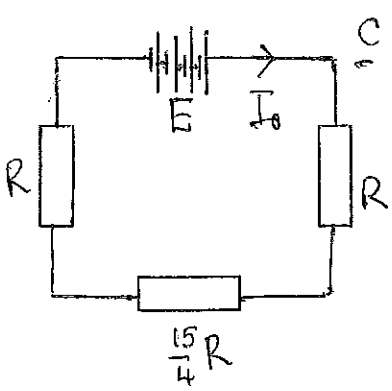
[5]

(b)(i)





R in ∥ with 3R gives $\left(\dfrac{1}{R} + \dfrac{1}{3R}\right)^{-1} = \dfrac{3}{4}R$ 1



$(2R + \frac{3}{4}R)$ in ∥ with R: $\left[\left(\frac{11}{4}R\right)^{-1} + (R)^{-1}\right]^{-1} = \frac{11}{15}R$

Gives $I_0 = \dfrac{E}{\frac{11}{15}R + 2R} = \dfrac{15E}{41R}$ 2

Thus from B

$$I = I_0\left(\frac{R}{2R + \frac{3}{4}R}\right) = \frac{4}{11} I_0$$

$$= \frac{4}{11}\left(\frac{15}{41}\frac{E}{R}\right) = \frac{60}{451}\frac{E}{R}$$

$I = 0.133\dfrac{E}{R}$ 2

Q7

b) (ii) This is a balanced Wheatstone Bridge with no current through GS. 1

So R_{GS} can be disconnected. Then $R_{CGK} = 2R$ and $R_{CDSFK} = 4R$. These two resistances are in parallel

Total resistance across CK = $\left(\frac{1}{2R} + \frac{1}{4R}\right)^{-1} = \frac{4}{3}R$ 1

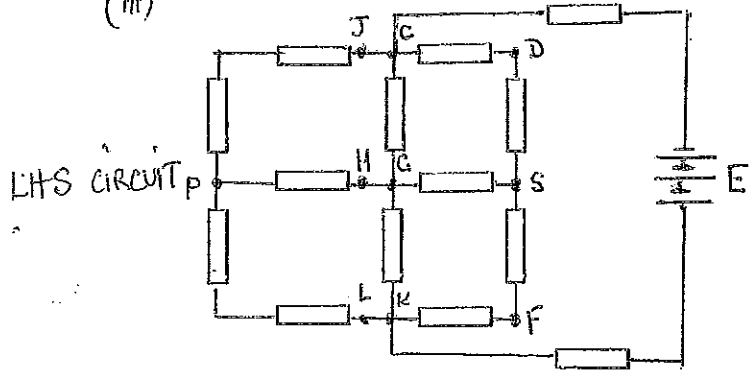
Current through the battery $= E \Big/ \left(\frac{4}{3}R + 2R\right)$

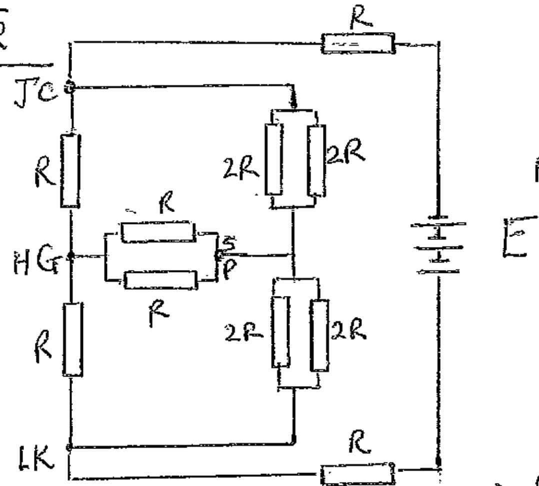
$$= \frac{3}{10}\left(\frac{E}{R}\right)$$

Current $\quad I = \frac{3}{10}\left(\frac{E}{R}\right)\left(\frac{4R}{2R+4R}\right)$ 1

$$\therefore \quad I = \frac{1}{5}\frac{E}{R}$$ 1

(iii)

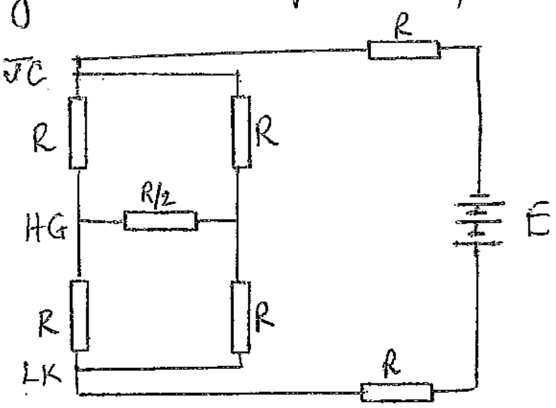




RHS CIRCUIT 2

By symmetry circuits on either side of CGK (excluding CEK) identical.
Hence S and P at same potential, so LHS circuit equivalent to RHS circuit above

Adding resistances in parallel, reduces the RHS circuit to



2

This is a balanced Wheatstone Bridge so R/2 can be removed as no current flows through it.

Total resistance across cells $= \left(\frac{1}{2R} + \frac{1}{2R}\right)^{-1} + 2R = 3R$ 2

Current through GC $\quad I = \frac{1}{2}\left(\frac{E}{3R}\right) = \frac{1}{6}\left(\frac{E}{R}\right)$

[15]

Q8

a) (i)

$$E = \tfrac{1}{2}mv^2 - \frac{GMm}{r}$$ 2

(ii)

$$v^2 = \frac{2E}{m} + \frac{2GM}{r}$$

v^2, and hence v, are a maximum when r is a minimum ie at A 1

v^2, and hence v, are a minimum when r is a maximum ie at B 1

(iii) Along the clockwise path from C to D, r is greater than along the clockwise path from D to C. Consequently v is smaller in the former case than in the latter case. Consequently the time to traverse C to D clockwise will be longer than D to C clockwise; both paths being of the same length. 2

(iv) For circular motion of the Earth about the Sun; $T = 1$ year
$a = 1$ AU

Therefore in these units, as $T^2 = ka^3$, $k = 1\,(\text{years})^{\frac{1}{2}}(\text{AU})^{-3/2}$ 2

(1AU = astronomical unit = R_{SE} = $1{\cdot}50 \times 10^{11}$ m)

(Alternative units acceptable if correct).

(v) For Halley's comet $T = 76$ years

So $a^3 = (76)^2$ where a in AU

$a = 17{\cdot}94$ AU 1

Length of Major axis $2a = 35{\cdot}9$ AU 1

[10]

(b) (i) Area swept out by radius vector, 'pivoted at O', during flight from L to T

$$A = \text{Area } \Delta TOL + \tfrac{1}{2}\text{ Area of Ellipse}$$ 2

$$= \tfrac{1}{2}R^2 \sin 2\theta + \tfrac{1}{2}(\pi ab)$$ 1

$$= \tfrac{1}{2}R^2 \sin 2\theta + \tfrac{1}{2}\pi R(R\sin\theta)$$ as $a = R$ and $b = R\sin\theta$

$$A = \tfrac{1}{2}R^2(\sin 2\theta + \pi\sin\theta)$$ 1

Q8

(b) (i) Now $T_0 = K\pi ab = K\pi R^2 \sin\theta$ as T_0 proportional to area swept out by complete ellipse, K is a constant 1

Let Time of flight from L to T be T, then

$$T = KA = K\ \tfrac{1}{2}R^2(\sin 2\theta + \pi\sin\theta)$$ 1

Thus

$$T = T_0 \frac{\frac{1}{2}R^2(\sin 2\theta + \pi \sin\theta)}{\pi R^2 \sin\theta}$$

$$\underline{T = T_0\left(\tfrac{1}{\pi}\cos\theta + \tfrac{1}{2}\right)}$$ 1

(ii) Yes. Limiting value gives

$$T = T_0\left(\tfrac{1}{2} + \tfrac{1}{\pi}\right)$$ 1

This is the limiting case of the ellipse; a straight line with foci at the extremities, at the centre of the planet and at the "top" of the flight. The height reached is equal to the radius of the planet. 2

[10]