BRITISH PHYSICS OLYMPIAD 2014-15

Round 1

Section 2

14th November 2014

Instructions

Questions: Only TWO of the eight questions in *Section 2* should be attempted.

Time: 1 hour 20 minutes on this section (approximately 40 minutes on each question).

Marks: The maximum mark for each of these questions is 20.

Answers
Answers and calculations can be written on loose paper or examination booklets. Graph paper and formula sheets are available.

Students should ensure their **name** and **school** is clearly written on all answer sheets.

Teachers' instructions
Section 1 and *Section 2* of *Round 1* may be sat in one session of 2 hour 40 minutes.

Alternatively, the paper may be sat in two sessions on separate occasions; with 1 hour 20 minutes for *Section 1* and 1 hour 20 minutes for *Section 2*. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected after the end of 1 hour 20 minutes and *Section 2* is to be handed out in the second session.

Important Constants

Speed of light	c	3.00×10^{8}	ms^{-1}
Planck constant	h	6.63×10^{-34}	Js
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Gravitational constant	G	6.67×10^{-11}	$Nm^{2}kg^{-2}$
Acceleration of free fall	g	9.81	ms^{-2}
Radius of the earth	R_E	6.38×10^{6}	m

Q2.

The circuits in this question have arms with resistance *R*, and currents *I*, i_1, i_2 and i_3 in the arms indicated in the Figures. An external potential difference, *V*, produces the current *I*. The symbol for resistance has been omitted from the Figures.

Use the symmetry of the circuit and apply the result that reversing *V*, reverses the directions of all the currents; the magnitudes remaining unaltered.

Kirchoff's first rule may be required. It states that at any vertex, the sum of the currents entering is equal to the sum of the currents leaving.

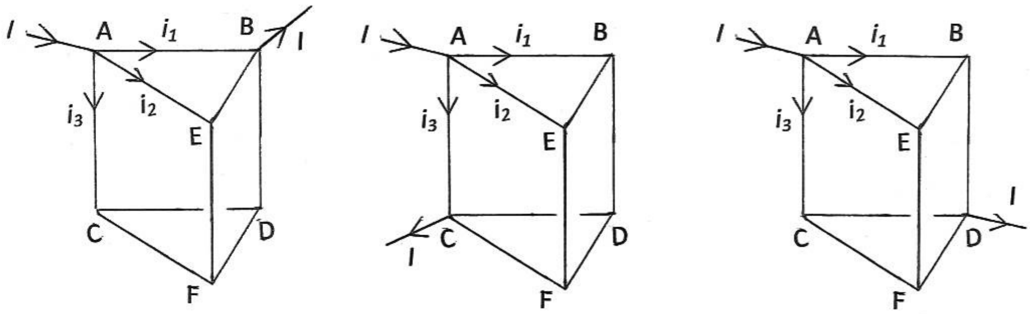

Figure 2.(a) Figure 2.(b) Figure 2.(c)

(a)

(i) Obtain the current in EF, i_{EF}, in the circuit in Figure 2.(a).

(ii) Determine the resistance across AB, R_{AB}, in terms of *R.*

[5]

(b) Determine the resistance across AC, R_{AC}, in terms of *R* in Figure 2.(b).

[5]

(c) Determine the resistance across AD, R_{AD}, in terms of *R* in Figure 2.(c).

This may require the application of Kirchoff's second rule. This states, for application here, that the clockwise sum of the currents in any closed loop is zero.

[10]

Q3.

Particles of mass *m* and charge *q* are projected with velocity *v*, with *v* much less than *c*, into a uniform magnetic field of flux density ***B***, at an angle θ to the field.

(a) Show, by considering the motion parallel and perpendicular to the magnetic field, that the particles have a helical trajectory, with their axes along the direction of the magnetic field.

[8]

(b) Determine the pitch of the helix x_{10}.

[3]

(c) Show that if θ is small, the trajectories for different θ but constant *v*, will all pass through a point P along the axis of the helix, after travelling a distance *L* along the magnetic field. Determine *L* and calculate its value if the particles are electrons and $v = 10^7\ \mathrm{ms^{-1}}$ and $B = 10^{-4}\ \mathrm{T}$.

[7]

(d) How would the analysis be modified if *v* was a relativistic velocity?

[2]

Q4.

(a)

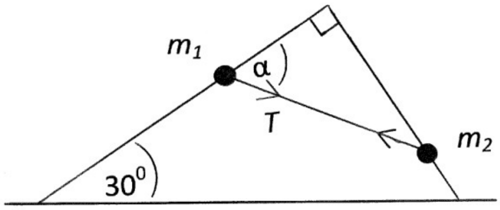

Figure 4.(a)

A smooth rod is bent into a right angle and mounted in a vertical plane on a horizontal base, Figure 4.(a). Beads of mass m_1 = 100 g and m_2 = 300 g slide without friction on the rod, and are connected by a cord of negligible weight with tension *T*. The angle between the horizontal base and the rod with the mass m_1 is 30^o. When the system is in static equilibrium, determine:

(i) the angle α between the cord with tension *T* and the 30^o slope, Figure 4.(a)

(ii) the tension *T*

[12]

(b) A smooth straw of length *L* and mass *m* is balanced on the edge of a smooth cylindrical container of diameter *d* which rests on a horizontal table, Figure 4.(b), at an angle α to the horizontal, as indicated in the Figure. Determine the angle α.

[8]

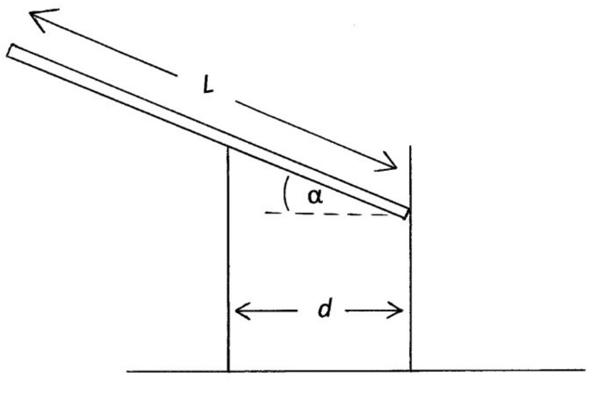

Figure 4.(b)

Q5.

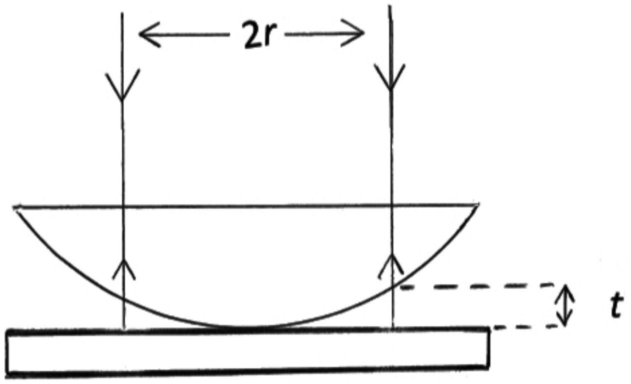

Figure 5.(1)

A Newton's rings interference experiment uses a plano-convex lens resting on a glass plate with light of wavelength λ incident normally on the lens, Figure 5.(1).

(a) Derive the conditions for constructive and destructive interference resulting from a region where the air gap has thickness *t.*

[5]

(b) A constructive interference ring, resulting from light passing through the air gap of thickness *t*, has a radius *r* . Show that the radius of curvature of the lens, *a*, is given by the equation.

$$r^2 = 2at - t^2$$

[3]

(c) By neglecting t^2 in the above formula, derive a formula for *a* in terms of *r* and the order of interference, *n*, that is independent of *t* for the case in which the n^{th} order destructive interference ring has radius *r*.

[2]

(d) If, using sodium light of wavelength λ = 598 nm, the diameter of the n^{th} order constructive interference ring is 0.582 cm and that of the $(n + 20)^{th}$ has diameter 1.36 cm, calculate the radius of curvature of the lens.

[4]

(e) If, in 5.0 s, the air gap between the lens and the plate is filled with methyl alcohol of refractive index μ = 1.33, what are the average speeds at which the radii of the constructive interference rings increase in (d) in this time interval?

[6]

Q6

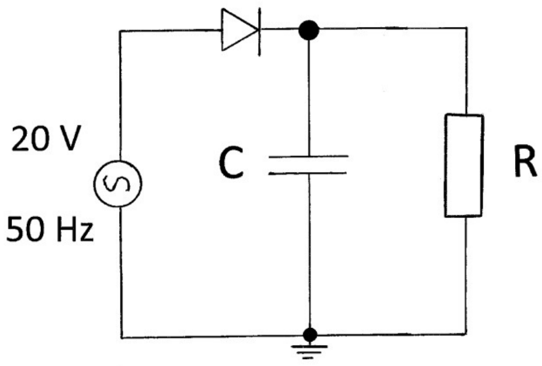

Figure 6.(1)

(a) The diode in the above circuit, Figure 6.(1), is used to rectify an ac voltage supply. The ac voltage supply at time t has the form,

$$V(t) = V_0 \cos(2\pi f t)$$

It has $f = 50$ Hz and $V_0 = 20$ V. The capacitor has capacitance $C = 50$ µF and the resistance, R, can take the values 10 kΩ, 1 kΩ and 100 Ω.
Determine the time constants for the discharge of the capacitor, for all three values of the resistance.

[2]

(b) Sketch a graph of the voltage forms across the resistance for the three values of R, superimposing the original ac source on each sketch.

[8]

(c) For the case $R = 10$ kΩ, deduce a relation for determining the time the diode conducts, T_D, in terms of f and T_1, where T_1 is the time the capacitor discharges per cycle. Show that it is the solution of the equation

$$\exp(-T_1/RC) = \cos(2\pi f T_1),$$

and verify that $T_1 = 0.019125$ s.

Determine the percentage of the time that the diode is conducting.

[6]

(d) Calculate the variation in voltage, from maximum to minimum, for $R = 10$ kΩ.

[4]

Q7

(a) Explain the difference between *random* errors and *systematic* errors. Give one example of each using either a clock to measure time or an ammeter to measure current.
Give a numerical example of:

(i) a correct measurement of g with low precision

(ii) the root mean square of the two length measurements, 65.0 m and 72.0 m

[6]

(b) If the maximum fractional error in the measurement of the period of a simple pendulum is 0.20% and that in measuring the length is 0.60%, what is the maximum percentage error in calculating the value of the gravitational acceleration due to free fall, g?

[3]

(c) Use the measurements in Table 7.(c) to plot a suitable straight line graph to verify that the numerical values of pd, V, and the current, I, varies as

$$10^V = A\, I^\alpha, \text{ where } A \text{ and } \alpha \text{ are constants.}$$

	1	2	3	4	5	6
V/V	3.0	3.6	4.6	5.0	6.2	6.6
I/A	3.2	9.9	20.4	31.6	97	193

Table 7.(c)

Determine α and A, with an estimate of their accuracy.

[7]

(d) Using the three sets of measurements, (1,4), (2,5) and (3,6) in Table 7.(c), determine the three associated values of α, and their mean value, $\alpha_{M,}$ together with the *magnitudes* of their deviations from α_M. Compare and comment on the accuracy of α_M with that of α determined in (c).

[4]

Q8

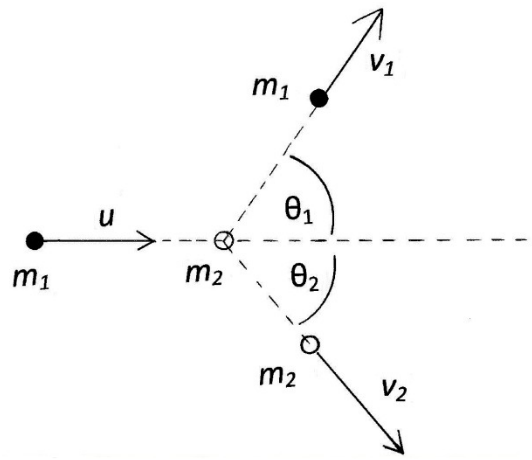

Figure 8.(1)

A particle of mass m_1 is incident with velocity u on a stationary particle of mass $m_2 = 4m_1$. The incident particle is scattered through an angle θ_1, with velocity v_1, and the other particle is scattered in a direction that makes an angle θ_2 with the direction of u, with velocity v_2, Figure 8.(1).

(a) Determine the velocity ratios (v_1/v_2) and (v_1/u) assuming momentum conservation only.

[5]

(b) Compare the initial kinetic energy, T_I, with the final kinetic energy, T_F, for the following situations:

(i) $\theta_1 = \theta_2 = 60.0^o$

(ii) $\theta_1 = \theta_2 = 56.0^o$

(iii) $v_1 = v_2$, $\theta_1 = 90^o$

In each case determine if the energy is conserved.

[12]

(c) If θ_1 and θ_2 are small, verify that energy is conserved if $3\theta_1 = -8\theta_2$.

(For small θ, $\sin\theta \approx \theta$)

[3]

Q9

(a) The energy levels, E_n , of the hydrogen atom are given by

$E_n = -13.6/n^2$ eV , where n = 1,2,3,....

What is the wavelength of the photon that results from a transition from the second excited state to the ground state?

[3]

(b) A 0.50 mW laser, of wavelength 590 nm producing a beam with a divergence angle of 1.5×10^{-3} radians, is pointing at the Moon. What is the maximum number of photons arriving per second per square metre on the Moon?

[5]

The Earth – Moon distance $R_{EM} = 3.84 \times 10^8$ m.

(c) A photon of frequency f_1 and wavelength λ_1, momentum h/λ_1 , is scattered by a stationary electron. A photon of frequency f_2 and wavelength λ_2 results. It travels in the opposite direction to the initial photon and the electron gains energy of 5.00 keV, with velocity v in the same direction as the incident photon. Determine numerically the value of λ_1.

[8]

(d) Monochromatic light of wavelength 380 nm is incident on a metal surface. A potential of 1.32 V is required to cut off the flow of photoelectrons.
What is the work function of the metal?

[4]

End of Questions