BRITISH PHYSICS OLYMPIAD 2016-17

BPhO Round 1

Section 2

18th November 2016

This question paper must not be taken out of the exam room.

Instructions

Time: 1 hour 20 minutes on this section.

Questions: Only two out of the seven questions in Section 2 should be attempted. Both Qu 5 and Qu 6 are on Gravity - you may choose to answer **EITHER** Qu 5 **OR** Qu 6 but **not both**.

Working: working, calculations and explanations, properly laid out, clearly legible, must be shown for full credit. The final answer alone is not sufficient. If derivations are required, they must be mathematically supported, with any approximations stated and justified.

Marks: Students are recommended to spend about 40 minutes on each question. The maximum mark for each question is 20.

Solutions: answers and calculations are to be written on loose paper or in examination booklets. Graph paper and formula sheets should also be made available. Students should ensure that their **name** and their **school** are clearly written on each and every answer sheet.

Setting the paper: There are two options for setting BPhO Round 1:

- *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes.
- *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes allocated for each section. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Answer TWO questions only from this section (and only one from Qu 5 and Qu 6 if you choose one of those)

***** A table of useful constants can be found inside the back page *****

Question 2.

(a) Ideal cells of EMF $\mathcal{E}_1, \mathcal{E}_2, \mathcal{E}_3$ are connected to a wire of resistance R and of length ℓ between ends A and B as shown in **Figure 2(a)**.

(i) With switches K_1 and K_2 initially open, write down the potential $\mathcal{E}$ at length $\ell_1 < \ell$ from A in terms of $\mathcal{E}_1, \ell$ and ℓ_1.

(ii) The cells $\mathcal{E}_2$ and $\mathcal{E}_3$ are chosen such that $\mathcal{E}_1 = \mathcal{E}_2 + \mathcal{E}_3$. When K_1 and K_2 are closed, the galvanometer G (a very sensitive ammeter) is connected between A and B at a distance ℓ_2 from A so that no current flows through it. Write down equations relating

a. $\mathcal{E}_1, \mathcal{E}_2, \ell$ and ℓ_2

b. $\mathcal{E}_1, \mathcal{E}_3, \ell$ and ℓ_2

[2]

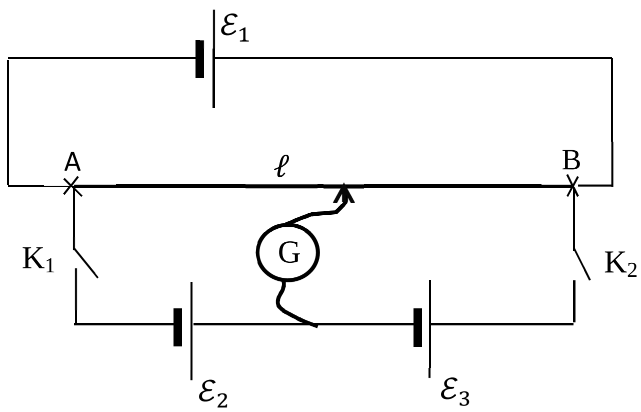

Figure 2(a). Cells connected to a resistance wire.

(b) The circuit in **Figure 2(b)** enables the measurement of the EMF, $\mathcal{E}$, of a thermocouple T (a temperature dependent source of a small EMF). AB is a uniform wire of length 1.00 m and resistance 2.00 Ω. K_1 and K_2 are switches.

No current flows through G when:

(i) K_1 is closed and K_2 is open and AC = 90.0 cm.

(ii) K_2 is closed and K_1 is open and AC = 45.0 cm.

Determine the EMF, ε, and the resistance R. **[8]**

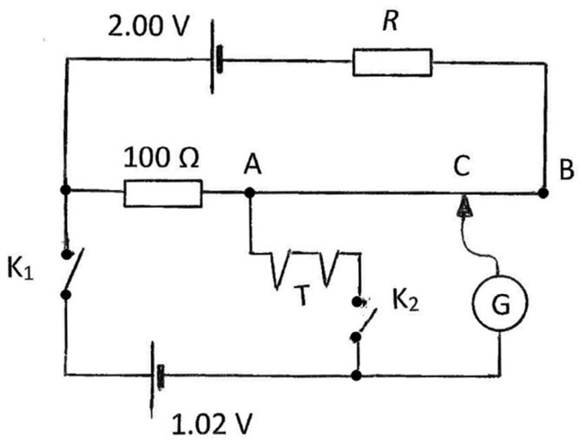

Figure 2(b).

(c) The potential difference across a filament lamp, V , is related to the current I through it by

$$V = 2I + 8I^2$$

The lamp is connected to a measurement arm of a Wheatstone bridge, the circuit shown in **Figure 2(c)**. The other arms are each of resistance 4 Ω. Determine the value of the voltage across the bridge, V_b (> 0), necessary for the bridge to be balanced i.e. when no current flows through G.

[5]

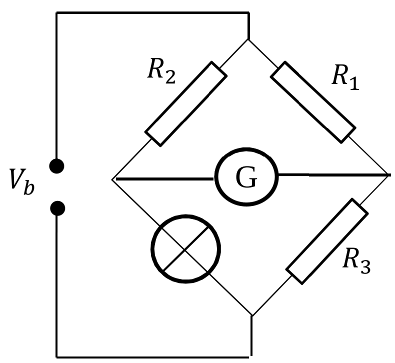

Figure 2(c). Wheatstone Bridge circuit.

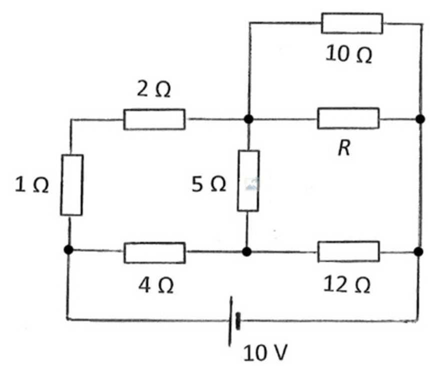

Figure 2(d).

(d) In the circuit, **Figure 2(d)**, determine the value of resistance R required to minimise the heat generated in the 5 Ω resistor.

[5]

Question 3.

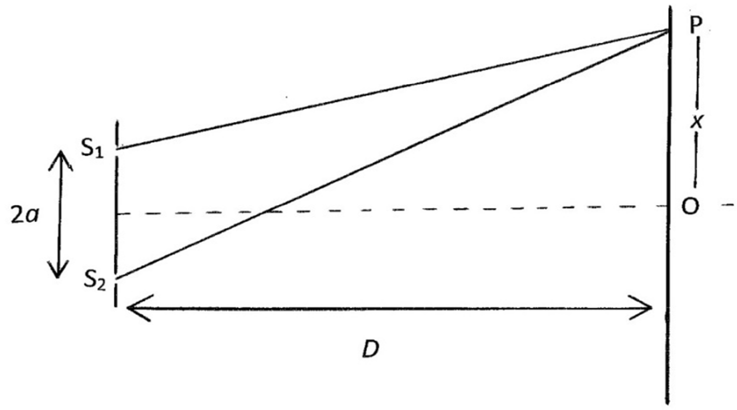

Figure 3(a).

(a) In a Young's slit experiment the slits, S_1 and S_2, are a distance $2a$ apart and the screen is at a distance D from the line joining the slits , **Figure 3(a)**. P is a point on the screen a distance x from the line of symmetry of the system.

(i) Derive an expression for the spacing between adjacent fringes, Δx, for light of wavelength λ, in terms of a, x and D in the approximation $x, a \ll D$.

[10]

(ii) If white light was passed through the slits, sketch or describe what would be seen near the centre of the screen.

(iii) A laser is often used to illuminate the two slits. Before the invention of the laser, a light bulb with a small filament and with a colour filter was used to illuminate a single slit in front of the double slit in **Figure 3(b)**. Explain the purpose of the single slit. Your answer should consider the effect of *coherence*.

[4]

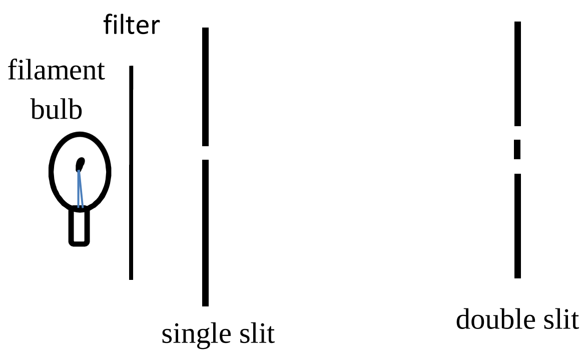

Figure 3(b). Filament bulb and single slit to illuminate the double slit.

(b) Two whistles, both of frequency f = 2.00 kHz, are situated 3.00 m apart and blown simultaneously. An observer travelling along a line parallel to the line joining the whistles, and at a distance approximately 20.0 m opposite the whistles, detects minima in sound intensity at a series of points spaced 1.14 m apart. Calculate the speed of sound in air. **[6]**

(For small ε, $(1+\varepsilon)^{\alpha} = 1 + \alpha\varepsilon + \cdots$)

Question 4.

(a) Fixed charges of +*Q* are situated at the corners of

(i) a square of side $2a$ and

(ii) a cube of side $2a$.

The square and cube each have a charge of $-Q$ at their centre. Determine the *total potential energy* of each system. Sketch diagrams of the charge arrangements for each example.

[9]

(b)

(i) Obtain the force, F, on the charge $-Q$ when it is displaced a distance x from the centre of the square along a line through its centre, and perpendicular to the plane of the square.

Sketch a graph of F against x.

[5]

(ii) If the charge $-Q$, mass m, in (b)(i), performs simple harmonic motion about the centre of the square, determine the period, T, of the motion. If the motion is simple harmonic, obtain the necessary condition for the amplitude of the motion, A.

[6]

(For small x, $(1+x)^{\alpha} = 1 + \alpha x + \ \ldots$)

Question 5. (You may answer **EITHER** Qu 5 **OR** Qu 6 but not both)

Galaxies are a large collection of stars and gas that are held together by their own gravity. One class of galaxies, spiral galaxies, are characterised by a spherical bulge and a flattened disk in which the stars form a spiral pattern. The stars and gas in a spiral galaxy trace out circular orbits around the centre of the galaxy. A schematic of a spiral galaxy, the Milky Way, can be seen below in **Figure 5**.

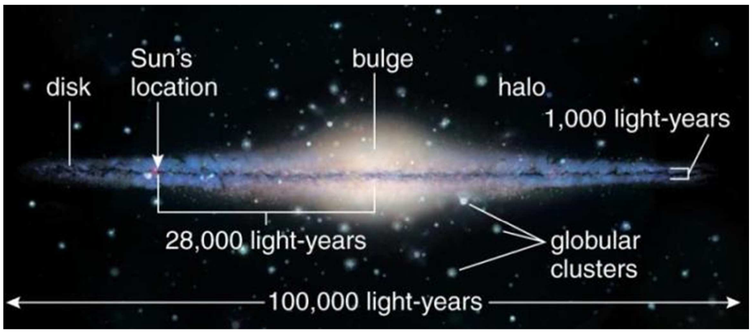

Figure 5. Schematic of the Milky Way galaxy.

(a) Using the gravitational force acing on a star moving in a circular orbit and assuming spherical symmetry, determine the rotational velocity (v_{rot}) of a star as a function of the mass enclosed within a radius r. Only the mass of material contained within the orbital radius of the star contributes to the gravitational force of attraction.

[2]

(b) Assuming that most of the mass of the galaxy is contained within the spherical bulge of radius r_0, with uniform density ρ_0, determine the shape of the rotation curve (the graph of v_{rot} against r) inside and outside r_0. Sketch the expected rotation curve of the galaxy.

[7]

(c) Astronomers have observed the rotation curves of galaxies and found a significant discrepancy between what is measured and the theoretical prediction. They found that the rotation velocity curve was flat at $r > r_0$, which cannot be explained by a centrally dominating mass. Another component was suggested to make up for the missing, invisible mass, called "dark matter". Assuming a more realistic density profile for the spherically distributed dark matter:

$$\rho(r) = \rho_0 \left(\frac{r}{r_0}\right)^{-\alpha}$$

Determine the exponent, α, needed such that the rotation curve is flat at $r > r_0$, as seen from measurements.

[4]

(d) At $r = 2.8 \times 10^5$ light years distant from the centre of the Milky Way, our Sun has a measured velocity of $v_{\mathrm{rot}} = 220\ \mathrm{km\ s^{-1}}$ and an expected velocity of $v_{\mathrm{exp}} = 70\ \mathrm{km\ s^{-1}}$. Calculate the visible as well as the true mass of the Milky Way. What is the percentage of dark matter in the galaxy? Express your answer in units of solar masses, $M_{\odot}$.

$M_{\odot} = 1.99 \times 10^{30}\ \mathrm{kg}$. 1 light year is the distance light travels in a year.

[3]

(e) Although dark matter is now the widely accepted explanation for the rotation curve problem, an alternative has been suggested: modifying the laws of gravity. Modified Newtonian Dynamics (MOND) postulates that Newton's third law can be re-written as $F = m\mu\left(\frac{a}{a_0}\right)a$ where μ is a function with the property that $\mu(x) = 1$ for $x \gg 1$ and $\mu(x) = x$, for $x \ll 1$, a is the acceleration and a_0 is a constant that marks the transition between Newtonian gravity and MOND. In the limits of small acceleration $(a \ll a_0)$, show that this formulation also leads to a flat rotation curve at large radii.

[4]

Question 6. (You may answer **EITHER** Qu 5 **OR** Qu 6 but not both)

(a) Kepler's 3rd Law requires that planets in orbit around the Sun, with radius R and period T, have the same value of $\frac{R^{\alpha}}{T^{\gamma}}$, where α and γ are constants. Assuming the orbits of the Earth and Jupiter are circular:

(i) Calculate the ratio $\frac{\alpha}{\gamma}$ by using the data provided below.

The radii of the orbits of the Earth and Jupiter are respectively 1.50 x 10^{11} m and 7.76 x 10^{11} m. Orbital period of Jupiter is 11.8 years.

(ii) Using the data above, show that the mass of the Sun is about 2.0 x 10^{30} kg.

[8]

(b) A satellite is launched into a synchronous orbit of the Earth. Determine its radius of orbit, R, and speed, v_s.

[4]

(c) Satellites launched from Earth speed up slowly as they take off from the launch pad, which uses up a lot of fuel (energy) in the process. This is inefficient but it is the only method at present. Physicists use the idea of escape velocity instead.

(i) What is meant by escape velocity?

(ii) Determine a value for the escape velocity, v_{E}, from the Earth.

(iii) In what way does the location of the launch site on the Earth's surface affect the escape velocity, v_{E} ?

(iv) Determine the minimum initial launch speed from the Earth.

[6]

(d) Ignoring any effect of the launch site location on the Earth in c(i), what is the minimum escape velocity when launching from the Earth to escape the gravitational field of the Sun?

[2]

Radius of the Earth $r_{\mathrm{E}} = 6.38 \times 10^{6}$ m, mass of the Earth $M_{\mathrm{E}} = 5.98 \times 10^{24}$ kg.

Question 7.

Millikan's oil drop experiment, illustrated in **Figure 7**, was the first experiment to determine the size of the elementary charge, e. Charged oil droplets are sprayed into an air filled chamber and observed through a microscope inserted in the side of the chamber. The droplets fall under gravity with a terminal velocity. A uniform electric field between the top and bottom plates could be used to hold the charged oil drops in a fixed vertical position in the electric field. The potential between the plates was measured. The polarity could be changed in order to pull them back up against gravity. The drag force on them due to air resistance is known as viscous drag. The charge on an oil drop may spontaneously change due to random ionisation.

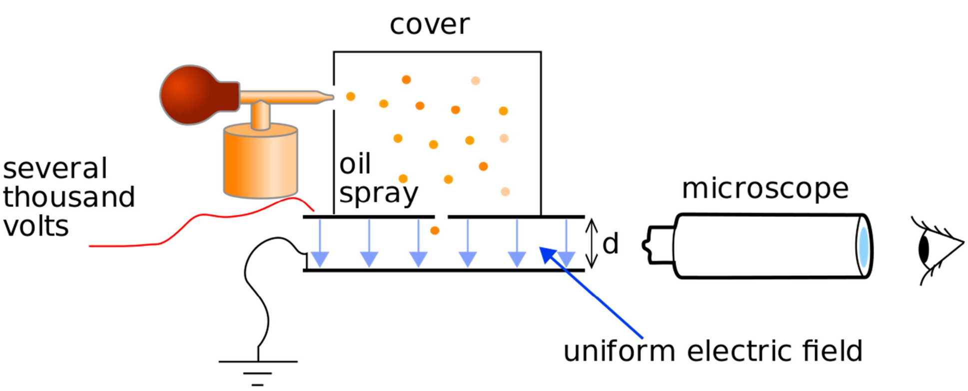

Figure 7. Millikan oil drop experiment apparatus (schematic).

Source: https://en.wikipedia.org/wiki/Oil_drop_experiment

(a) The viscous drag force on a small spherical droplet is given by $F_v = 6\pi\eta a u$ where a is the radius of the drop, u is the terminal velocity of the drop, η is the viscosity of the air. $\eta = 1.82 \times 10^{-5}\ \mathrm{kg\ m^{-1}\ s^{-1}}$.

In two successive measurements of an oil droplet, the rise times were 42 s and 78 s. The distance travelled upwards at constant velocity was 1.00 cm, the potential difference across the plates was 5000 V, and the plate separation was 1.50 cm. The radius of the drop was $a = 2.76 \times 10^{-6}$ m.

Calculate the change in the number of electrons in the drop between the two observations.

[8]

(b) Two drops of the same density, with radii r_1 and r_2, both with identical charges Q and terminal velocities u_1 and u_2 respectively, coalesce. How is the final terminal velocity v of the drop related to the initial velocities?

[5]

(c) A separate experiment to measure e/m_e for an electron could be used to enable the electron mass, m_e, to be determined.

A potential difference of 3000 V is maintained between deflector plates, with separation 2.0 cm, in an evacuated tube. A magnetic field of 2.5×10^{-3} T at right angles to the electric field gives no deflection to an electron beam entering the fields. The electron beam received an initial kinetic energy by being accelerated through a potential difference of 10 000 V. Calculate the ratio e/m_e.

[7]

Question 8.

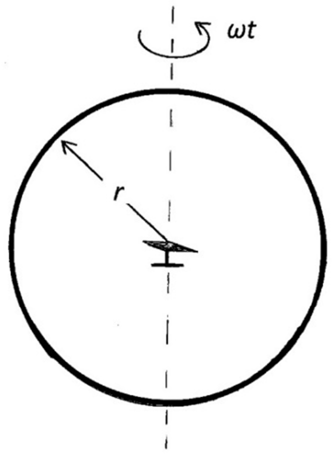

Figure 8.

A circular copper ring, of radius $r = 0.125$ m and resistance R, rotates about a vertical diameter with constant angular velocity, ω, radians per second. A small magnetic compass needle, which can rotate about a vertical axis, is situated in the middle of the ring, **Figure 8**. When the ring is stationary, the needle points in the direction of the horizontal component of the Earth's magnetic field, $\boldsymbol{B}$. However, when it rotates at the rate of 10 revolutions per second, the compass needle deviates by an average angle of $\alpha = 2.00^{\mathrm{o}}$ from this orientation. At time $t = 0$ the ring is in a plane perpendicular to $\boldsymbol{B}$.

Determine, algebraically:

(a) the magnetic flux ,Φ, through the ring at time t. **[2]**

(b) the current, I, in the ring. **[3]**

(c) the magnetic field components of the ring, parallel and perpendicular to $\boldsymbol{B}$. **[3]**

(d) the average values, over time, of these components. **[7]**

(e) Hence determine, numerically, R. **[5]**

$$(\ 2\sin^2\theta = 1 - \cos 2\theta, \quad \sin 2\theta = 2\sin\theta\cos\theta \ )$$

field at the centre of a circular loop $\quad B = \dfrac{\mu_0 I}{2\pi r}$

END OF SECTION 2

Important Constants

Speed of light	c	3.00×10^{8}	m s^{-1}
Planck constant	h	6.63×10^{-34}	J s
Electronic charge	e	1.60×10^{-19}	C
Mass of electron	m_e	9.11×10^{-31}	kg
Gravitational constant	G	6.67×10^{-11}	N m^2 kg^{-2}
Acceleration of free fall	g	9.81	m s^{-2}
Permittivity of a vacuum	ε_0	8.85×10^{-12}	F m^{-1}
Avogadro constant	N_A	6.02×10^{23}	mol^{-1}

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