BPhO
British Physics Olympiad

British Physics Olympiad 2017-18

BPhO Round 1

Section 2

17th November 2017

This question paper must not be taken out of the exam room

Instructions

Time: **5 minutes reading time (NO writing)** and **1 hour 20 minutes** for writing on this section.

Questions: Only **two questions** out of the five questions in *Section 2* should be attempted.
Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations and diagrams, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified.

Marks: Students are recommended to spend about 40 minutes on each question. The maximum mark for each question in *Section 2* is 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Graph paper and formula sheets should also be made available. Students should ensure that their **name** and their **school** are clearly written on each and every answer sheet.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes plus 10 minutes reading time.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes plus 5 minutes reading time allocated for each section. If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.67 \times 10^{-27}\,\mathrm{kg}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of Earth	M_{E}	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$

Section 2

Question 2

a) A $50\,\mathrm{Hz}$ alternating current of $10\,\mathrm{A}$ (rms) passes along a copper wire whose diameter is $1.0\,\mathrm{mm}$. If copper contains 9.0×10^{22} free electrons per cm^3, estimate

(i) the maximum speed of the electrons, and

(ii) the maximum displacement of the electrons associated with the current.

(6)

b) A wire of length l and resistance R is extended by an amount ε such that $\varepsilon \ll l$. If the volume of the wire and its resistivity remain unchanged,

(i) Give an expression for the fractional change in $R, i.e.\ \dfrac{\delta R}{R}$.

(ii) Calculate the new resistance of the wire if $R = 100.0\,\Omega$ and $\varepsilon/l = 0.001$.

(6)

c) A non-linear electrical component is connected in a circuit in series with a supply of emf $\mathcal{E} = 300\,\mathrm{V}$ and negligible internal resistance, and a resistor R of $4.0\,\mathrm{k\Omega}$. The relation between the current I through the non-linear component, and the potential difference across it, V_c, is given by $I = AV_\mathrm{c} + BV_\mathrm{c}^2$ where $A = 0.070\,\mathrm{mA\,V^{-1}}$, $B = 0.0050\,\mathrm{mA\,V^{-2}}$.

(i) Obtain an algebraic expression for V_c.

(ii) Determine the current I through the non-linear component.

(6)

d) A metal hemisphere of radius $0.10\,\mathrm{m}$ is immersed near the centre of a large conducting tank containing a liquid of resistivity $60\,\Omega\,\mathrm{m}$. The plane surface of the hemisphere is level with the surface of the liquid.

(i) Find an expression for the resistance between two hemispherical shells within the liquid, concentric with the centre of the metal hemisphere, having radii r and $(r + \delta r)$ respectively.

(ii) Calculate the resistance between the hemisphere and the tank.

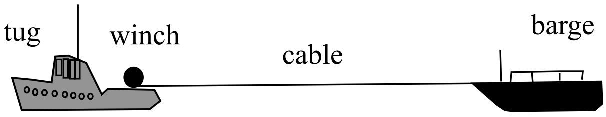
(7)

[25 marks]

Question 3

A tug boat winch is fitted with friction brakes that slip when the tension in the winch cable reaches $50\,\text{kN}$. The winch is mounted on a 450 tonne tug which is to tow a floating barge of mass 6300 tonne. The barge is initially at rest while the tug moves away from it at. The speed of the tug at the moment that the cable becomes taut is $2.5\,\text{m}\,\text{s}^{-1}$. At that moment the winch friction brakes start slipping in order to prevent the cable from breaking. The thrust exerted on the tug by its propeller is $35\,\text{kN}$. The winch and arrangement might look something like **Figure 1a** and **Figure 1b**:

(a) *The winch can be seen in the lower part of the picture. Photo: Casey Ocean Systems*



(b)

Figure 1

a) Calculate

(i) The time during which the brakes are slipping,

(ii) The common velocity of the tug and the barge when the brakes stop slipping.

(iii) What length of cable runs out of the drum of the winch during the time the brakes are slipping?

(9)

b) In a different geometry the winch is used to haul sand up from the bottom of a dry dock using a crane constructed from a very light framework of girders. The crane geometry can be seen in the diagram of **Figure 2**. The crane is fixed to the horizontal dock side at points A and B. The dimensions of the sides CB, CD and BD are as shown, the girder CB is vertical, and the winch is hauling a load of weight $15W$.

(i) Find the magnitude of the force exerted on D by the member BD,

(ii) The tension in the member CD.

(iii) If the force on the crane at A by the ground acts vertically downwards and has a magnitude $31W$, find the distance AB in terms of L.

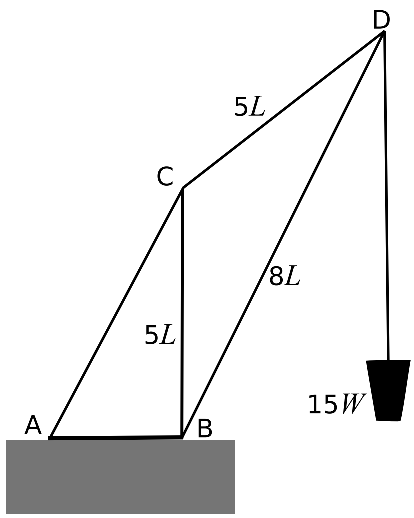


Figure 2

(8)

c) The bucket of mass m is drawn up by the winch cable which exerts a steady force F upwards, shown in **Figure 3**. The bucket starts from rest and initially contains a mass m_0 of sand. The sand leaks out at a constant rate so that the bucket is empty after time t, which occurs before it reaches the top of the dry dock wall.

In terms of F, m_0, m, t and g (the gravitational field strength), what is the velocity v of the bucket when it is just empty? The friction brakes do not slip.

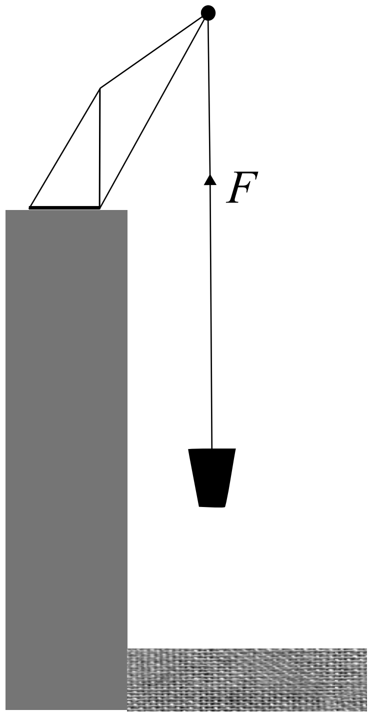


Figure 3

(8)

$$\text{You may use} \quad \int \frac{\mathrm{d}x}{(kx+a)} = \frac{1}{k}\ln(kx+a) + \mathrm{c}$$

[25 marks]

Question 4

a) Calculate the smallest angle of incidence for light incident on the face of a $60°$ prism of refractive index 1.5, such that all of the light on the second face is totally reflected.
Draw a large diagram to show the path of the light and the angles.

(5)

b) Consider the step index optical fibre shown in **Figure 4**.

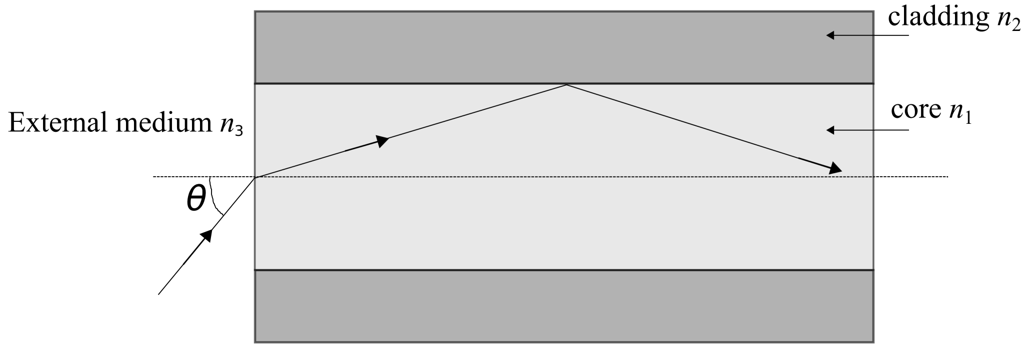


Figure 4

The core, cladding and external medium have refractive indices n_1, n_2, and n_3, respectively. The acceptance angle θ for light entering the core of the fibre will allow transmission without loss providing the angle θ between the incident ray and the axis is smaller than the maximum acceptance angle θ_m.

(i) Obtain an expression for $\sin\theta_m$ in terms of n_1, n_2, and n_3.
Hint: $\cos^2\theta = 1 - \sin^2\theta$

(ii) Light enters the fibre optic at angles between $\theta = 0°$ and θ_m. Given a step index fibre with a core refractive index of 1.48585, and a cladding refractive index of 1.45641, determine the time broadening of the leading edge of a square pulse over a distance of 100 km.

(iii) If multiple, very brief, square pulses of light are sent down the cable at frequency f_p, calculate the maximum possible pulse frequency over this 100 km distance, such that the pulses do not overlap.

(iv) Show that in general the time spread of a pulse Δt, in a fibre of length L, is given by $\Delta t = \dfrac{L}{c' n_2}\Delta n$, with Δn the difference in refractive indices between core and cladding, and c' the speed of light in the optical fibre.

(12)

c) A ray of light from a point source **O** on the principal axis of a thin converging lens, passing through the lens, is brought to a focus at the image point **P** also on the principal axis, and shown in **Figure 5**.

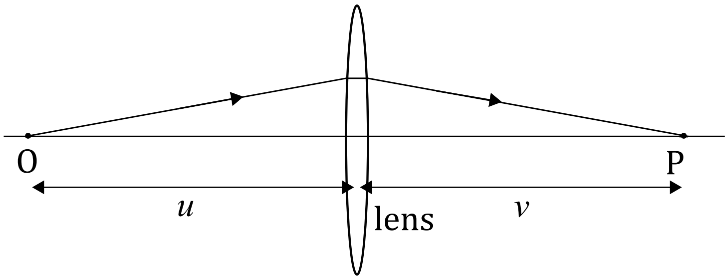


Figure 5

One form of an equation for a thin lens, relating object and image distances, u, v to the focal length f is

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

By taking the limit $u \to \infty$, explain what the length f represents for a lens.

(2)

d) Rays of light from a point source on the principal axis, passing through a converging lens by different paths and reaching a focus point on the principal axis, can be considered to do so by following paths which all have an **equal time of travel**. (This is a simplified form of a more general principle — Fermat's Principle of Least Time).

A flat, thin, circular disc of transparent material is made so that its refractive index n varies with the distance r from the principal axis normal to the centre of the disc, according to the equation $n = n_\mathrm{o}(1 - kr^2)$, where n_o and k are constant.

(i) By considering the times taken by light travelling by different paths (take a path not on the principal axis), show for rays travelling very close to the principal axis that the thin disc can behave as a lens. i.e. obtain an expression for $\frac{1}{u} + \frac{1}{v}$, if the thickness of the disc is d, with $d \ll u, v$.

(ii) Derive an expression for its focal length in terms of d, k and n_o.

(6)

[25 marks]

Question 5

a) A simple pendulum of string length l, with a bob of mass m, containing charge Q, is in a horizontal electric field of magnitude E.

(i) Determine the angle of the string with respect to the vertical when the bob is in equilibrium.

(ii) If the electric field now points vertically downwards and there is a small negative charge on the bob, $-Q$, such that the string remains taut, what would be the period of oscillation of the bob once perturbed?

(iii) Give the condition relating Q, E, m and g, such that oscillations will occur when the bob is perturbed.

(6)

b) Equal and opposite charges $+Q$ and $-Q$ are a distance of $2a$ apart. Taking the midpoint between them as the origin O, find the electric potential and the magnitude of the field strength, at a point P which is distance $r(r \gg a)$ from O, when

(i) P is on the line passing through the charges;

(ii) When P is on a perpendicular bisector of the line joining the charges.

(10)

c) Equal and opposite charges separated by a small distance constitute a dipole. A hydrogen chloride molecule may be treated as a dipole in which an electron is separated from a positive charge of equal magnitude by a distance of $2 \times 10^{-11}\,\mathrm{m}$. Calculate the work done needed to turn the molecule, which is lying parallel to a field of $3 \times 10^{5}\,\mathrm{V\,m^{-1}}$, through an angle of $180°$.

(2)

d) In a simple model of a thundercloud, an upper cylindrical shaped region (axis vertical) contains a net positve charge and the lower cylindrical shaped region of the cloud a net negative charge, as shown in **Figure 6**. The upper region, from 9 km to 12 km height, with a radius of 5 km has a charge density of $0.21\,\mathrm{nC\,m^{-3}}$, whilst the lower region from 2 km to 9 km high, also with a radius of 5 km, has a charge density of $-0.18\,\mathrm{nC\,m^{-3}}$.

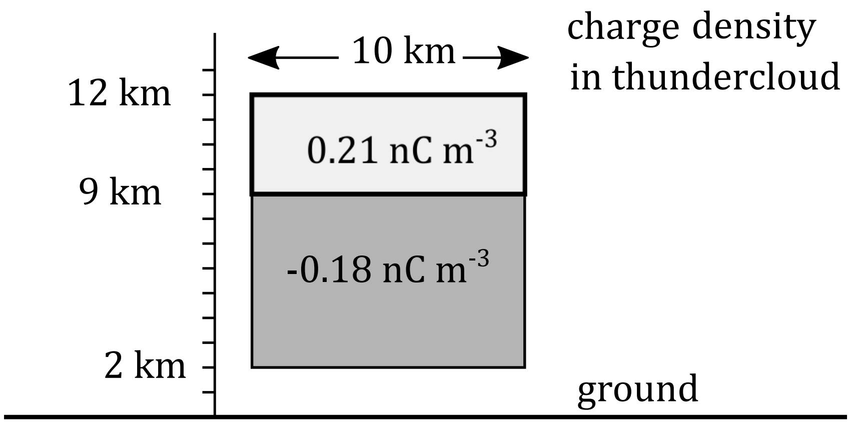


Figure 6: ref. Mazur V, LH Runhke: JGR, v103, D18, pp23299-23308, 1998

(i) Calculate the magnitudes of the positive and negative charges in each region.

In order to determine the field strength at the ground due to these charges, we can aproximate these two volumes of charge as two point charges at the centres of each of the two regions (at heights 10.5 km and 5.5 km respectively). The ground is a good conducting plane surface and so charges will be induced in the surface. The effect of this induced charge can be simulated by considering a charge of equal magnitude but opposite sign at the same distance h below the surface as the real charge at height h above the surface. The field at the surface is then calculated from the field of the real charge at the surface added to the field due to the image charge at the surface.

(ii) For the two charges calculated, with their two image charges of opposite signs, calculate the strength of the electric field at the surface of the Earth directly below the thundercloud.

(iii) If a two metre tall person stands with his feet on the ground in this region, what is the magnitude and polarity of the potential at his head? Take the potential of the ground to be 0 V.

(7)

[25 marks]

Question 6

a) Sea waves of frequency f are moving North with velocity c. A small boat moves South with a velocity u, both velocities being measured with respect to the land. By counting the number of waves moving past the boat in a time Δt or otherwise, show that the boat will be moving up and down with a frequency f' given by

$$f' = \frac{(u+c)f}{c}$$

(4)

b) (i) A source of sound, emitting a single note of frequency f is moving towards a stationary observer with speed v_{s}. Show that if the air is stationary, the frequency f' detected by the observer is given by

$$f' = \frac{f}{(1 - \frac{v_{\mathrm{s}}}{c})}$$

where c is the speed of sound in air. You may wish to consider a time Δt, and how many waves are emitted by the source, how far the waves and the source each move in this time, the wavelength the observer then detects, or any other approach.

(ii) What would be the value of $\Delta f = f' - f$ if the source is moving away from the observer at a speed $v_{\mathrm{s}} = c/2$?

(iii) If $v_{\mathrm{s}} \ll c$, show that $\dfrac{\Delta f}{f} \approx \dfrac{v_{\mathrm{s}}}{c}$

(6)

c) The above formula can be applied to electromagnetic waves.

(i) Show that the observed frequency of an absorption line of emitted frequency f in the spectrum of the Sun should vary between $f + \delta f$ and $f - \delta f$ where

$$\delta f = \frac{2\pi R f}{Tc}$$

in which R is the radius of the Sun, and T the period of rotation.

(ii) Given that the density of the Sun is $1.4 \times 10^3\,\mathrm{kg\,m^{-3}}$, the mass is $2.0 \times 10^{30}\,\mathrm{kg}$ and the equatorial rotation period is 24.5 days, estimate the fractional frequency shift, $\dfrac{\delta f}{f}$, for the peak in the visible spectrum at $550\,\mathrm{nm}$.

(5)

d) Two stars labelled A and B separated by a distance $a = 19.8\,\mathrm{AU}$ form a binary pair, gravitationally bound, orbiting a common centre of mass with a period $T = 50.1\,\mathrm{y}$. Star A, with mass m_{A} is at radius R_{A} from the centre of mass, and star B, a very faint white dwarf, of mass m_{B} orbits at radius R_{B}. The stars have circular orbits with the plane of the orbit lying along the line of sight from the Earth. Only the brightest of the two stars can be observed through a telescope, and it is found that a spectral line of 656.3 nm in the lab varies by $\pm 0.0235\,\mathrm{nm}$ when observed in the bright star.

(i) Show that $T^2 = \dfrac{4\pi^2}{GM} a^3$ where $M = m_{\mathrm{A}} + m_{\mathrm{B}}$ is the total mass of the binary pair.

(ii) From the figures given, calculate a value for the total mass of the system.

(iii) Show that $v_{\mathrm{A}} = \dfrac{GT}{2\pi a^2} m_{\mathrm{B}}$ where v_{A} is the orbital velocity of star A.

(iv) From the spectral line shift, calculate mass m_{B} and hence the mass of star A, m_{A}.

(v) Hence calculate the ratio of the orbital radii, $\dfrac{R_{\mathrm{A}}}{R_{\mathrm{B}}}$.

$$1\,\mathrm{AU} = 1.50 \times 10^{11}\,\mathrm{m}$$

(10)

[25 marks]

END OF SECTION 2

BPhO Sponsors