BPhO Round 1 Marking - November 2015

- Positive marking is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- Significant figures. A leeway of ±1 sig fig is allowed; if the published solution gives 3, allow 2 or 4 in the students answer. This only applies to the final answers as intermediate answers may be recorded to greater precision to avoid rounding errors. Candidates should not lose more than 1 mark per question for this even if they have got it wrong in more than one place in the question (they might lose all their hard earned marks otherwise). Question 1 (a) is different in this respect as it is specifically about sf and dp.
- Units should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Some allowance may be made if it is clear that the unit has been used a line or two earlier.
- In one or two places the units are a required part of the answer for the mark, and so must be there.
- Error carried forward (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks.
- You are not required to spend time deciphering scribble.

~~If you need support, email [illegible]~~
~~You can send a phone photo or just ask a question. We want to be fair in the marking, so I think that your good judgment should be acceptable.~~

<u>**Question 1 (Section 1)**</u>

(a)

- The **<u>decimal places</u>** on result and its uncertainty must agree for full marks
- The uncertainty can be 0.0007 or 0.0006 or 0.0005 depending on how it is calculated and whether rounded up or down.
- The result is 0.0621 (i.e 4 dp)
- Missing unit (s) does not matter
- 0.062 ± 0.001 loses a mark (So 2/3 for this answer)
- 0.062 ± 0.0007/6/5 etc loses a mark.
- 0.0621 ± 0.001 loses a mark

(b)

- If the $(1 \pm \alpha)$ is inserted as just $\pm\alpha$ then an answer for L_C and L_M cannot be obtained as the θ dependence remains. The mark can be obtained if it is clear that L_M and L_C are to be related by two equations.

(c)

- If the candidate uses $4/3\pi R^2$ for the surface area of a sphere then that is OK. Full marks, even though it results in 849 m

(d)

- They do not have to separately calculate the KE of the proton for the mark. It may be included within the subsequent calculation using the KE = PE.
- If the gold recoils, the value of r is greater by 180/179. Ignore this effect.

(e)

- Various pseudo calculations will appear. Resolving horizontally and vertically is required.

(f)

- A factor of 2 missing from the answer results in 18.59^o, loses one mark.

(g)

- The derivation of the Doppler result for a moving away observer must be clear for the mark. Not just jottings and arrows.
- A mark for the correctly expressed answer, but it must have the correct sign, and not be an approximation which they get a formula sheet).
- An approximation formula can be used in the subsequent calculation.
- The calculation may be done numerically as opposed to the algebraic formulation in the solutions.

(h) (i) (j)

(k)

-

As there are 8 marks for the question it must be properly constructed. The answer alone does not obtain the marks as that can easily be obtained with rw^2/g. The digarm itself is not required, but the formaulae and to what they apply, so that the pole is g_P is distinguished from g_E . The sign must be correct in the $g_E = g_P - rw^2$ and also a statement as to which is larger.

(l)

- For the direction, a mention of Lenz's Law is not enough; a reasoned statement as to how the direction is obtained is needed.

(m) The derivation can be much briefer than the solution.

<u>Question 2.</u>

The three electric fields are uniform and independent of each other, being perpendicular. So that the there is no complication due to any interaction. They are oriented in the X, Y, Z directions and there is no rotation of the coordinates.

Cosines appear at the end because it is the angle of the final velocity with X, Y, Z that is required, not the relative V_x, V_Y, V_Z to each other (which would give tan)

Question 3

Parts c and d may be intertwined. There are 2 marks for the theory stating about F_{60}.

There are 2 marks for determining what graph to plot 60/v against v^2 and how to obtain the value of V_C from the graph (in the intercept of the two graphs). If this is done without explanation, then they get the two marks. i.e V_C obtained graphically gets these two marks without an explanation being required.

There are two marks for drawing the graph itself.

There is a mark for getting V_C itself, which may be obtained from the graph or a calculator.

One mark for the unit and one mark for the uncertainty, which is only easily obtained from the graph.

Question 4

The marks are awarded as 7, 7, 6 as in the mark scheme (the question has 6, 7, 7 which should be ignored)

If you are missing page 15 of the mark scheme then ask Lena for that page. It was lost in the scanning.

Question 5

Gases; probably not too many candidates will do this question.

Question 6

Students can get a little bit muddled about the maximum potential when they are all negative. So careful about interpreting what they are trying to say. They may state the answer, such as the potential at W or U is the minimum because it is the maximum of "-GM/R" or something like that.

(a) They do not have to give the formula as $(1/r_A + 1/r_B)$ if they state in words what the potential does at different distances from A and B.

(b)

(c) They do not need to write the formula as an inequality but can use an equals sign for the relation between the terms.
If they choose the wrong point on the planet (the near side rather than the far side) then they will have GM/5r rather than GM/7r. If so, they lose one mark, but get the rest of the marks if the calculation is correct (ECF). The result is then, I believe, $v_{min}^2 = 16GM/15r$
If they do not do the calculation in (c), then
1 mark for explanation without calculation
1 mark for the idea of relating the energy of GPE to the KE of escape velocity.

Q1 BPhO 2016 SOLUTIONS

MARKS

(a) Time exposure, τ, using film given by

$$\tau_{\pm} = \frac{(12.4 \pm 0.1)}{360} \frac{60}{(33.3 \pm 0.1)} \text{ s}$$

$$= \tau \pm \Delta\tau$$

Giving

$$* \tau = 0.0621 \text{ s}$$ 1

$$\Delta\tau = \left(\frac{12.5}{360}\right)\left(\frac{60}{33.2}\right) - 0.0621$$

$$\underline{\Delta\tau = 0.0007 \text{ s}}$$ 1

$$\underline{\tau_{\pm} = 0.0621 \pm 0.0007}$$

ALTERNATIVE 1 DETERMINATION OF $\Delta\tau$.

$$* \tau = 0.0621 \quad (\text{as above})$$ 1

$$\frac{\Delta\tau}{\tau} = \frac{0.1}{12.4} + \frac{0.1}{33.3}$$

$$= 0.011$$

Giving

$$\Delta\tau = (0.062)(0.011)$$ 1

$$\underline{\Delta\tau \simeq 0.0007}$$ 1

3

ALTERNATIVE 2 using RMS value

$$* \tau = 0.0621 \quad (\text{as above})$$ 1

$$\frac{\Delta\tau}{\tau} = \sqrt{\left(\frac{0.1}{12.4}\right)^2 + \left(\frac{0.1}{33.3}\right)^2}$$

$$\frac{\Delta\tau}{\tau} \simeq 8.6 \times 10^{-3}$$ 1

$$\underline{\Delta\tau = 0.0005}$$ 1

3

Q1

(b) Let L_c and L_M be the lengths of the constantan and manganin wires. Then

$$6.3\,L_c + 5.3\,L_M = 5 \qquad (1)$$ 1

At temperature $\theta\,^{\circ}C$ we require

$$6.3\,L_c + 5.3\,L_M = 6.3\,L_c(1 - 3.0\times10^{-5}\theta) + 5.3\,L_M(1 + 1.4\times10^{-5}\theta)$$ 1

Thus

$$5.3\,(1.4\times10^{-5})\,L_M = 6.3\,(3.0\times10^{-5})\,L_c$$

Giving

$$L_M = \frac{6.3\,(3.0)}{5.3\,(1.4)}\,L_c = 2.55\,L_c \qquad (2)$$ 1

Substituting (2) into (1)

$$6.3\,L_c + 5.3\,(2.55)\,L_c = 5$$

Thus

$$\underline{L_c = 0.25\,\text{m}}$$ 1

Substituting into (2) $$\underline{L_M = 0.64\,\text{m}}$$ 1

5

(c) Number of photons emitted in time Δt, $(f = c/\lambda)$

$$n = \frac{100\,\Delta t}{hf} = \frac{100\,(6.0\times10^{-7})}{6.63\times10^{-34}\,(3.0\times10^{8})}$$ 1

$$= 3.02\times10^{20}\,\Delta t$$ 1

Volume of a shell, thickness ΔR, at distance $R = 4\pi R^2(c\Delta t)$ 1

Thus density of photons at R

$$10^6 = \frac{3.02\times10^{20}\,\Delta t}{4\pi R^2\,(3.00\times10^{8})\,\Delta t}$$ 1

Giving $$R^2 = \frac{3.02\times10^{20}}{12\pi\times10^{14}} = 8.01\times10^4$$

$$\underline{R = 283\ \text{m}}$$ 1

5

Q1

(a) Energy gained by acceleration through pd $V = 2\times10^6$ V
$= 2\times10^6 e$ J.

Conservation of energy at distance R of closest approach requires

$$Ve = \frac{Ze^2}{4\pi\varepsilon_0 R}$$ 1

Giving

$$R = \frac{Ze}{4\pi\varepsilon_0 V}$$ 1

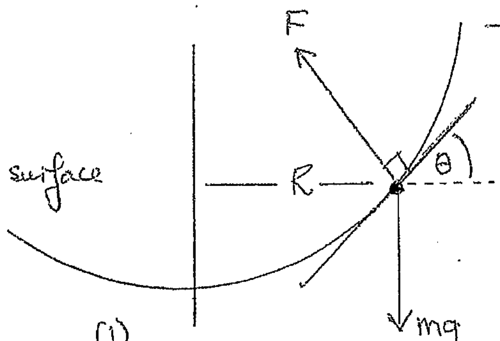
$$= \frac{79(1.60\times10^{-19})}{4\pi(8.85\times10^{-12})\,2\times10^6}$$ 1

Thus

$$R = 5.7\times10^{-14}\ \text{m}$$ 1

4

(e) Let sphere have mass m
F is force of bowl on mass normal to surface



Resolving vertically

$$F\cos\theta = mg \qquad (1)$$ 1

Resolving horizontally, resultant force is $mR\omega^2$,

$$F\sin\theta = mR\omega^2 \quad (2)$$ 1

Dividing (2) by (1) $$\tan\theta = \frac{R\omega^2}{g} \quad (3)$$ 1

However as gradient of bowl $2ax$,

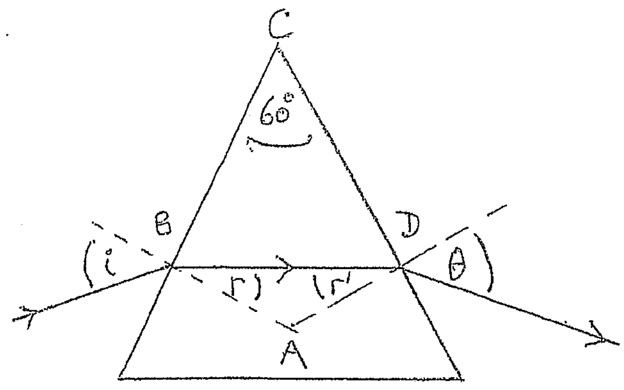
$$\tan\theta = 2aR \quad (4)$$

Substituting into (3) $$\omega = (2ag)^{1/2}$$ 1

4

Q1

(f)
(i)



Using notation in the diagram

$$\frac{\sin i}{\sin r} = 1.500$$ 1

$$\sin r = \sin(48.59°)/1.500 = 0.5000$$

Giving $\underline{r = 30.0°}$ 1

Now $\hat{A} = 120°$, so $r' = 30.0°$.
Consequently

$\underline{\theta = 48.59°}$ (as $r = r'$) 1

(ii) $\delta = (i - r) + (\theta - r') = 2(i - r)$ 1

$= 2(48.59° - 30.00°)$

$\underline{\delta = 37.18°}$ 1

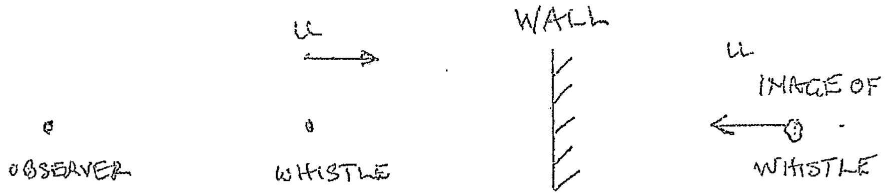
5

(g)

In 1 s whistle travels a distance (c+u) and generates f vibrations. Consequently wavelength

$$\lambda_1 = \frac{(c+u)}{f}$$ 1

with frequency $\underline{f_1 = \frac{c}{(c+u)} f}$ as "$f\lambda = c$" (1) 1



Frequency received directly by observer is given by (1)

Frequency produced by reflection at the wall is equivalent to image of whistle at a distance on the other side of the wall equal to its distance in front of the wall. The image travels at u towards observer, giving frequency

$$f_2 = \frac{c}{c-u} f$$ 1

ie replacing u by (−u) in (1)
Thus beat frequency F given by

$$F = f_2 - f_1$$ 1

$$= \left[\frac{c}{c-u} - \frac{c}{c+u}\right] f = \frac{2cu}{c^2-u^2} f$$

$$= \frac{2(330)(2.00)}{(330)^2-(2.00)^2} \; 500$$ 1

Giving

$$\underline{F = 6.06 \, s^{-1}}$$ 1

6

(h)
(i) $R_{DA} + R_{AC} = 35\ \Omega$
35 Ω in parallel with 7 Ω giving a total resistance

$$R = \left(\frac{1}{35}+\frac{1}{7}\right)^{-1} = \frac{35}{6}\ \Omega$$ (1) 1

$$R_{BD} + R = 3 + \frac{35}{6} = \frac{53}{6}\ \Omega$$

$\frac{53}{6}\ \Omega$ is in parallel with 2 Ω
This gives a total resistance $R_{TBC} \left(\frac{6}{53}+\frac{1}{2}\right)^{-1} = \frac{106}{65}\ \Omega$

$$\underline{R_{TBC} = 1.63\ \Omega}$$ 1

Q1

(h) (ii) R_{TBD} is $3\,\Omega$ in parallel with $(2+\frac{35}{6})\,\Omega$, from (i)

Thus

$$R_{TBD} = \left(\frac{1}{3}+\frac{6}{47}\right)^{-1} = \frac{141}{65}.$$

$$\underline{R_{TBD} = 2.17\ \Omega}$$ 1

(iii) For a potential across AB, as $\frac{2}{3} = \frac{14}{21}$, we have a balanced Wheatstone bridge, with zero current in DC. Thus the R_{TAB} consists of $(3+21)\,\Omega$ in parallel with $(14+2)\,\Omega$ giving 1

$$R_{TAB} = \left(\frac{1}{24}+\frac{1}{16}\right)^{-1} = \frac{48}{5}\ \Omega$$

$$\underline{R_{TAB} = 9.60\ \Omega}$$ 1

(Alternative derivations accepted). 6

(i) $N = N_0 e^{-\lambda t}$, $\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$ or $\left|\frac{dN}{dt}\right|_{t=0} = \lambda N_0$. 1

Substituting data at $t=0$ using $\left|\frac{dN}{dt}\right| = \lambda N_0$ at $t=0$,

$$10 = N_0 \frac{\ln 2}{1620 \times 365 \times 24 \times 3600}$$

$$\underline{N_0 = 7.37 \times 10^{11}}$$ 1

Mass of one nucleus

$$m = \frac{226 \times 10^{-3}}{6.02 \times 10^{23}}\ \text{kg} = 3.75 \times 10^{-25}$$ 1

Thus total mass

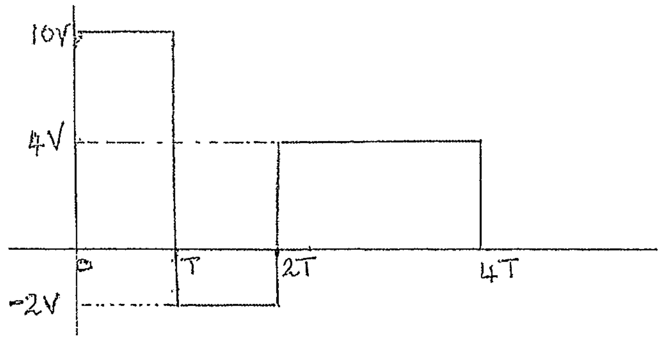
$$M = N_0 m = (7.37 \times 10^{11})(3.75 \times 10^{-25})$$

$$\underline{M = 2.76 \times 10^{-13}\ \text{kg}}$$ 1

4

Q1

(j)



$$\langle V^2 \rangle = \frac{T(10)^2 + T(-2)^2 + 2T(4)^2}{4T} \qquad 1$$

$$= 34$$

RMS $V_{rms} = \langle V^2 \rangle^{1/2} = \sqrt{34} = 5.83\,V$ 1

MEAN $V_m = \bar{V} = \langle V \rangle = [\,10T - 2T + 4(2T)\,]/4T = \underline{4V}$ 1

$$\langle (V-\bar{V})^2 \rangle = [\,T6^2 + T(-6)^2 + 2T(0)\,]/4T$$
$$= 18$$

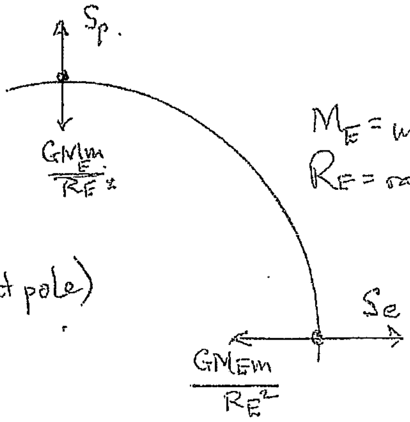
$V_{rmsm} = \langle (V-\bar{V})^2 \rangle^{1/2} = 4.24\,V$ 1

4

(Alternatively use result $\langle (V-\bar{V})^2 \rangle = \langle V^2 \rangle - \bar{V}^2$)

(k) For mass m at the pole with, normal reaction S_p, equating forces

$$S_p = -\frac{GM_E m}{R_E^2}$$



M_E = mass of Ear[th]
R_E = radius of Ear[th]

But $S_p = -mg_p$ giving (g_p is 'g' at pole)

$$mg_p = \frac{GM_E m}{R_E^2}$$

$$g_p = \frac{GM_E}{R_E^2} \qquad 1$$

If the Earth has an angular velocity ω the resultant force

$$m R_E \omega^2 = \frac{GM_E m}{R_E^2} - S_e \qquad 2$$

If 'g' at equator is g_e then $S_e = mg_e$, giving

$$g_e = \frac{GM_E}{R_E^2} - R_E\omega^2 \qquad 1$$

Q1 /8

(k) So

$$\Delta = \frac{g_p - g_e}{g_p} = \frac{R_E \omega^2}{GM_E / R_E^2} \quad \& \; g_p > g_e.$$ 1

$$= \frac{R_E^3 \omega^2}{G M_E}$$

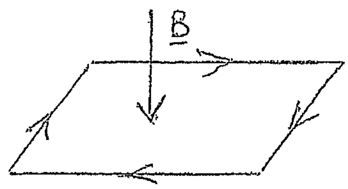
$$= \frac{(6.38 \times 10^6)^3}{(6.67 \times 10^{-11})(5.98 \times 10^{24})} \left[\frac{2\pi}{24 \times 3600}\right]^2$$ 1

$$= \frac{(2.60 \times 10^{20})(5.29 \times 10^{-9})}{3.99 \times 10^{14}}$$

$$\underline{\Delta = 3.45 \times 10^{-3}}$$ 1

8

(l)



Area $16 \times 10^{-4}\ m^2$

(i) ~~The flux through the loop~~ Φ.

$$\frac{d\Phi}{dt} = IR$$ 1

Substituting the data

$$\frac{0.700 (16 \times 10^{-4})}{0.800} = I (2.00 \times 10^{-3})$$ 1

Thus

$$I = \frac{14.0 \times 10^{-4}}{(2.00 \times 10^{-3})} \text{ amps}$$

$$\underline{I = 0.700 \text{ amps}}$$ 1

(ii) Energy dissipated in the loop in $t = 0.800$ s, E, is given by

$$E = I^2 R t = (0.700)^2 (2.00 \times 10^{-3})(0.800)$$ 1

$$\underline{E = 7.84 \times 10^{-4}\ J}$$ 1

Q1

(l) Direction of current opposes the change taking place in the magnetic field, so will tend to reestablish the field.
The current is thus clockwise, as indicated in the diagram, producing a field in the direction of B

2 marks for correct explanation and arrows in diagram. 2
Zero marks for correct direction but NO explanation

7

(m) When spheres connected they have a common potential V_f. 1
If initial charge Q_1 on 200V sphere and Q_2 on 400V. sphere, after connection, by symmetry, each sphere has a charge $\frac{1}{2}(Q_1+Q_2)$ as charge is conserved. Let capacity 1
of spheres be, respectively, C_1 and C_2 with initial potentials V_1 and V_2.
Capacity of sphere

$$C = C_1 = C_2 = 4\pi\varepsilon(10\times10^{-2}) = 4\pi\varepsilon 10^{-1}$$ 1

Initial energy, E_i, given by

$$E_i = \tfrac{1}{2}C_1V_1^2 + \tfrac{1}{2}C_2V_2^2 = \tfrac{1}{2}(4\pi\varepsilon 10^{-1})(2\times10^5)\text{J} \quad (1)$$ 1

Final energy, E_f, given by

$$E_f = \tfrac{1}{2}(C_1+C_2)V_f^2$$

$$= 4\pi\varepsilon 10^{-1} V_f^2 \quad (2)$$ 1

Conservation of charge gives

$$Q_1+Q_2 = C(V_1+V_2)$$ 1

$$= CV_f + CV_f = 2CV_f$$ 1

Thus

$$\underline{V_f = \tfrac{1}{2}(V_1+V_2) = 300\text{V}}$$ 1

Thus energy released, from (1) and (2),

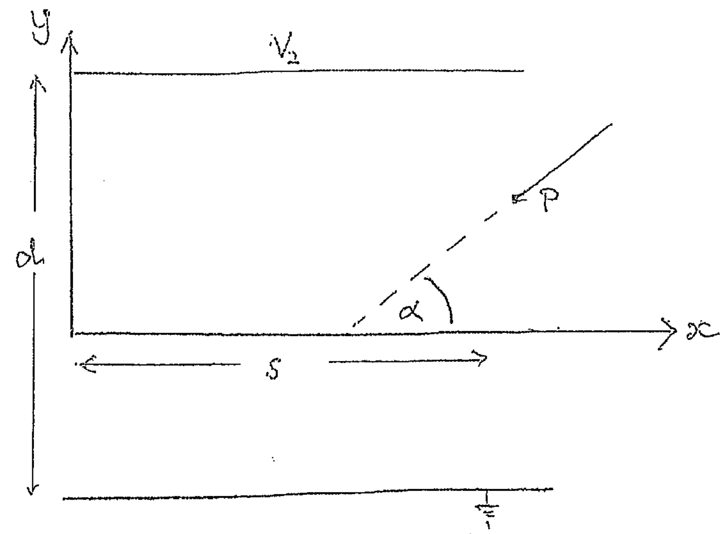
$$E = E_i - E_f = \tfrac{1}{2}(4\pi\varepsilon 10^{-1})(2\times10^5) - (4\pi\varepsilon 10^{-1})(9\times10^{+4})$$ 1

$$= 4\pi(8.85\times10^{-12}\times10^{-1})(10-9)10^4\ \text{J}$$

$$\underline{E = 1.11\times10^{-7}\ \text{J}}$$ 1

10

Q2



(i)

Initial energy

$$\frac{1}{2} m_e v_0^2 = eV_1 \qquad (1)$$

$$v_0 = \sqrt{\frac{2eV_1}{m_e}}$$ 1

(ii) If v_y and v_x are the velocity components at P

$$\tan\alpha = \frac{v_y}{v_x} = \frac{v_y}{v_0}$$ 1

If a_y is the acceleration in y direction for distance s in time $t = \frac{s}{v_0}$,

$$v_y = a_y t$$

$$= \left(\frac{eE}{m_e}\right)\left(\frac{s}{v_0}\right)$$ where E is the electric field 1

Substituting $E = \frac{V_2}{d}$, $$= \frac{eV_2 s}{m_e d v_0}$$ 1

Giving $$\tan\alpha = \frac{eV_2 s}{m_e d v_0^2}$$

Substituting for v_0 from (1)

$$\tan\alpha = \frac{V_2 s}{2V_1 d} \quad \text{or} \quad \alpha = \tan^{-1}\left(\frac{V_2 s}{2V_1 d}\right)$$ 1

(iii)

$$y_e = \frac{1}{2} a_y t^2$$ 1

$$= \frac{eEs^2}{2m_e v_0^2}$$ 1

$$y_e = \frac{eV_2 s^2}{2m_e d v_0^2} \qquad (v_0^2 \text{ given by } (1))$$ 1

8

Q2

(b) $T = \frac{s}{v_0}$

$= s\sqrt{\frac{m_e}{2eV_1}}$ from (1) 1

Substituting the data

$T = 1.51 \times 10^{-9}$ s 1

2

(c) On entering the final electrical field the electrons have zero velocity component in the z-direction. When they emerge they will have the same expression for velocity component in the z-direction as that for the previous velocity gained in the y-direction namely

$$\frac{eV_2 s}{d m_e v_0}$$ 2

Thus the Cartesian components of the electrons' emergent velocity from the system is

$$\left(\sqrt{\frac{2eV_1}{m_e}},\ \frac{eV_2 s}{d m_e v_0},\ \frac{eV_2 s}{d m_e v_0}\right)$$ 3

where v_0 is given by (1)

The resultant speed is v given by

$$v^2 = \left(\frac{2eV_1}{m_e}\right) + \left(\frac{eV_2 s}{d m_e v_0}\right)^2 + \left(\frac{eV_2 s}{d m_e v_0}\right)^2$$ 1

Thus

$$v = \sqrt{\frac{2eV_1}{m_e} + 2\left(\frac{eV_2 s}{m_e v_0 d}\right)^2}$$ 1

The angles the final velocity makes with the x, y and z axes are, respectively,

$$\cos^{-1}\left(\frac{1}{v}\sqrt{\frac{2eV_1}{m_e}}\right),\ \cos^{-1}\left(\frac{eV_2 s}{v\, m_e v_0 d}\right),\ \cos^{-1}\left(\frac{eV_2 s}{v\, m_e v_0 d}\right)$$ 3

10

Q3 12

(a) Statement $P = Fv$ for determination of F 1

Plot of F against v^2 1

Good straight line graph with axes labelled 2

Determination of $A = 0.19$ kgm^{-1} giving units kgm^{-1} 1

Accuracy ± 0.01 kgm^{-1} acceptable 1

Intercept $B = 4.0$ N giving units N 1

Accuracy ± 0.6 N acceptable 1

8

(b) Av^2 air resistance 1

B friction 1

2

(c) $F = \frac{P}{v} = \frac{P}{(v^2)^{1/2}}$

For max. power $P = 60$ W

$F_{60} = \frac{60}{(v^2)^{1/2}}$ 2

If one plots F_{60} against v^2, on the same graph as above, where it intersects the linear plot determines V_c and satisfies $\frac{60}{V_c} = AV_c^2 + B$ 2

1

5

(d) Plot of $\frac{60}{(v^2)^{1/2}}$ against v^2 on the original graph 2

Value of $V_c = 5.8$ ms^{-1} 1

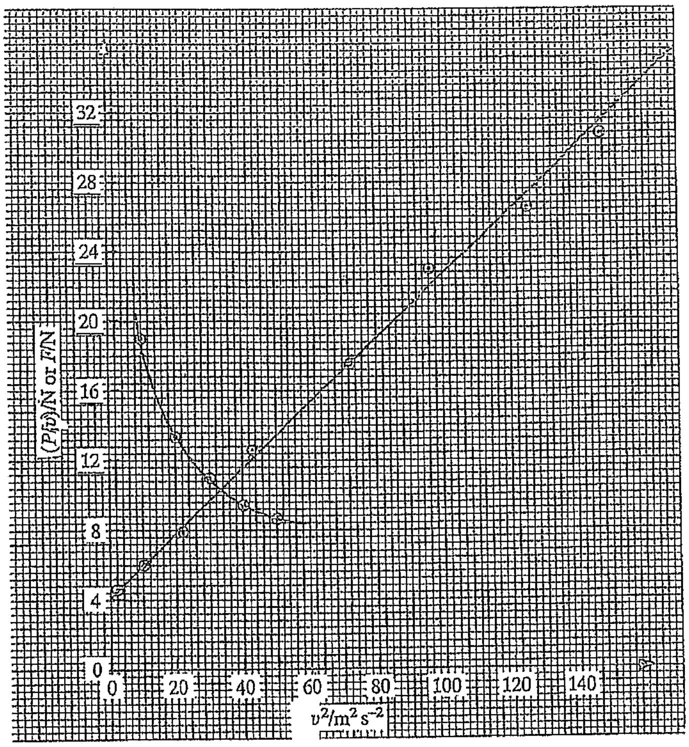
Correct units 1

Accuracy ± 0.4 ms^{-1} 1

5

Q3

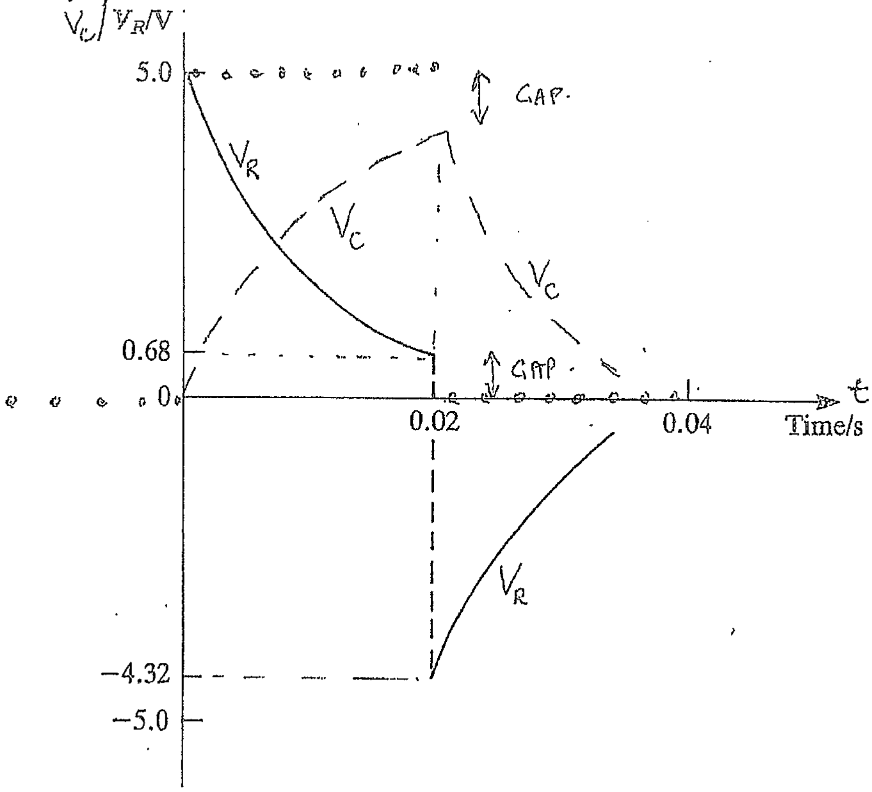
13



Q4

(a) KEY: RESISTOR VOLTAGE FULL CURVE
CAPACITOR VOLTAGE BROKEN CURVE
TOTAL VOLTAGE

Only qualitative behavior required



RESISTOR VOLTAGE: FULL CURVE
One mark for each section correct 2
CAPACITOR VOLTAGE: BROKEN CURVE
One mark for each correct section 2
One mark for TOTAL VOLTAGE correct 1
Two marks for 'GAP' VOLTAGES 2

7

NOTE

4 marks available
for correct transformations
of circuit

Q4 16

(c)

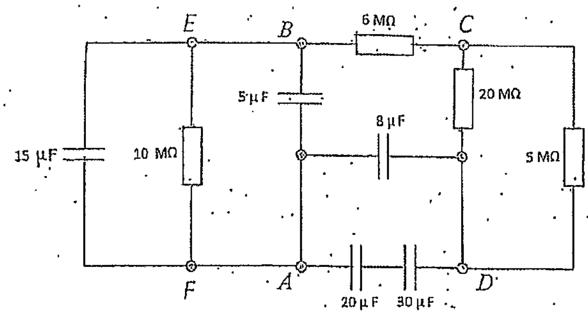


Figure 4.c(i)

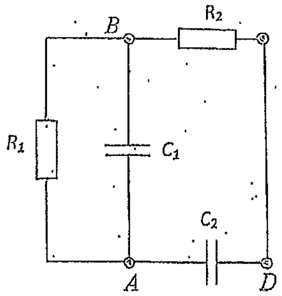
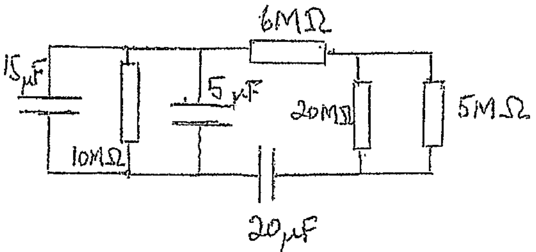


Figure 4.c(ii)

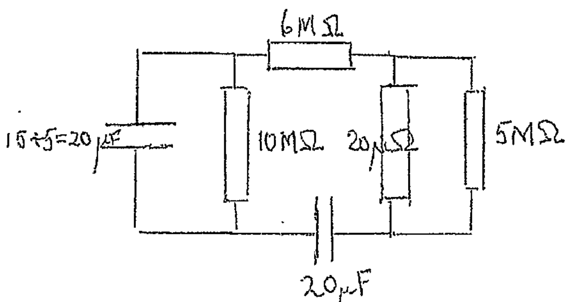
AN EXAMPLE OF CIRCUIT SIMPLIFICATIONS RESULTING IN CIRCUIT FIGURE 4. c(ii)

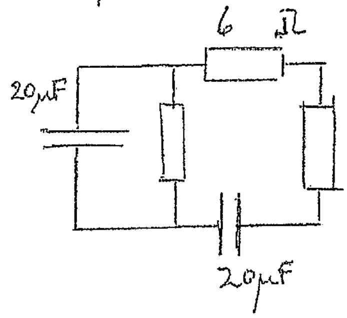
4 marks are available for four correct circuit simplifications

$\left(\frac{1}{20}+\frac{1}{30}\right)^{-1}+8 = 20\mu F$



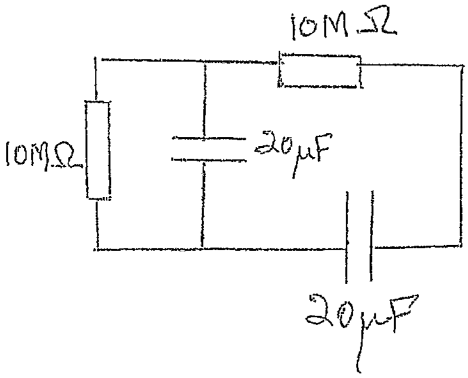
1





$\left(\frac{1}{5}+\frac{1}{20}\right)^{-1} = 4M\Omega$

2



1

$R_1 = R_2 = 10 M\Omega$ 1

$C_1 = C_2 = 20\mu F$ 1

6

Q5

(i) (a) If air expands by length x, A being area of cross-section of tube

$$V_R = \frac{V}{V_0} = \frac{A(x_0+x)}{A\,x_0}$$

$$V_R = 1 + \frac{x}{x_0} = (1+x_R) \qquad (1) \qquad 1$$

As atmospheric pressure $x_0 \rho g$,

$$P_R = \frac{P}{P_0} = \frac{(2x_0 - x)\rho g}{x_0 \rho g}$$

Giving

$$P_R = (2 - x_R) \qquad (2) \qquad 1$$

From (1) and (2)

$$P_R = 2 - (V_R - 1)$$

$$P_R = 3 - V_R \qquad (3) \qquad 1$$

(ii).

$$PV = nRT$$

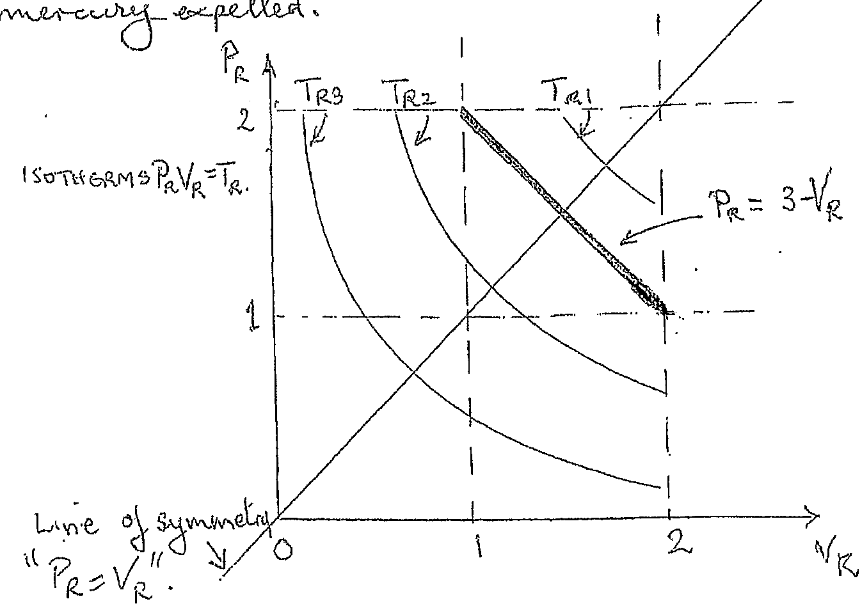
$$\frac{PV}{P_0V_0} = \frac{nRT}{P_0V_0}$$

Giving

$$P_RV_R = T_R \qquad (4) \qquad 1$$

4

(b) In the $P_R - V_R$ graph (3) is represented by a straight line, begining at $V_R = 1$ and ending at $V_R = 2$, when all mercury expelled.



Q5

(b) $T_{R1} > T_{R2} > T_{R3} \ldots$

(i) Thick straight line between (2,1) and (1,2) gives behaviour of air in the tube 2

(ii) Isotherms indicated by curves "$P_R V_R = T_R$", $T_{R1} > T_{R2} > T_{R3} \ldots$ 2

The isotherms and the line $P_R = 3 - V_R$ are symmetrical about the line $P_R = V_R$, which passes through the origin. 2

The highest temperature isotherm will touch the line $P_R = 3 - V_R$ at its mid point.
This occurs at

$$\underline{P_R = V_R = \tfrac{3}{2}}$$

Thus from (4) this occurs at

$$\underline{T_R = \tfrac{9}{4}}$$

2

8

(c)

$$\tfrac{5}{2}\Delta T_R = S_R \Delta T_R - P_R \Delta V_R. \qquad (5)$$

From (3) and (4)

$$T_R = V_R(3 - V_R)$$

Hence

$$\Delta T_R = \Delta V_R (3 - 2V_R)$$

2

Substituting into (5)

$$\tfrac{5}{2}\Delta T_R = S_R \Delta T_R - \left(\frac{3 - V_R}{3 - 2V_R}\right)\Delta T_R$$

1

Thus

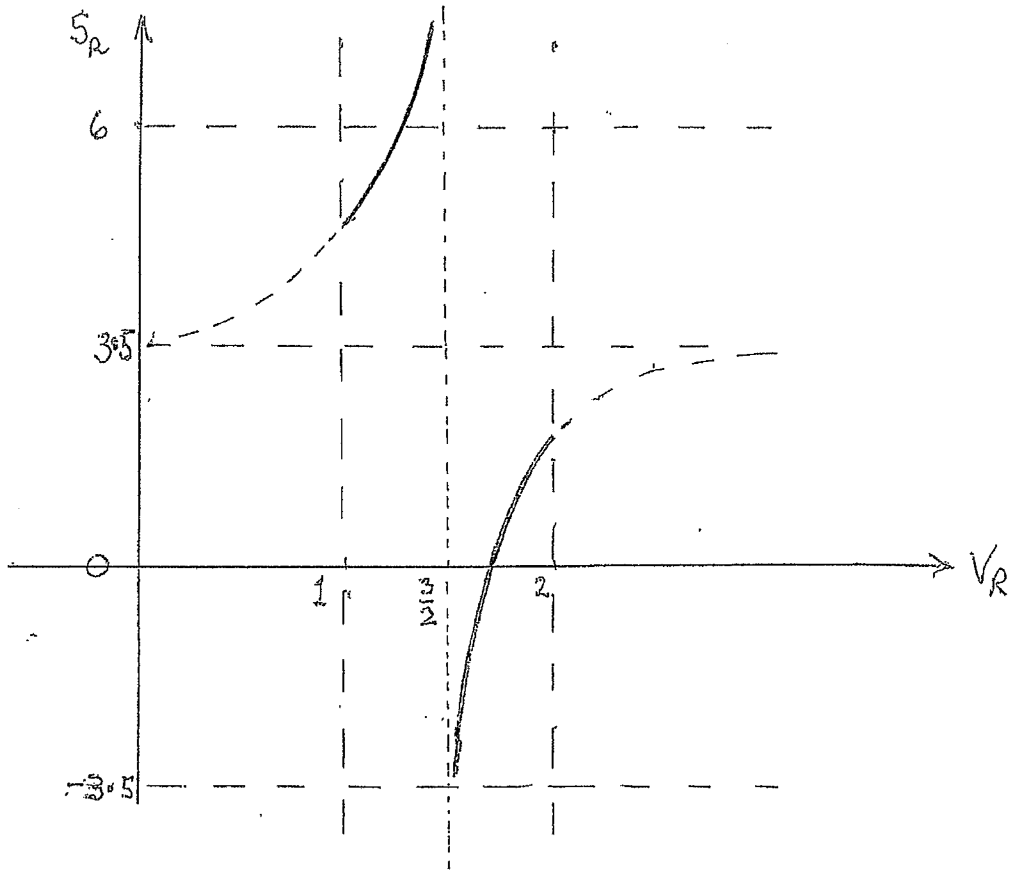
$$\underline{S_R = \left(\frac{21 - 12V_R}{6 - 4V_R}\right)}$$

1

4

Q5

(d)



Physical regions bold curves

Correct sketch asymptotic to $V_R = \frac{3}{2}$ (above and below) 2

Bounded by $V_R = 1$ and $V_R = 2$ 2

4

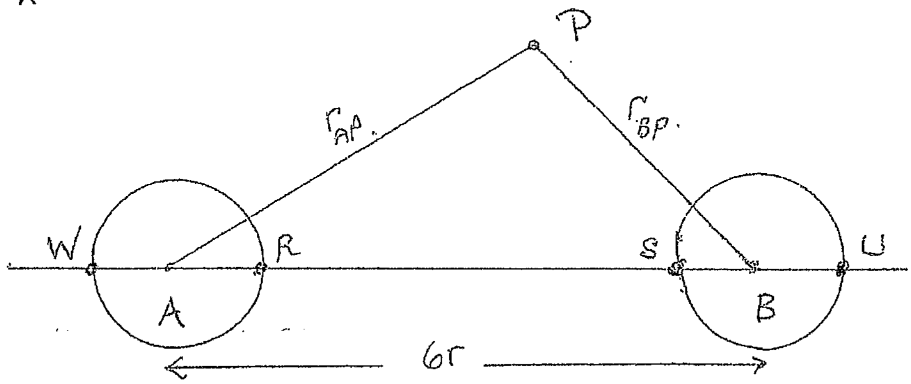
Q6

(i) (a) The surface potential of B, V_B, depends on the sum of the potentials from B and A. If r_A is the distance from the centre of A to a point on the surface of B then

$$V_B = \frac{-GM}{r} - \frac{GM}{r_A} = -GM\left(\frac{1}{r} + \frac{1}{r_A}\right) \quad (1)$$ 1

This will vary over the surface of B due to the variation in r_A. 1

(ii)



At any point P, external to the stars, the potential V_P is given by; using notation in the diagram,

$$V_P = \frac{-GM}{r_{AP}} + \frac{-GM}{r_{BP}} = -GM\left(\frac{1}{r_{AP}} + \frac{1}{r_{BP}}\right)$$ 1

NOTE: V_P is negative

This will be largest when r_{AP} and r_{BP} are largest. 1
This occurs when r_{AP} and r_{BP} are infinite and $V_P = 0$

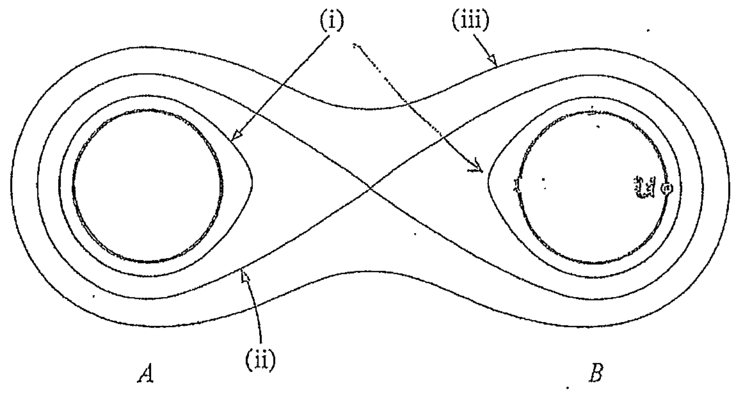
It will be smallest when $\left(\frac{1}{r_{AP}} + \frac{1}{r_{BP}}\right)$ is largest, as V_P negative. This occurs when r_{AP} and r_{BP} are small, and P is along the line RS at R or S. 2

(iii) From (1) this will occur at U when r_{AU} is largest and similarly on B at W, where r_{BW} is largest 2

(iv) $V = -\frac{2GM}{3r}$ (V potential at midpoint) 1

9

(b)



EQUIPOTENTIALS

(i) $V_1 = -10GM/3r$ 1

(ii) $V_2 = -2GM/3r$ 2

(iii) $V_3 = -GM/3r$ 1

4

(c). Launch from U, which has the largest surface potential. 1
To be captured by A it must be able to overcome the potential at (ii), on diagram. So conservation of energy requires it has a speed v such that 1

$$\tfrac{1}{2}mv^2 + V_U m \geqslant -\tfrac{2}{3}\frac{GMm}{r}$$ 1

where V_U is the potential at U – location of max. potential.

Now

$$V_U = -\frac{GM}{r} - \frac{GM}{7r} = -\frac{8GM}{7r}$$ 1

Thus

$$\tfrac{1}{2}mv^2 \geqslant \frac{8GMm}{7r} - \frac{2}{3}\frac{GMm}{r}$$ 1

$$v^2 \geqslant \frac{20GM}{21r}$$ 1

Giving minimum v,

$$v_{min} = \sqrt{\frac{20GM}{21r}}$$ 1

7