BPhO Round 1 Marking- November 2018

Thank you for taking part in marking the scripts. It is of enormous benefit to young students to be able to take part in these competitions, tackling much harder problems than they normally get, and be able to have them marked by physicists who know what they are doing. It is because of your expertise in the subject that it is possible to do this. Exams marked by non-specialists are more of a tick box exercise, which is of little value in stretching students to grasp the subject at a deeper level. The layout of the work may be annoying. That is a national problem and we are not going to change that easily, although we do our best, as do the teachers of these students.

- Positive marking is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- Significant figures. This is not a test of significant figures. A leeway of ±1 sig fig is generally allowed, but we are not being strict at all in penalising for sig figs. ; if the published solution gives 3, allow 2 or 4 in the students answer. However, unless they have put down all of the figures on their calculator, there are no questions this year that are likely to invoke a sig fig penalty.
- Some answers can be left in fractional form.
- Units should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Some allowance may be made if it is clear that the unit has been used a line or two earlier.
- If the units are a required part of the answer for the mark, and so must be there.
- Error carried forward (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks. You do not need to spend time working through laborious arithmetic calculations if there is a possible ecf. Just make a decision as to whether they should have the single mark or not.
- You are not required to spend time deciphering scribble.
- There may be a lot of working for the answer. If they are almost there, you may give the mark even if there is a numerical mistake in the last line. Use your judgement. The ticks for the marks are not exact i.e. they are for the idea and almost getting there.
- Full marks are awarded for the correct answer, provided that there is some supporting working and it is not a "show that" question.

If you need advice, email Robin Hughes robin.hughes@physics.ox.ac.uk

You can send a phone photo or just ask a question. We want to be fair in the marking, so your good judgment should be acceptable.

But: 1. Do not mix up students' names when entering marks

2. Add up the marks correctly. Check your addition.

3. Do not leave papers in any unsecured place. The loss of papers would be serious.

Question 1

(a)

$$V = \frac{2\pi r}{T}$$

$$= 2\pi \frac{26 \times 1\,\text{year} \times 3\times10^{8}}{240\times10^{6} \times 1\,\text{year}}$$ ✓ appropriate units converted, or as shown,

$$= 2\pi \times \frac{78\times10^{8}}{2.4\times10^{8}}$$ ✓ correct ratio, or evaluated.

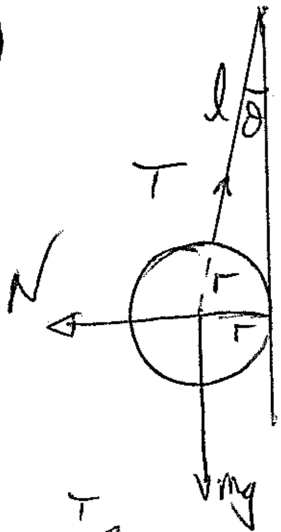
$$= 2\pi \times 32.5$$

$$= \underline{204}\ \text{ms}^{-1} = \underline{200\ \text{ms}^{-1}}$$ ✓

$$= \underline{65\pi\ \text{ms}^{-1}}$$

(3)

(b)



Smooth wall: mg through centre,
so N through centre
so T through centre.

so $\sin\theta = \frac{r}{r+l}$

$$= \frac{6}{6+9} = \underline{0.4}$$ ✓

Resolving vertically: $mg = T\cos\theta$ ✓

$$T = \frac{mg}{\cos\theta} = \frac{0.5\times9.81}{\cos 23.6^\circ}$$

$$= 5.35 = \underline{5.4\ N}$$ ✓

Angle wrong, lose a mark if used correctly.

(3)

(c) (i) $\frac{ds}{dt} = 6t^2 - 18t + 12$

$\dot{s} = 0 \Rightarrow \quad t_0^2 - 3t_0 + 2 = 0$

$t_0 = 1\,s,\ 2\,s$ ✓

(ii) $\frac{d^2s}{dt^2} = 12t - 18$

$\ddot{s} = 0 \Rightarrow \quad t_{a=0} = \frac{3}{2} = 1.5\ s$ ✓

(iii) $\dot{s}$ at $t = \frac{3}{2}\ s$

$\dot{s}_{a=0} = V = 6\left(\frac{3}{2}\right)^2 - 18 \cdot \frac{3}{2} + 12$

$= -\frac{3}{2} = -1.5\ ms^{-1}$ ✓

(iv) $\ddot{s}$ at $t = 1\,s, 2\,s.$

$a_1 = 12t - 18 = -6\ ms^{-2}$ ✓

$a_2 = 12t - 18 = +6\ ms^{-2}$

(4)

(d) Substituting $100 = a \cdot 40 + b \cdot 40^2$ ①

$280 = a \cdot 80 + b\ 80^2$ ②

① × 2 and subtract $80 = b(80^2 - 2 \cdot 40^2)$

$1 = b(80 - 40)$

$b = \frac{1}{40} \frac{m}{(km\,h^{-1})^2}.$ ✓

In eqn. ① $100 = a \cdot 40 + \frac{1}{40} \cdot 40^2$ ✓

$a = 1.5 \frac{m}{km\,h^{-1}}$

Now $500 = 1.5\,V + \frac{1}{40} \cdot V^2$ ✓ – correct approach here even if a, b incorrect.

$40 \cdot 500 = 60V + V^2 \Rightarrow V^2 + 60V - 20000 = 0$

$V = 114.6 = 115\ km/h$ ✓

$(\equiv 31.8\ ms^{-1} \doteq 32\ ms^{-1})$ (4)

(e)

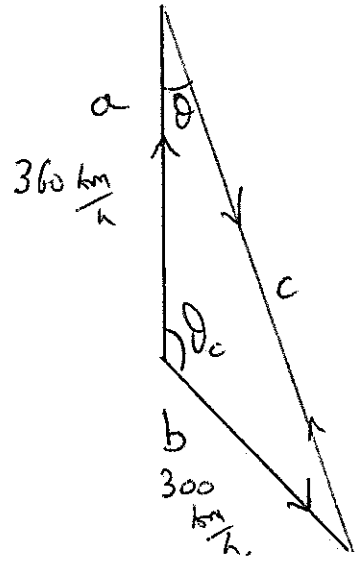


Diagram ✓

length, $a = \frac{2}{3} \times 360 = 240$ km

$b = \frac{2}{3} \times 300 = 200$ km ✓

$\theta_c = 90 + 45 = 135°$

Cosine rule.

$c^2 = 240^2 + 200^2 - 2 \cdot 240 \cdot 200 \cos 135$

$c = 406.80$ km

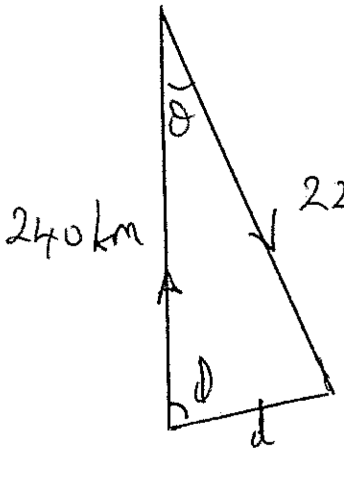
combined speed of approach along c is 660 km/h ✓

Time taken to meet is $\frac{406}{660} = 0.616$ h ✓

$= 36.98$ minutes

In this time, plane 'a' travels $360 \times 0.616 = 221.9$ km

meeting point found. ✓



From the top diagram using sine rule

$\frac{c}{\sin 135} = \frac{b}{\sin \theta}$ $\theta = 20.34°$

Then cosine rule,

$d^2 = 240^2 + 221.9^2 - 2 \cdot 240 \cdot 221.9 \cos 20.34°$

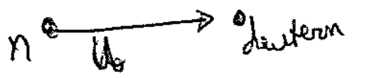
$d = 83.5$ km ✓

Then $\frac{d}{\sin \theta} = \frac{221.9}{\sin \phi}$ $\phi = 67.5°$ ✓

[Need find two answers for (7) marks.
Either one or other gives (6) marks].

(7)

(f) $M = M_n$ $\quad 2m = M_d$.



(i) Cons. of Mom. $\quad Mu_0 = M_d V_d + M_n V_n$

$$u = 2V_d + V_n \quad (1)$$

Cons. of KE $\quad \frac{1}{2}Mu^2 = \frac{1}{2}2mV_d^2 + \frac{1}{2}mV_n^2$

$$u^2 = 2V_d^2 + V_n^2 \quad (2)$$

Solve: from (1), $\quad u - V_n = 2V_d \quad (3)$

(2), $\quad (u - V_n)(u + V_n) = 2V_d^2 \quad (4)$

Since $u \neq V_n$ or V_d is zero and there has been no interaction,

then divide (3) and (4) to obtain $\quad u + V_n = V_d$

With (3) to eliminate V_n, $\quad u - 2V_d = V_n = V_d - u$

$$2u = 3V_d$$

$$\frac{V_d}{u} = \frac{2}{3}$$

(ii) Initial KE is $\frac{1}{2}mu^2$

deuteron KE is $\frac{1}{2}2m \cdot V_d^2 = \frac{1}{2}2m\frac{4}{9} \cdot u^2$

$$\therefore \frac{KE_d}{KE_0} = \frac{8}{9} = 89\% = 0.889$$

(iii) $\frac{8}{9}KE_0$ is given to deuteron.

$\frac{1}{9}KE_0$ left with neutron after each collision.

After N collisions, $\quad \left(\frac{1}{9}\right)^N \cdot \times 10 \times 10^6\ eV = 10^{-2}\ eV$

$$\frac{1}{9^N} = 10^{-9}$$

$$9^N = 10^9$$

$$N\log_{10}9 = 9$$

$$N = 9.4$$

$\therefore$ 10 collisions required

(This result is for head on collisions, but is similar to the general result)

(6)

(g)

R = weight of chain on table + rate of change of momentum. ✓

(ignore ± sign for direction of R).

In free fall, the links of the chain are accelerating down at the same rate so there is no force acting between the links. i.e. there is no tension to consider in the chain.

Starting from rest, when a length l of chain has fallen, it arrives at the table at speed v, and for const. accel, ✓

$$v^2 = 2gl.$$

Now, force due to impact, $F = \frac{dp}{dt} = V\frac{dm}{dt}$

[in a short time interval a mass dm arrives, but V changes to $V+dV \simeq V$]

so $F = \sqrt{2gl} \cdot \frac{(\mu V dt)}{dt} \quad = \mu V^2$

$F = 2gl\mu \quad = \underline{\mu g^2 t^2}$ ["$V = at$"].

$= 2g\mu \cdot \frac{1}{2}gt^2$

["$s = \frac{1}{2}at^2$"]

$$\underline{F = \mu g^2 t^2}$$ ✓

Weight of chain on table, is $\mu l g$ ✓

$= \mu \frac{1}{2} g^2 t^2$

so $R =$ weight + rate of mom. change

$= \mu g^2 t^2 (\frac{1}{2} + 1)$

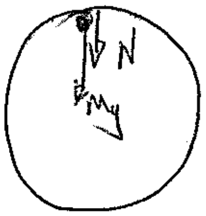
$R_{max} = \frac{3}{2}\mu g^2 t_{fin}^2$

$= \underline{3W}$ ✓ occurs at end of fall ✓

[there are several ways of solving this question]

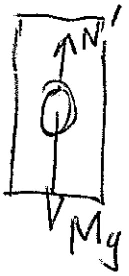
(6)

(h)



Resolving forces on the particle at the top,

$$N + mg = \frac{m v_{top}^2}{r}$$ ✓



For the block, the minimum force N' to raise the block is $N' = Mg$ ✓

by NIII, $N - N' = 0$

Hence $$Mg + mg = \frac{m v_{top}^2}{r}$$

and so $$v_{top}^2 = \frac{(M+m)gr}{m}$$ ✓

From conservation of Energy, for the particle, (object M may only move with negligible KE as we are finding the limiting speed).

$$\frac{1}{2}mv^2 = mg2r + \frac{1}{2}mv_{top}^2$$ ✓

i.e. $$v^2 = 4gr + v_{top}^2$$

Hence $$v^2 = 4gr + \frac{(M+m)gr}{m}$$
$$= 4gr + \left(\frac{M}{m} + 1\right)gr$$
$$= 5gr + \frac{M}{m} \cdot gr$$
$$= gr\left(5 + \frac{M}{m}\right)$$

} any of these ✓

(5)

(i)

(i) By Kirchhoff II,

$$I = \frac{\text{sum of emfs}}{\text{sum of series resistors}}$$

$$= \frac{2.0 + 1.5}{5.0 + 3.0 + 1.0 + 0.5}$$

$$= \frac{7}{19} = 0.368 = 0.37 \text{ A}$$ ✓

(ii)

$$V_{2.0} = \mathcal{E} - Ir$$

$$= 2.0 - \frac{7}{19} \cdot 1$$

$$= 1.6(3) \text{ V}$$ ✓

$$V_{1.5} = 1.5 - \frac{7}{19} \times 0.5$$ ✓

$$= 1.3(2) \text{ V}$$ ✓

(iii)

$$V_A = +\frac{7}{19} \times 5 = +1.8(4) \text{ V}$$

$$V_B = -\frac{7}{19} \times 3 = -1.1(1) \text{ V}$$

} both. ✓

sign required.

(4)

(j)

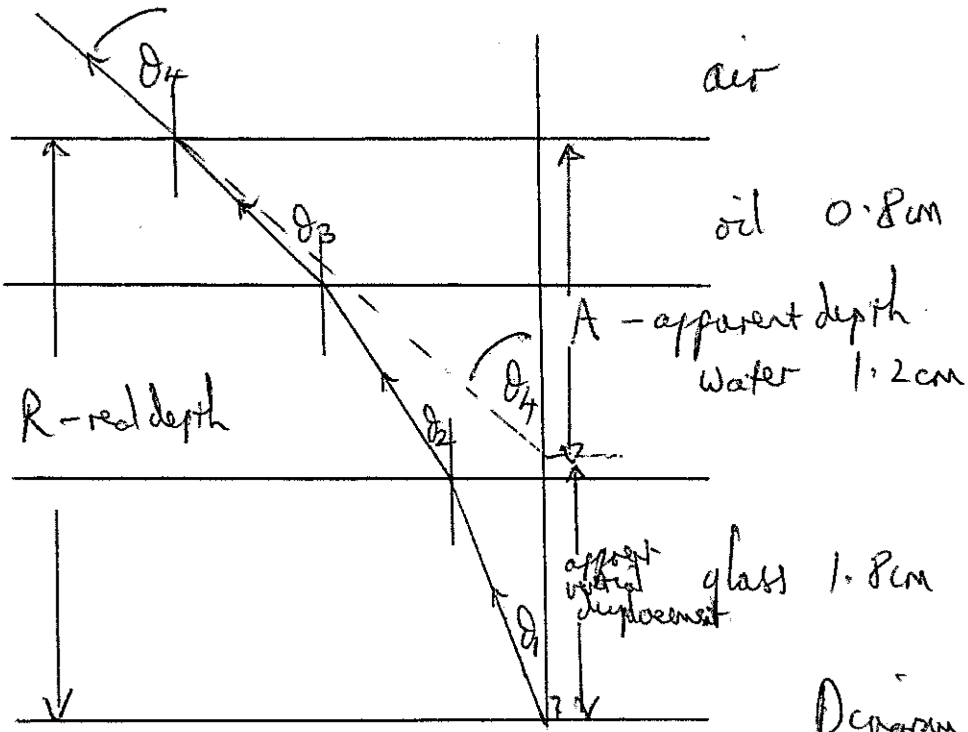


Diagram ✓ with ray drawn.
$\theta_1 < \theta_2 < \theta_3 < \theta_4$, and indication of apparent depth ✓

From Snell's Law.

$$1.5 \sin\theta_1 = 1.3 \sin\theta_2$$
$$1.3 \sin\theta_2 = 1.1 \sin\theta_3$$
$$1.1 \sin\theta_3 = 1.0 \sin\theta_4$$
✓

So, $\sin\theta_4 = 1.5 \sin\theta_1$

Now, $A \cdot \tan\theta_4 = 1.8 \sin\theta_1 + 1.2 \sin\theta_2 + 0.8 \sin\theta_3$ ✓

$$= \frac{1.8}{1.5}\sin\theta_4 + \frac{1.2}{1.3}\sin\theta_4 + \frac{0.8}{1.1}\sin\theta_4$$

For small angles, $\tan\theta_4 \simeq \sin\theta_4$ ✓

$$\therefore A = \frac{1.8}{1.5} + \frac{1.2}{1.3} + \frac{0.8}{1.1}$$
$$= \underline{2.85 \text{ cm}}$$

$R = 1.8 + 1.2 + 0.8 = \underline{3.8 \text{ cm}}$

Hence, vertical displacement is $\underline{0.95 \text{ cm}}$

} any two of these ✓✓

Alternative solution.

Diagram required – Diagram ✓ with ray drawn

$\theta_1 < \theta_2 < \theta_3 < \theta_4$ and indication of apparent depth } ✓

A_g apparent depth in glass $= \frac{n_w}{n_g} \cdot R_g \leftarrow$ real depth in glass. ✓

Then,
$$A_w = \frac{n_{oil}}{n_w}\left(R_w + A_g\right)$$

$$A_{oil} = \frac{n_{air}}{n_{oil}}\left(R_{oil} + A_w\right)$$

Putting these together,

$$A = A_{oil} = \frac{n_{air}}{n_{oil}}\left(R_{oil} + \frac{n_{oil}}{n_w}\left(R_w + \frac{n_w}{n_g}R_g\right)\right). \checkmark$$

$$= \frac{n_{air}}{n_{oil}} \cdot R_{oil} + \frac{n_{air}}{n_w} R_w + \frac{n_{air}}{n_{oil}} \cdot \frac{n_{oil}}{n_w} \cdot \frac{n_w}{n_g} \cdot R_g$$

$$= \frac{R_{oil}}{n_{oil}} + \frac{R_w}{n_w} + \frac{R_g}{n_g} \checkmark$$

$$= \frac{0.8}{1.1} + \frac{1.2}{1.3} + \frac{1.8}{1.5}$$

$= \underline{2.85 \text{ cm}}$

$R = 1.8 + 1.2 + 0.8 = \underline{3.8 \text{ cm}}$

Vertical displacement $= \underline{0.95 \text{ cm.}}$] any two of these ✓✓

(k)

Supply the same energy means the same mgh ✓

$$g_E = \frac{GM_E}{R_E^2}$$

$$M_E = \rho_E \cdot \frac{4}{3}\pi R_E^3$$

mark for either but not both.

$$\therefore \quad \frac{g_E}{g_P} = \frac{G}{R_E^2} \cdot \rho_E R_E^3 \times \frac{R_P^2}{G \rho_P R_P^3}$$

✓ expression for g_E, g_P.

$$= \frac{\rho_E R_E}{\rho_P R_P}$$

$$= \frac{\rho_E}{\frac{2}{3}\rho_E} \cdot \frac{R_E}{2R_E}$$

$$= \frac{3}{4}.$$ ✓

And $g_E h_E = g_P h_P$

So, $\frac{h_P}{h_E} = \frac{g_E}{g_P}$

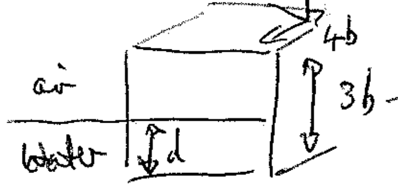
$$h_P = \frac{3}{4} \cdot 1$$

$$= \frac{3}{4} = 0.75 \text{ m}$$ ✓

(4)

$$g_P = G \cdot \frac{2}{3}\left(\frac{M}{R^3}\right) \cdot (2R) = \frac{4}{3} g_E.$$

(l) Method A. The oil has the same density of the block, so a height b of the block is neutrally buoyant in depth, d, of oil. So Consider only a block 3b high floating in water alone.



Length of block is l.

weight of block $= 3 \times 4 . b^2 . l \frac{2}{3} \rho . g$ ✓

weight of water displaced $= d . 4b . l \rho g$ ✓

By Archimedes, these are equal in equilibrium. ✓

Hence $3 . 4 . b^2 l \frac{2}{3} \rho g = d . 4b . l \rho g$ ✓

$d = 2b$.

And b lies within the oil,

so 3b is submerged, 75% ✓

(4)

Method B Including the oil.

Weight of block $= 4 . b^2 l . \frac{2}{3} \rho g$ ✓

Weight of liquid displaced $= b . 4b l \frac{2}{3} \rho g + d \, 4b l \rho g$
(oil) (water)

equating, as these two are equal in equilibrium by Archimedes. ✓

Then $b . \frac{2}{3} + d = 4b . \frac{2}{3}$ ✓

$d = 2b$

and depth of oil is b.

so 3b is submerged, 75% ✓

(4)

(M) Assumption:
For the two systems at any given equal temperature, the cooling method is the same, so the rate of loss of thermal energy is the same at that temperature. ✓
The time taken is proportional to the thermal energy lost. (assumption)
Not rate of energy loss $\propto T$ ✗

Water + calorimeter:

$\Delta Q_{w+c} = \rho_w \cdot V_w \cdot c_w (40-15) + m_c \cdot c_c (40-15)$

$= 1.0 \times 80 \times 4.2 \times 25 + 150 \times 0.4 \times 25$

$= 8400 + 1500$ ✓

$= 9900\ J$

$\Delta Q_{e+c} = 0.8 \times 80 \times c_e (40-15) + 150 \times 0.4 \times 25$ ✓

$= 1600\, c_e + 1500$

and using the rate of cooling assumption.

$\Delta Q_{w+c} = R \times 12$ at 'average' rate R.

and $\Delta Q_{e+c} = R \times 8$

} use of rate to relate cooling. ✓

So $\frac{12}{8} = \frac{9900}{1600 c_e + 1500}$

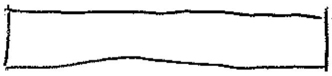
$\therefore 12.16\, c_e + 12.15 = 8.99$

$c_e = \frac{612}{192} = 3.19\ \frac{J}{g\,°C}$

$= 3190\ \frac{J}{kg\,°C}$ ✓

(5)

(m)



girder expands by δl with a temp rise. linear exp. = $\frac{\delta l}{l} \cdot \frac{1}{\Delta T}$

δl = linear exp × $l \, \Delta T$

$= 1.2 \times 10^{-7} \times 4.0 \times (20 - 5)$

$= \underline{7.2 \times 10^{-6} \text{ m}}$ ✓

Young Modulus, $E = \frac{F}{A} \frac{1}{\delta l / l} = \frac{F}{A} \frac{l}{\delta l}$.

$\therefore F = \frac{E A \delta l}{l}$

correct unit change ✓

$= 2.0 \times 10^{11} \times (30 \times 10^{-4}) \times \frac{7.2 \times 10^{-6}}{4.0}$

$= 1080 \text{ N}$

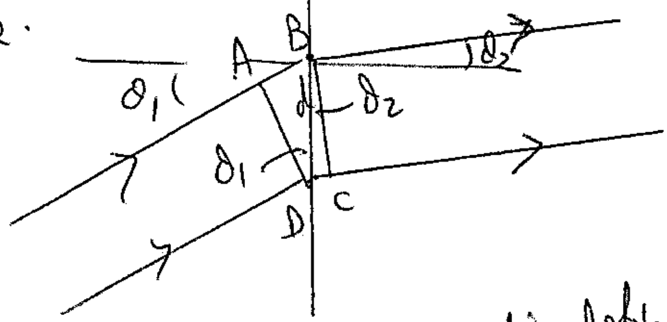
$= \underline{\underline{1100 \text{ N}}}$ ✓

Idea relating the thermal expansion, δl, to the compressive force via Young Modulus ✓

(4)

(c) Alternative on next page.

(i)



[angles marked
labelled
rays parallel
• proper legible diagram]

Separation of slits, $d = \frac{1}{1.2\times10^6}$ m

(ii) path difference $= AB - CD$
$= d(\sin\theta_1 - \sin\theta_2)$
$= n\lambda$ for maxima.

(iii) for $\sin\theta_1 - \sin 73° = n\lambda\, 1.2\times10^6$
and $\sin\theta_1 - \sin 14° = (n\pm1)\cdot\lambda\cdot 1.2\times10^{-6}$

Subtracting. $-\sin 14° + \sin 73° = \pm1\cdot\lambda\cdot 1.2\times10^{-6}$

Thus $\sin 73° - \sin 14° = +1\cdot\lambda\cdot 1.2\times10^{-6}$

So $\lambda = 595$ nm

$\lambda \times 1.2\times10^{-6} = 0.714$

(iv) Hence $\sin\theta_1 = \sin 73° + 0.714\cdot n$ — No positive value of n works.
and $\sin\theta_1 = \sin 14° + 0.714(n\pm1)$

So the diffracted rays must be the other side of the normal (below the horizontal in the top diagram.
[See the diagram on the next page].

So, $\sin\theta_1 + \sin 73° = 2\times0.714$ (n = 2)
$\sin\theta_1 + \sin 14° = 1\times0.714$ (n = 1)

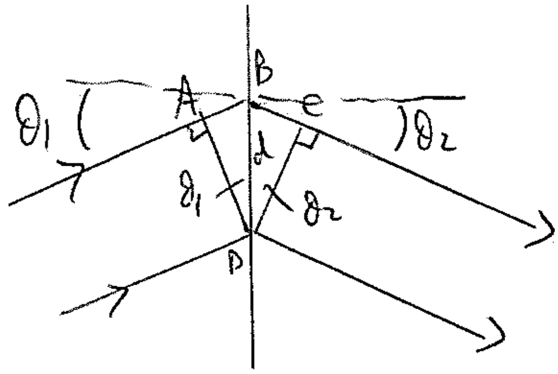
$\theta_1 = 28(2)°$.

(v) For $n = 3$, $\theta_2 > 90$ so no diffracted beam.
Or, the undeflected beam emerging at $\theta_2 = 28°$
Or, ½.

(6)

Alternative

(i)



angles marked / labelled / parallel rays } ✓
• proper legible diagram ✓

(ii) separation of slits, $d = \frac{1}{1.2\times10^{6}}$ m

path difference = AB + BC
$= d(\sin\theta_1 + \sin\theta_2)$
$= n\lambda$ for maxima } ✓

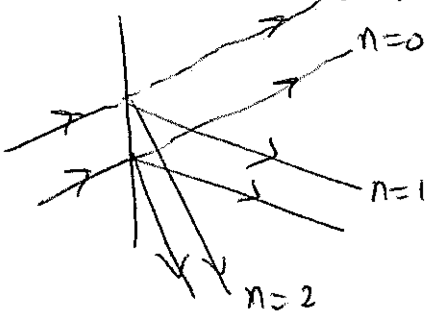
(iii) So, $\sin\theta_1 + \sin 73 = (n+1)\lambda \times 1.2\times10^{6}$
$\sin\theta_1 + \sin 14^\circ = n\lambda \times 1.2\times10^{6}$

subtracting $\sin 73 - \sin 14 = \lambda \times 1.2\times10^{6}$

$\lambda = 595$ nm ✓

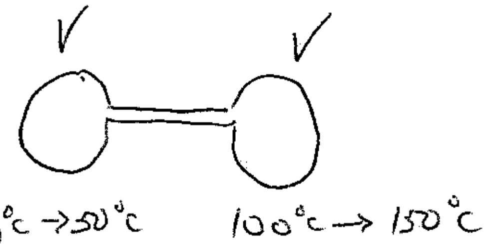
(iv) $(\lambda \times 1.2\times10^{6} = 0.714)$
$\sin\theta_1 + \sin 14^\circ = n\, 0.714$
only possible value of n is 1 (or $\theta_1 > 90^\circ$)
$\sin\theta_1 = 1\times 0.714 - \sin 14^\circ$
$\theta_1 = 28^\circ$ ✓

(v) For $n = 3$ $\theta_2 > 90^\circ$. So no 3rd diffracted beam ✓
and none on the other side of the normal
as $\sin\theta_2 - \sin\theta_1 = n \times 0.714$
$1 - \sin 28 = 0.53$ so no n value possible.
or
But the undiffracted beam at $\theta_2 = 28^\circ$ may be allowed as a 3rd beam.



(6)

(p)



Initially, $P_i \cdot 2V = nRT_i$ n is total moles of gas.

so $n = \frac{P_i \cdot 2V}{RT_i}$

Finally, $n = \frac{P_f \cdot V}{RT_{50}} + \frac{P_f V}{RT_{150}}$ same final mass of gas.

Hence $\frac{P_i 2V}{RT_i} = \frac{P_f V}{RT_{50}} + \frac{P_f V}{RT_{150}}$ ✓ relating initial and final through fixed mass (n) of gas.

$\frac{2P_i}{T_i} = \frac{P_f}{T_{50}} + \frac{P_f}{T_{150}}$

Thus, $\frac{2P_i}{P_f} = T_i\left(\frac{1}{T_{50}} + \frac{1}{T_{150}}\right)$ ① ✓

And we also require the same P_f with temp T_f.

Then $\frac{P_i 2V}{RT_i} = \frac{P_f 2V}{RT_f}$

so, $\frac{P_i}{P_f} = \frac{T_i}{T_f}$ ②

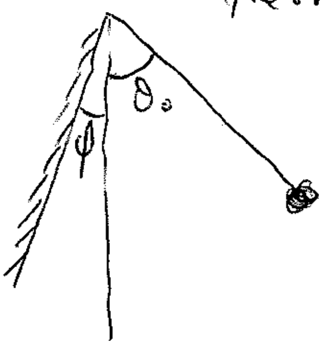
From ① and ② $\frac{2T_i}{T_f} = T_i\left(\frac{1}{T_{50}} + \frac{1}{T_{150}}\right)$

$\frac{2}{T_f} = \frac{1}{323} + \frac{1}{423}$ ✓ use of kelvin temperatures.

so that $T_f = 366.3\ K$
$= 93.3°C$
$= 93°C$ ✓

(4)

(9) The small angles in the question mean this is SHM.



[A] $\theta = \theta_0 \cos(\omega t)$ ✓

$[\theta = \theta_0 \text{ when } t = 0]$

$\omega = 2\pi f = \frac{2\pi}{T_0}$ ✓

Now, $-7 = 14 \cos\left(\frac{2\pi}{T_0} t\right)$ ✓

i.e. $-\frac{1}{2} = \cos\left(\frac{2\pi t}{T_0}\right)$

$\cos \frac{2\pi}{3} = \cos(120°) = -\frac{1}{2}$ ✓

$\therefore \quad \frac{2\pi}{3} = \frac{2\pi t}{T_0}$

$t = T_0/3$ is the time to fall from $+14°$ to $-7°$.

The full period is $2t = \frac{2}{3} T_0$. ✓ (4)

(or other methods).

[B] Use of $\sin(\omega t)$. At $t = 0$, $\theta = 0$. $\theta = \theta_0 \sin \omega t$ ✓

So we want the time to rise from $\theta = 0$ to $\theta = 7°$

$7° = 14° \sin\left(\frac{2\pi t}{T_0}\right)$ ✓

$\sin \frac{\pi}{6} = \frac{1}{2}$. ✓

$\therefore \quad \frac{\pi}{6} = \frac{2\pi t}{T_0}$

So $t = T_0/12$.

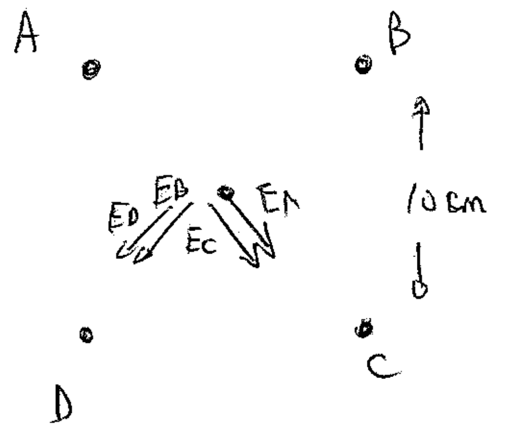
Double this to fall back down, and for the other half of the swing, add $T_0/2$.

So $t = 2 \times \frac{T_0}{12} + \frac{T_0}{2} = \frac{2}{3} T_0$. ✓

or variations on either of these approaches. (4)

(r)

(i)



Fields from A and C are parallel and along diagonal
" " B and D " " " " "

$$E_{AC} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{(10+12)\times10^{-9}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 8.99\times10^{9} \frac{\times 22\times10^{-9}}{50\times10^{-4}} = \underline{39556 \ N/C}$$

✓

$$E_{BD} = \frac{1}{4\pi\varepsilon_0} \frac{(8+6)\times10^{-9}}{(5\sqrt{2}\times10^{-2})^2}$$

$$= 8.99\times10^{9} \frac{(14)\times10^{-9}}{50\times10^{-4}} = \underline{25172 \ N/C}$$

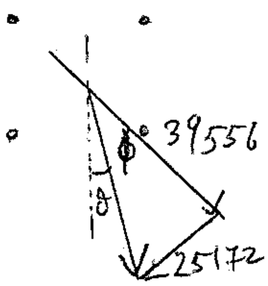
✓

Resultant field strength $= \sqrt{E_{AC}^2 + E_{BD}^2}$

$= 46886 \ N/C.$

$= \underline{4.7\times10^{4} \ N/C.}$

✓



$$\tan\phi = \frac{25172}{39556}$$

$\phi = 32.5°.$

$\therefore \underline{\theta = 12.5°}$

✓

May quote as a bearing of 167° or 168°

May miss out the 8.99×10^{9} of $\frac{1}{4\pi\varepsilon_0}$ ⇒ $E_{total} = 4.7 \ N/C$ (−1 mark)

or just use cm., losing a factor of 10^4 (−1 mark)

(ii)

$$V_{centre} = \frac{1}{4\pi\varepsilon_0} \frac{[10+8-12-6]\times10^{-9}}{5\sqrt{2}\times10^{-2}}$$

$$= \frac{8.99}{5\sqrt{2}\times10^{-2}} \times [0] = \underline{\underline{0}}$$

✓

$$V_{midpoint} = \frac{1}{4\pi\varepsilon_0} \frac{[-12-6]\times10^{-9}}{5\times10^{-2}} + \frac{1}{4\pi\varepsilon_0} \frac{[10+8]\times10^{-9}}{(10^2+5^2)^{\frac{1}{2}}\times10^{-2}}$$

$$= \frac{8.99}{10^{-2}} \left(\frac{-18}{5} + \frac{18}{\sqrt{125}} \right)$$

$$= 8.99 \times 1800 \, (-0.1106)$$

$$= \underline{\underline{-1789 \text{ V}}}$$

WD in moving electron from centre to midpoint is

$$WD = -1789 \times 1.6 \times 10^{-19}$$

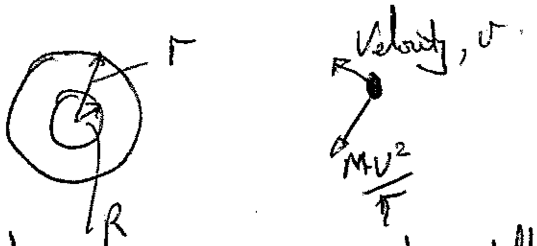
$$= \underline{\underline{~~2~~.86 \times 10^{-16} \text{ J}}}$$ (ignore the sign)

✓

(7)

Question 2

(a)



The force acting on the satellite is gravitational and given by $\frac{GM_p . M}{r^2}$. Newton's 2nd Law states that this unbalanced force is equal to $F = ma$ with $a = \frac{v^2}{r}$ for a circular orbit.

Use of NII and gravity. ✓

$$\therefore \frac{Mv^2}{r} = \frac{GM_p . M}{r^2}$$

but $v = \frac{2\pi r}{T}$

Hence $T^2 = \frac{4\pi^2}{GM_p} . r^3$ ✓

And we also have at the surface, $Mg = \frac{GM_p . M}{R^2}$

hence $GM_p = gR^2$ ✓

and $T^2 = \frac{4\pi^2}{gR^2} . r^3$

(b) $\frac{4\pi^2}{gR^2}$ is fixed for a central gravitating body

$\therefore T^2 = k r^3$

For the Earth, $1^2 = k . 1^2$ in years and AU.

$\therefore k = 1 \left(\frac{y^2}{AU^3}\right)$

Hence, for Jupiter. $11.9^2 = 1 . r^3$ ✓

$r = 5.21$ AU

(4)

(c) Radius of core ≈ 3.2×10^6 m ✓

Density of core ≈ $(10-11) \times 10^3$ kg m^{-3}

The density must be less than 11×10^3 (since the volume $\propto r^3$ and hence the density at larger r is more significant).

Radius of mantle ≈ $6.2 - 6.4 \times 10^6$ m

Density of mantle ≈ $3-5 \times 10^3$ kg m^{-3} ✓

Mass ≈ $4-6 \times 10^{24}$ kg ✓

The volume of the mantle must be given by $\frac{4}{3}\pi(r_E^3 - r_{core}^3)$.

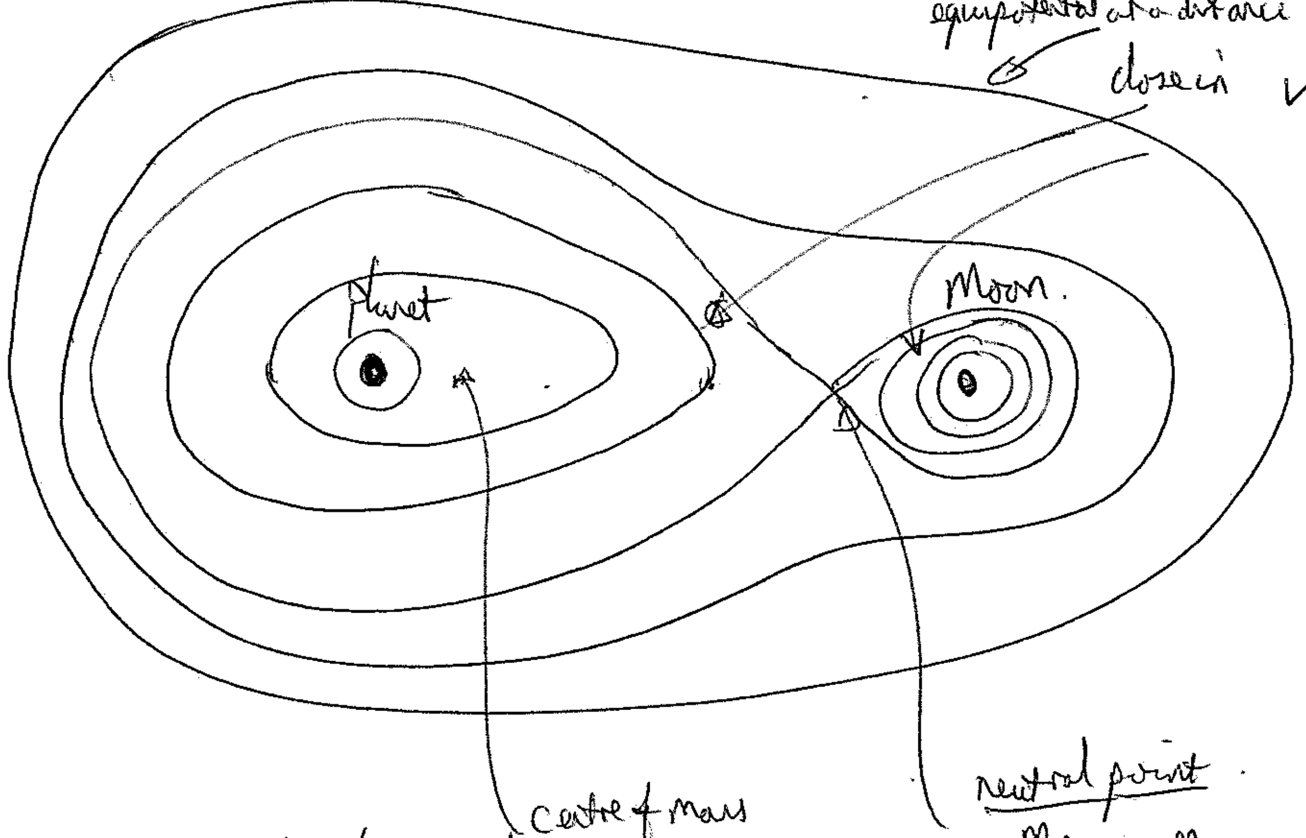
③

(d)(i)

Centre of mass closer to planet } n.p. closer to moon } ✓

equipotential at a distance ✓

close in ✓



centre of mass

$M_P r_1' = M_m r_2'$

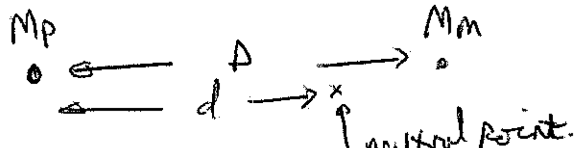
C of M ≈ $\frac{1}{9}$ of separation distance from planet

neutral point

$\frac{M_P}{r_1^2} = \frac{M_m}{r_2^2}$

n.p. is about $\frac{1}{4}$ of the separation distance from the moon

(ii)



At the neutral point, the field strengths from the planet and moon are equal and in opposite directions. So the rocket only has to reach height d, and it can fall to the moon from speed of zero.

At the n.p., $\frac{\not{G} M_p}{d^2} = \frac{\not{G} M_m}{(D-d)^2}$ ✓

$$\therefore \sqrt{\frac{M_m}{M_p}} \cdot d = D - d.$$ ← Take opposite roots

$$\left(1 + \sqrt{\frac{M_m}{M_p}}\right) d = D$$

$$d = \frac{D}{\left(1 + \sqrt{\frac{M_m}{M_p}}\right)}$$ ✓

Evaluating, $d = \frac{3.84 \times 10^8}{1 + \sqrt{\frac{7.5}{60}}}$

$$d = 2.84 \times 10^8 \text{ m}$$ ✓

idea here! or here ✓

(iii) Equating KE of launch and change of grav. p.e. at neutral point where KE is zero,

$$\therefore \tfrac{1}{2} m v^2 = \underbrace{\frac{GM_p}{R_p} + \frac{GM_m}{D-R_p}}_{\text{p.e. when on planet}} - \underbrace{\frac{M_p}{d} - \frac{M_m}{D-d}}_{\text{p.e. at n.p.}}$$ ✓

So, $\frac{v^2}{2G} = \frac{M_p}{R_p} + \frac{M_m}{D-R_p} - \frac{M_p}{D}\left(1 + \sqrt{\frac{M_m}{M_p}}\right) - \frac{M_p}{D}\left(\sqrt{\frac{M_m}{M_p}} + \frac{M_m}{M_p}\right)$

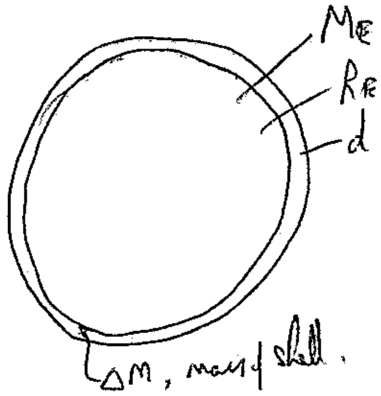
$$\frac{v^2}{2G} = \frac{M_p}{R_p} + \frac{M_m}{D-R_p} - \frac{M_p}{D}\left(1 + \sqrt{\frac{M_m}{M_p}}\right)^2$$ } not required

$$\frac{v^2}{2G} = \frac{M_p}{R_p} + \frac{M_m}{D-R_p} - \frac{M_p \cdot D}{d^2}$$ ✓

Substituting $v = 11049$ m/s $= 11 \times 10^3$ m/s $= 11$ km/s

(9)

(e)



$$g = \frac{GM_E}{R_E^2}$$ ✓

N.B the density given is that of the Mantle, ρ_m and not of the whole Earth.

Mass of the thin spherical shell is $4\pi R_E^2 . d . \rho_m$. ✓

$\therefore$ at the bottom of the shaft, $$g' = \frac{G(M_E - 4\pi R_E^2 . d . \rho_m)}{R_E^2}$$ ✓

$$= g - G4\pi . d . \rho_m .$$ ✓

$$\therefore \frac{g - g'}{g} = \frac{\Delta g}{g} = \frac{G 4\pi d . \rho_m}{G M_E / R_E^2}$$

$$= 4\pi \frac{d \rho_m . R_E^2}{M_E}$$

$$= 6.37 \times 10^{-4}$$

$$= 6.4 \times 10^{-2} \%$$

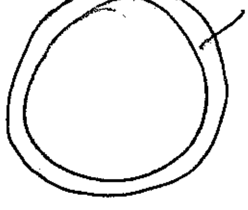
$$= 0.064 \%$$ ✓

(4)

If the mistake is made of stating $\frac{4}{3}\pi R_E^3 \rho_m = M_E$, then $\frac{\Delta g}{g}$ becomes $\frac{3d}{R_E}$, and this gives 0.14%

This is allowed [3 marks] if the above steps are present.

(f) (i)



shell of thickness δr and mass δm collapses onto a [illegible] of radius, r, and mass M.

The energy lost when δm falls from a large distance (∞) to radius, r, attracted by mass, M

is given by $\delta W = \frac{GM.\delta M}{r}$ ✓

But M depends upon, r, so we need to express in terms of a single variable.

$$M = \frac{4}{3}\pi r^3 \rho$$

and $\delta M = 4\pi r^2 \rho \, \delta r$ — mass of a shell of area $4\pi r^2$ and thickness δr.

so $$\delta W = G \frac{\frac{4}{3}\pi r^3 \rho \cdot 4\pi r^2 \rho}{r}$$ ✓

$$= G \frac{4^2 \pi^2 \rho^2}{3} \cdot r^4 \, \delta r$$ ✓

(ii) We need to ~~integrate this from 0 to~~ R.

$$\int_0 dW = G \frac{4^2\pi^2\rho^2}{3} \int_0^R r^4 \, dr$$ ✓

$$E = G \frac{4^2\pi^2\rho^2}{3} \cdot \frac{R^5}{5}$$

and substituting back in for M (M^2 in fact)

$$E = \frac{3}{5}\frac{GM^2}{R}$$ ✓

(iii) Using $E = Mc\Delta T$

then $$\frac{3}{5}\frac{GM^2}{R} = Mc\Delta T$$

and $$\Delta T = \frac{3}{5}\frac{GM}{Rc}$$

$$= \frac{3}{5} \times \frac{6.67 \times 10^{-11} \times 4 \times 10^{26}}{6.4 \times 10^6 \times 700}$$

$$= 3.6 \times 10^6 \text{ K}$$ ✓

(5)

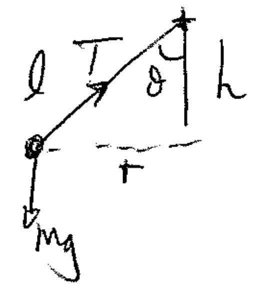
END OF QUESTION 2.

QUESTION 3

Comments required

(a)

(i)



Resolve V: $T\cos\theta = mg$ ①

" H: $T\sin\theta = ma = \frac{mv^2}{r}$ ②

where for circular motion, $a = \frac{v^2}{r}$

Dividing, $\tan\theta = \frac{v^2}{rg}$

But, geometrically, $\tan\theta = r/h$ ③

Hence $\frac{r}{h} = \frac{v^2}{rg}$ ④

and since $V = \frac{2\pi r}{P}$ ⑤

then $P = 2\pi\sqrt{\frac{h}{g}}$ ✓

(ii)

From ① $\frac{T}{mg} = \frac{1}{\cos\theta}$ and $\frac{T}{mg} = 3$

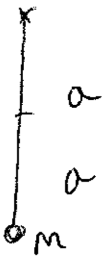
$\therefore\ 3 = \frac{1}{\cos\theta}$

with $\cos\theta = \frac{h}{l} = \frac{1}{3}$

so, $l = 3h$ ✓

⑤

(b) (i)



for a linear elastic, $F = kx$

Here $mg = ka$ ①

so $k = \frac{mg}{a}$ ② ✓

when displaced and released, there is a restoring force $k(a+x)$

which by NII is $F = ma$

$\therefore \quad ma = -k(a+x) + mg$.

and from ① $\quad ma = -kx$

SHM with solution such that $\omega^2 = \frac{k}{m} = \frac{g}{a}$ (from ②)

$$\therefore \quad T = 2\pi\sqrt{\frac{a}{g}}$$ ✓

(ii) The particles move towards each other with equal speeds / i.e. centre of mass remains fixed ✓

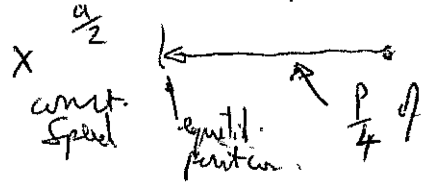
They accelerate towards each other (but the force is not constant) as part of a cycle ($\frac{1}{4}$) of SHM. } ✓

At a separation 'a' they travel at constant speed.

The string has a fixed point in the centre. So each half behaves as $\frac{a}{2}$ is natural length, and will stretch with $\frac{a}{2}$ with a weight mg attached

so $k = 2\frac{mg}{a}$

Here (for one mass) $P = 2\pi\sqrt{\frac{a}{2g}}$



So, accelerated time is $\frac{P}{4} = \frac{\pi}{2}\sqrt{\frac{a}{2g}}$ ✓

During constant speed, energy stored, $\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

i.e. $\frac{1}{2}\left(\frac{2mg}{a}\right).\, a^2 = \frac{1}{2}mv^2$

$\left(\frac{3}{2}a - \frac{a}{2}\right)^2$ (half the elastic) ✓

($V = 1.98$ m/s)

$v = \sqrt{2ga}$

Constant speed time is $t = \frac{a}{2} \cdot \frac{1}{v} = \frac{a}{2\sqrt{2ga}} = \frac{1}{2}\sqrt{\frac{a}{2g}}$.

Time up to collision is $\frac{1}{2}\sqrt{\frac{a}{2g}} + \frac{\pi}{2}\sqrt{\frac{a}{2g}} = \frac{1}{2}\sqrt{\frac{a}{2g}}(1+\pi) = 0.209$

$= 0.21$ s ✓

⑦

(c) (i) $E = \frac{F}{A} / \frac{\delta l}{l_0} = \frac{F}{A}\frac{l}{\delta l}$

E is constant and l_0 is the initial length.

$\therefore F = \frac{AE\,\delta l}{l_0}$

In the final state, $A \to \frac{A}{2}$ as doubling the length halves A ✓

$\delta l \to l_0$

the denominator (l) remains the same as that is the initial l specified for reference.

$\therefore F_f = \frac{A_0}{2} E . \frac{l_0}{l_0} = \frac{A_0 E}{2}$ ✓

(ii) In stretching the elastic, with extension, x.

$WD = \delta W = F\,\delta x$

$= A\frac{Ex\delta x}{l_0}$ l_0 is the initial length.

$dW = \frac{AE}{l_0} x\,dx$ ✓

Now $A_0 l_0 = Al$ for constant volume.

So $A = \frac{A_0 l_0}{l}$. ✓

and $l = l_0 + x$

So, $dW = \left(\frac{A_0 l_0}{l_0(l_0+x)}\right) Ex . dx$

$= A_0 E . \frac{x}{(l_0+x)} dx.$ ✓

Let $p = l_0 + x$, $dp = dx$ } $\int_0^W dW = A_0 E \int_{x=0}^{x=l_0} \frac{(p - l_0)}{p} dp$

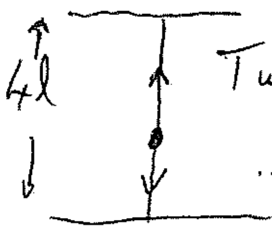
$W = A_0 E \left[p - l_0 \ln p \right]_{x=0}^{x=l_0}$

$W = A_0 E l_0 - A_0 E l_0 \ln (l_0 + x) \Big|_{x=0}^{x=l_0}$

$W = A_0 E l_0 (1 - \ln 2)$

(5)

(d) (i)



$T_{up} = k x_u$ ← extension of top elastic

$T_d = k x_d$ ← extension of bottom elastic

Elastic : $mg = kl$

$k = \frac{mg}{l}$

We know that $x_u + x_d + 2l = 4l$

$x_u + x_d = 2l$ ①

Equating forces $T_{up} = T_d + mg$

So, $k x_u = k x_d + mg$ ②

Hence from ① and ②

$k2l - k x_d = k x_d + mg$

$2mg - mg = 2 k x_d$

$mg = 2 \frac{mg}{l} \cdot x_d$

$x_d = l/2$ ✓

$\therefore \; x_u = \frac{3l}{2}$

Hence ball is $l + \frac{3l}{2} = \frac{5l}{2}$ below ceiling at equilibrium. ✓

(ii) Displace ball a distance y down from the equilibrium position [Down ↓ is positive]

Then $m\ddot{y} = -k(\frac{3l}{2} + y) + k(\frac{l}{2} - y) + mg$

So $m\ddot{y} = -k\frac{3l}{2} - ky + k\frac{l}{2} - ky + mg$

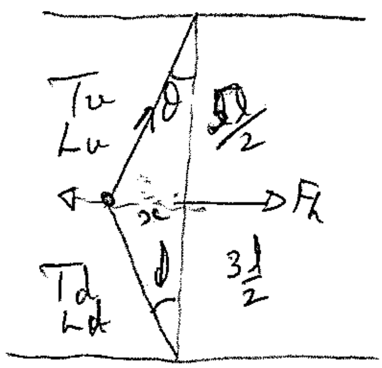
$= -kl - 2ky + mg$ but $kl = mg$ for the elastic

Hence $m\ddot{y} = -2ky$

$\omega^2 = \frac{2k}{m}$ $\Rightarrow T = 2\pi\sqrt{\frac{m}{2k}} = \pi\sqrt{\frac{2m}{k}}$

$= 2\pi\sqrt{\frac{l}{2g}} = \pi\sqrt{\frac{2l}{g}}$ } any of these ✓

(iii)



Restoring force, $F_h = T_u \sin\theta + T_d \sin\phi$ ✓

Now $L_u = \frac{5l}{2\cos\theta}$

$L_d = \frac{3l}{2\cos\phi}$

Hence $T_u = k\left(\frac{5l}{2\cos\theta} - l\right)$

and $T_d = k\left(\frac{3l}{2\cos\phi} - l\right)$

$\therefore F_h = kl\left(\frac{5}{2}\cdot\frac{\sin\theta}{\cos\theta} - \sin\theta\right) + kl\left(\frac{3}{2}\cdot\frac{\sin\phi}{\cos\phi} - \sin\phi\right)$

$= kl\left[\frac{5}{2}\tan\theta - \sin\theta + \frac{3}{2}\tan\phi - \sin\phi\right]$ ✓

But $\tan\theta \simeq \sin\theta$ and $\tan\theta = \frac{x.2}{5l}$

$\tan\phi \simeq \sin\phi$ and $\tan\phi = \frac{x}{3l}.2$

Hence, $F_h = kl\left[\frac{3}{2}\cdot\frac{2x}{5l} + \frac{1}{2}\frac{2x}{3l}\right]$

$= k\left[\frac{3}{5}x + \frac{1}{3}x\right]$

$F_h = k\cdot\frac{14}{15}x$ ✓

Various approaches possible here.

Hence $m\ddot{x} = -k\cdot\frac{14}{15}x$

and $\omega^2 = \frac{k}{m}\frac{14}{15}$, so $T = 2\pi\sqrt{\frac{15m}{14k}} = 2\pi\sqrt{\frac{15l}{14g}}$ ✓

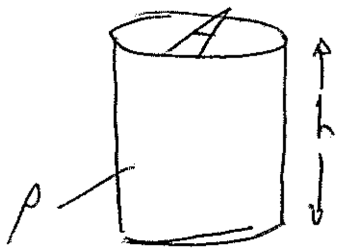
(iv) $\frac{T_h}{T_V} = \sqrt{\frac{15l}{14g}\cdot\frac{2g}{l}} = \sqrt{\frac{15}{7}} = \underline{1.46}$ ✓

(8)

END OF QUESTION 3.

Question 4

(a) [i] (i)



$P_{bottom} = \frac{force}{area} = \frac{weight}{A}$ ✓

$= \frac{mg}{A} = (\rho \frac{A h}{A}) g$

$P = \rho g h$ ✓

(ii) LHS. $\frac{N}{m^2} = m\,kg\,s^{-2} m^{-2} = m^{-1} kg\, s^{-2}$

RHS $kg\,m^{-3} . m\,s^{-2} . m = m^{-1} kg\, s^{-2}$ } ✓

explicitly shown or $kg\,m^{-3}.\,N\,kg^{-1}.\,m = N\,m^{-2}$ for RHS is OK.

(iii) Average density is $\frac{\rho_t + \rho_b}{2}$ ✓

$\therefore P = (\frac{\rho_t + \rho_b}{2}) g h.$

(iv) g is almost the same at ground level and 200 km high.
The air is compressible, and is very nonlinear when it is compressed under its own weight. An average density would be difficult to specify. ✓ ✓
(using ground air density of 1 kg m^-3, the ht of the atmosphere is 10 km).

[ii]

(i) Energy: $\frac{1}{2} m v^2 = m g h'$

$v^2 = 2 g h'$

Outside, in free fall.

Vertically $h - h' = \frac{1}{2} g t^2$

Horizontally $H = v t$ } ✓

$H = \sqrt{2gh'} . \sqrt{\frac{2(h-h')}{g}}$

$H = 2\sqrt{h'(h-h')}$ ✓

(ii) Momentum :

NII - force = rate of change of momentum

$$\frac{d(mv)}{dt} = P.a = \rho g h'.a$$

Momentum of jet of liquid emerging from hole in can

is $v \frac{dm}{dt}$

so $v \frac{\Delta m}{\Delta t} = v \rho a \frac{\Delta x}{\Delta t}$

and this is equal to $\rho g h' a$.

$$\therefore \quad v \rho a \frac{\Delta x}{\Delta t} = \rho g h' a$$

$$v^2 = g h'$$

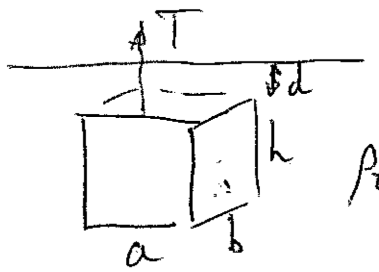
So, now we have $H = \sqrt{gh'} \cdot \sqrt{\frac{2(h-h')}{g}}$

$$H = \sqrt{2h'(h-h')}$$

✓

(8)

(b) (i)



pressure on top $P_t = \rho_t \cdot g d \quad (+P_0)$

" " bottom $P_b = \rho_t \cdot g(d+h) \quad (+P_0)$

$\therefore T = Mg + P_t \cdot ab - P_b \cdot ab$

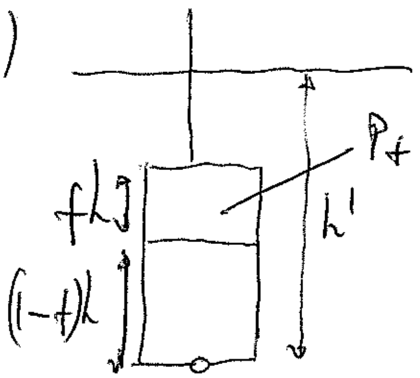
$= Mg + \rho_t \cdot g\, abd - \rho_t g(abd + abh)$

lose 1 mark if they leave out the (ab) factor

$= Mg - \rho_t g\, abh$

$= Mg -$ "weight of liquid displaced"

(ii)



$P_f = P_0 +$ depth of liquid − depth of liquid in container.

$P_f = P_0 + \rho g h' - \rho g(1-f)h.$

For an (isothermal) gas,

$P_0 h = P_f \cdot fh$

$\therefore P_f = \frac{P_0}{f}.$

Then, $\frac{P_0}{f} = P_0 + \rho g(h' - (1-f)h)$

$P_0\left(\frac{1-f}{f}\right) = \rho g(h' - (1-f)h)$

$\frac{P_0}{\rho g}\left(\frac{1-f}{f}\right) = h' - (1-f)h.$

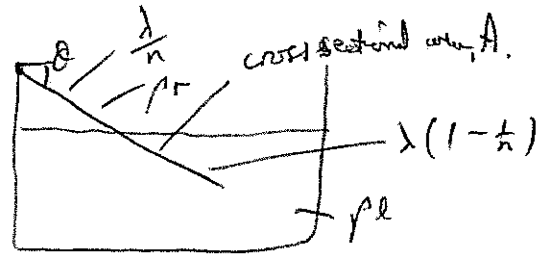
$h' = \frac{P_0}{\rho g f}(1-f) + (1-f)h$

$h' = \left(\frac{P_0}{\rho g f} + h\right)(1-f)$

(7)

(c)

(alternative overleaf)



Take moments about the pivot. ✓

At equilibrium,
moment of rod = moment produced by upthrust/buoyancy ✓

$$\lambda A \cdot \rho_r \cdot g \cdot \tfrac{1}{2}\lambda \cdot \cos\theta = \lambda\left(1-\tfrac{1}{n}\right) A \rho_l \cdot g \cdot \left(\tfrac{\lambda}{n} + \tfrac{1}{2}\lambda\left(1-\tfrac{1}{n}\right)\right)\cos\theta$$ ✓

cancelling $\lambda^2, A, g, \cos\theta$ on each side

$$\frac{\rho_r}{2} = \left(1-\frac{1}{n}\right)\rho_l\left(\frac{1}{n} + \frac{1}{2} - \frac{1}{2n}\right)$$ ✓

$$\frac{\rho_r}{2} = \left(1-\frac{1}{n}\right)\rho_l \,\frac{1}{2}\left(1+\frac{1}{n}\right)$$

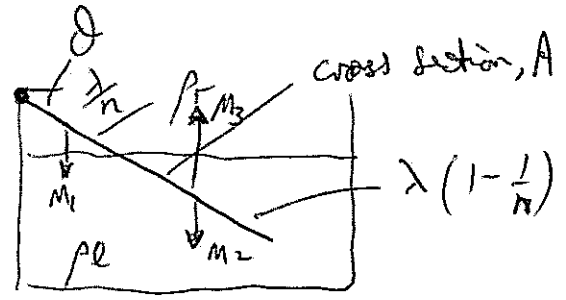
$$\rho_r = \left(1-\frac{1}{n^2}\right)\rho_l$$

$$\frac{1}{n^2} = \frac{\rho_l-\rho_r}{\rho_l}$$ ✓

$$n = \sqrt{\frac{\rho_l}{(\rho_l-\rho_r)}}$$

(5)

(Q)



Taking moments about the pivot:

(M1) weight of rod × horizontal distance = − vertical moment of wt of rod (M2) + weight of liquid displaced (M3)

$$\lambda \cdot A\rho_r \cdot g \times \frac{1}{2}\frac{\lambda}{n} \cdot \cos\theta = \left\{ -\lambda\left(1-\frac{1}{n}\right)A\rho_r \cdot g + \lambda\left(1-\frac{1}{n}\right)A\rho_l \cdot g \right\} \cdot \left(\frac{\lambda}{n} + \frac{1}{2}\lambda\left(1-\frac{1}{n}\right)\cos\theta\right)$$

distance of centre point of submerged rod.

$$\lambda \rho_r \cdot \frac{1}{2}\frac{\lambda}{n} = \left[-\lambda\left(1-\frac{1}{n}\right)\rho_r + \lambda\left(1-\frac{1}{n}\right)\rho_l \right] \times \frac{1}{2}\lambda\left(1+\frac{1}{n}\right)$$

$$\frac{\rho_r}{2} = (\rho_l - \rho_r)\left(1-\frac{1}{n}\right)\left(\frac{1}{2}\left(1+\frac{1}{n}\right)\right)$$

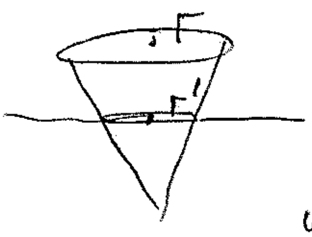
$$\rho_r = (\rho_l - \rho_r)\left(1-\frac{1}{n^2}\right)$$

$$n^2 = \frac{\rho_l}{\rho_l - \rho_r}$$

(5)

(d) weight of cone, $Mg = \frac{1}{3}$ weight of liquid displaced at equilibrium.

∴ radius of cone at liquid level, r' is given by $\frac{h}{n} \cdot \frac{1}{r'} = \frac{h}{r}$



∴ $r' = \frac{r}{n}$

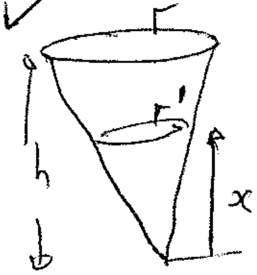
weight of liquid displaced $= \rho_l \cdot \frac{1}{3}\pi\left(\frac{r}{n}\right)^2 \frac{h}{n} g$ ✓

So $Mg = \rho_l \frac{1}{3}\pi \frac{r^2}{n^3} \cdot h \cdot g$

$m = \frac{\pi}{3} \cdot \rho_l \frac{r^2 h}{n^3}$ ✓

In sinking, the gpe lost is against WD by the upthrust. ✓

$$2mg\left(h - \frac{h}{n}\right) = \int_{h/n}^{h} \frac{\pi}{3}\rho_l \, r'^2 x g \, dx$$



given $\frac{x}{h} = \frac{r'}{r}$ so that $r' = \frac{xr}{h}$

$$2mg\left(h - \frac{h}{n}\right) = \int_{h/n}^{h} \frac{\pi}{3}\rho_l g \, \frac{x^3 r^2}{h^2} \, dx$$ ✓

so $2mh\left(1 - \frac{1}{n}\right) = \frac{\pi}{3}\rho_l \frac{r^2}{h^2}\left[\frac{x^4}{4}\right]_{h/n}^{h}$

substitute for m from equilibrium result above,

$$2h\left(1 - \frac{1}{n}\right) = \frac{n^3}{h^3} \cdot \left(\frac{h^4}{4} - \frac{h^4}{4n^4}\right)$$ ✓

$$2\left(1 - \frac{1}{n}\right) = \frac{n^3}{4}\left(1 - \frac{1}{n^4}\right)$$

$$8(n-1) = (n^4 - 1)$$

$$(n-1)\left(8 - (n^2+1)(n+1)\right) = 0$$

$$(n-1)\left(8 - n^3 - n^2 - n - 1\right) = 0$$

$$n^3 + n^2 + n = 7$$

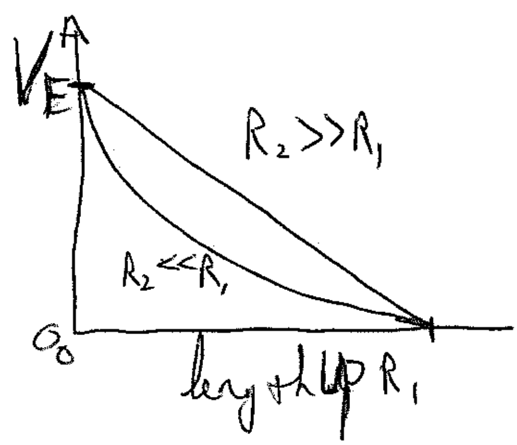
(This can be done by forces).

(5)

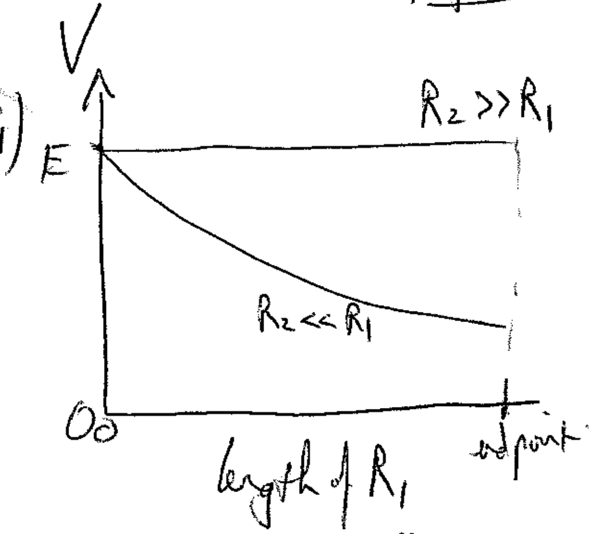
END OF QUESTION 4.

Question 5

(a) (i)

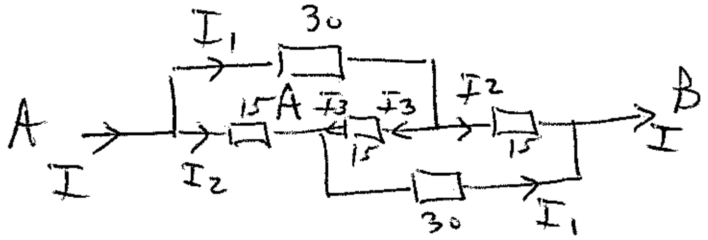


(ii)



Circuit (a) has a linear aspect and covers the full range of E. A current is always flowing through R_1.
circuit (b) does not cover the range E to 0 V.

(b)



currents marked according to KI. (and symmetry of the circuit).

$$I_1 = I_2 + I_3 \quad (1)$$

KI $\quad I = I_1 + I_2 \quad (2)$

KII. $\quad 30 I_1 + 15 I_3 - 15 I_2 = 0 \quad$ (loop A) (3)

substitute from (2) for I_3.

$$30 I_1 + 15 I_1 - 15 I_2 - 15 I_2 = 0$$

so $\quad 45 I_1 = 30 I_2$

$$3 I_1 = 2 I_2$$

From (2) $\quad I = \frac{5}{2} I_1$

For AB, $\quad V = I_1 30 + I_2 15$

$$= \frac{2}{5} 30 I + \frac{3}{5} 15 I$$

$$= 21 I$$

$\Rightarrow$ $\underline{R = 21 \Omega}$

$I_1 = 4$ mA.
$I_2 = 6$ mA
$I_3 = -2$ mA.

(4)

(c)

(i) $R = \rho \frac{l}{A} = \frac{5 \times 10^{-7} \times 0.3}{\pi (0.014 \times 10^{-3})^2} = 244\,\Omega$

$(243.60\,\Omega)$

(ii) $\Delta R = \frac{1}{300} R = 0.81\,\Omega$

$\therefore R_{stretch} = 244.4\,\Omega$.

(iii) $A \cdot l$ is constant.

$$Al = (A + \delta A)(l + \delta l)$$
$$\approx Al + l\,\delta A + A\,\delta l.$$
$$l\,\delta A = -A\,\delta l$$
$$\frac{\delta A}{A} = -\frac{\delta l}{l}.$$

So $\Delta R = \frac{2}{300} R = 1.624\,\Omega$

$R_{constant} = 245.2\,\Omega$

(iv) $V_{top} = \frac{R_1}{R_1 + R_2} \times 9.0 - \left(9.0 - \frac{R_4 \times 9.0}{R_3 + R_4}\right)$ idea of calculating potential change from 4.5 V

$$= 9.0 \left(\frac{244.41}{244.41 + 243.60} - \left(1 - \frac{244.41}{243.60 + 244.41}\right)\right)$$

$$= 9.0 \left(0.50083 - (1 - 0.50083)\right)$$

$$= 9 \times 1.66 \times 10^{-3}$$

$$= 0.015\ V$$

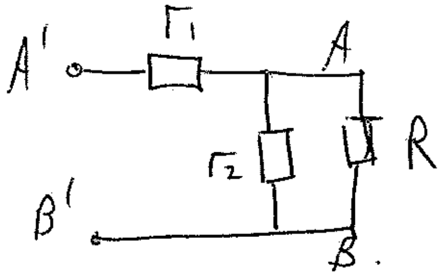
If they have used the constant volume, then the answer is 0.030 V – which is acceptable.

(6)

(d)

Consider the resistance at AB to be R.

Then add on another section



✓

$$R_{A'B'} = r_1 + \frac{R r_2}{R + r_2}$$

(parallel resistors) ✓

but $R_{A'B'} = R$ since it is infinite series and adding a term should make no change.

$\therefore\ R(R + r_2) = r_1 + R r_2$ ✓

$R^2 + R r_2 - R r_1 - r_2 r_1 - R r_2 = 0$

ie $R^2 - r_1 R - r_1 r_2 = 0$

So $R = \dfrac{r_1 \pm \sqrt{r_1^2 + 4 r_1 r_2}}{2}$

Then $R = \dfrac{4 \pm \sqrt{4^2 + 4.4.15}}{2}$

$\underline{\underline{R = 10\ \Omega}}$ ✓

(4)

(e) consider a cell and resistor to represent the infinite series:

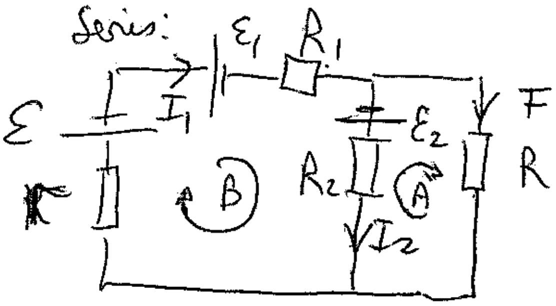
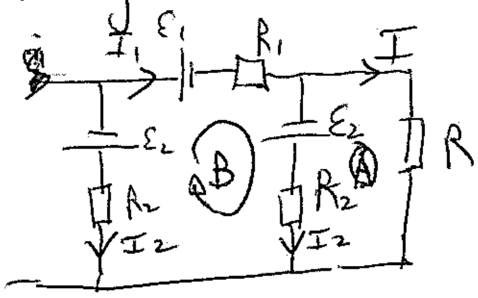


Diagram with currents ✓

or similarly can consider



KII for loop A: $IR - I_2R_2 = -\varepsilon_2$

$\therefore \varepsilon_2 = I_2R_2 - IR$ ① ✓

for loop B. $I_1R_1 + I_2R_2 - I_2R_2 = -\varepsilon_1 + \varepsilon_2 - \varepsilon_2$ ✓

so $I_1R_1 = -\varepsilon_1$

and $\varepsilon_1 = -I_1R_1$ ②

KI $I_1 = I + I_2$ ③ ✓

so $I + \frac{\varepsilon_2 + IR}{R_2} = -\frac{\varepsilon_1}{R_1}$ (linking) ✓

Hence $IR_1R_2 + \varepsilon_2 R_1 + IRR_1 = -\varepsilon_1 R_2$

$I(R_1R_2 + RR_1) = -(\varepsilon_1R_2 + \varepsilon_2R_1)$

$I = -\frac{(\varepsilon_1R_2 + \varepsilon_2R_1)}{(R_1R_2 + RR_1)}$

$\varepsilon_1 = 1.0\text{ V}, \varepsilon_2 = 2.0\text{ V}$
$R_1 = 4.0\,\Omega, R_2 = 6\,\Omega$
$R = 1.0\,\Omega$

$I = -\frac{(1.6 + 2.4)}{(4 \cdot 6 + 1 \cdot 4)} = \frac{(-)14}{28} = (-)0.5\text{ A}$ ✓

⑥

END OF QUESTION 5

Question 6

(a) (i) a charge Q flows, so that $\underline{Q_1 = Q_2}$ ($\equiv Q$) ✓

(ii) $V_1 = \frac{Q}{C_1}\left\{=\frac{VC_1}{(C_1+C_2)}\right\}$ $V_2 = \frac{Q}{C_2}\left\{=\frac{VC_1}{(C_1+C_2)}\right\}$ ✓ ✓ Simple form only "$V = \frac{Q}{C}$"..

(iii) and $V = V_1 + V_2$,

so $V = \frac{Q}{C_{eqv}} = \frac{Q}{C_1} + \frac{Q}{C_2}$

Hence $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ ✓

(iv) $E_1 + E_2 = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2}$

$= \frac{Q^2}{2}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

$= \frac{Q^2}{2C_{eqv}}$ ✓

$= \underline{\underline{\frac{1}{2}QV}}$.

Battery supplies QV, so half of the energy is dissipated ("lost") ✓

(6)

(b) (i) With plates disconnected, doubling the separation doubles the potential difference between them.
Charge remains fixed.
so $\frac{1}{2}QV$ is doubled. i.e. $\frac{1}{2}CV^2$ doubled.

$WD = \frac{1}{2} \times 15 \times 10^{-9} \times 70^2$

$= \underline{3.7 \times 10^{-5}\ J}$ ✓

(ii) Potential fixed, but twice separation, so E field halved
Hence Q is halved.

or use $C = \varepsilon_0 \frac{A}{d}$

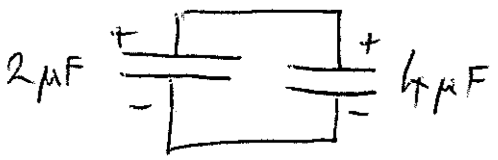
$E_f - E_i = \left(\frac{1}{2} - 1\right) 3.7 \times 10^{-5}\ J$

$= \underline{-\ 1.8 \times 10^{-5}\ J}$ ✓

↑ need an indication of the sign.

(2)

(c) (i)



total charge is $120 \times 2 = 240\,\mu C$

on $6\,\mu F \Rightarrow V = 40\,V$ ✓

Initial energy $= \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2}$

$= \frac{1}{2}(120\,\mu C)^2 \left(\frac{1}{2.0\,\mu F} + \frac{1}{4.0\,\mu F}\right)$

$= 5.4 \times 10^{-3}\,J$

Final energy $= \frac{1}{2} C V^2$

$= \frac{1}{2}\, 6 \times 10^{-6} \times 40^2$ ✓

$= 4.8 \times 10^{-3}\,J$ ✓

change of energy is $6 \times 10^{-4}\,J$ ✓

(ii) On first contact, the potentials are the same on sphere + plate.

i.e. $\frac{Q}{C_{sphere}} = \frac{q - Q}{C_{plate}}$ ① ✓

Final potential on the two, V_f is given by

$V_f = \frac{Q_f}{C_{sphere}} = \frac{q}{C_{plate}}$ ②

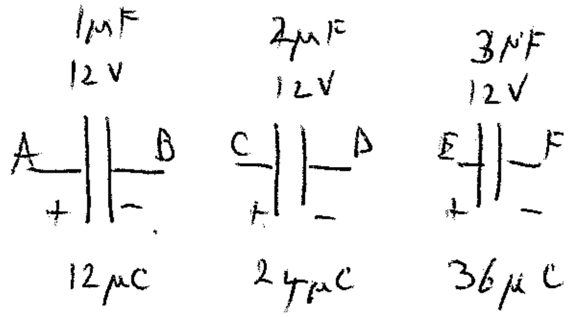
substituting in ①

$\frac{Q}{Q_f} = \frac{q - Q}{q}$

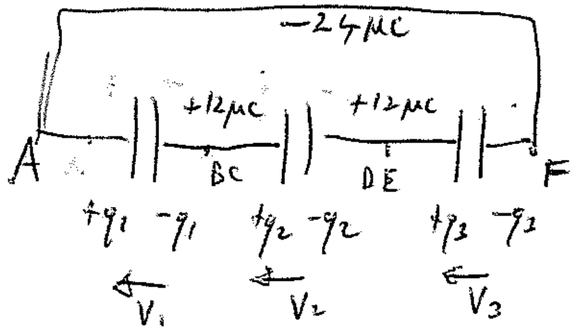
$\frac{1}{Q_f} = \frac{1}{Q} - \frac{1}{q}$ ✓

or $Q_f = \frac{qQ}{q - Q}$

(iii)



Connected



By conservation of energy (Kirchhoff II)

$$V_1 + V_2 + V_3 = 0 \quad (1)$$

Isolated plates:

$$-q_1 + q_2 = 12\,\mu C \quad (2)$$

$$-q_2 + q_3 = 12\,\mu C \quad (3)$$

$$-q_3 + q_1 = -24\,\mu C \quad (4)$$

} any of these.

From (1) we may write $(V = \frac{Q}{C})$

$$q_1 + \frac{q_2}{2} + \frac{q_3}{3} = 0 \quad [q \text{ in } \mu C]$$

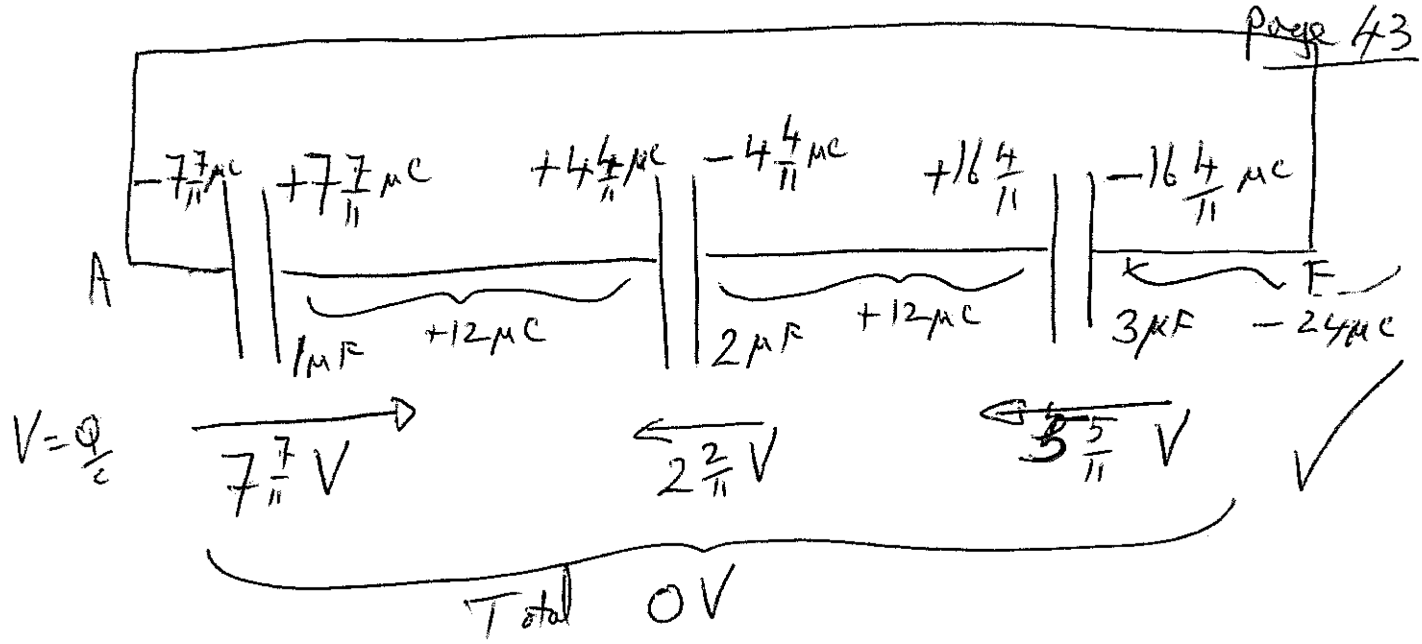
or $\times 6$ $\quad 6q_1 + 3q_2 + 2q_3 = 0$

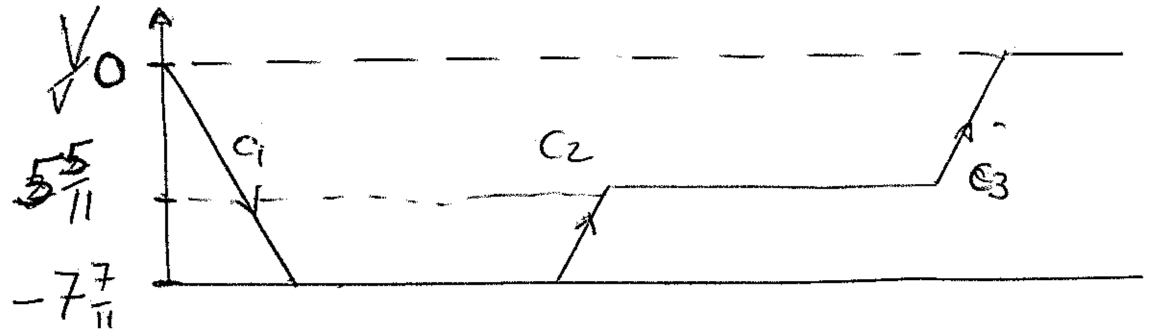
so, substituting for q_1 from (4) and q_2 from (3)

$$6(q_3 - 24) + 3(q_3 - 12) + 2q_3 = 0$$

$$6q_3 - 6.24 + 3q_3 - 36 + 2q_3 = 0$$

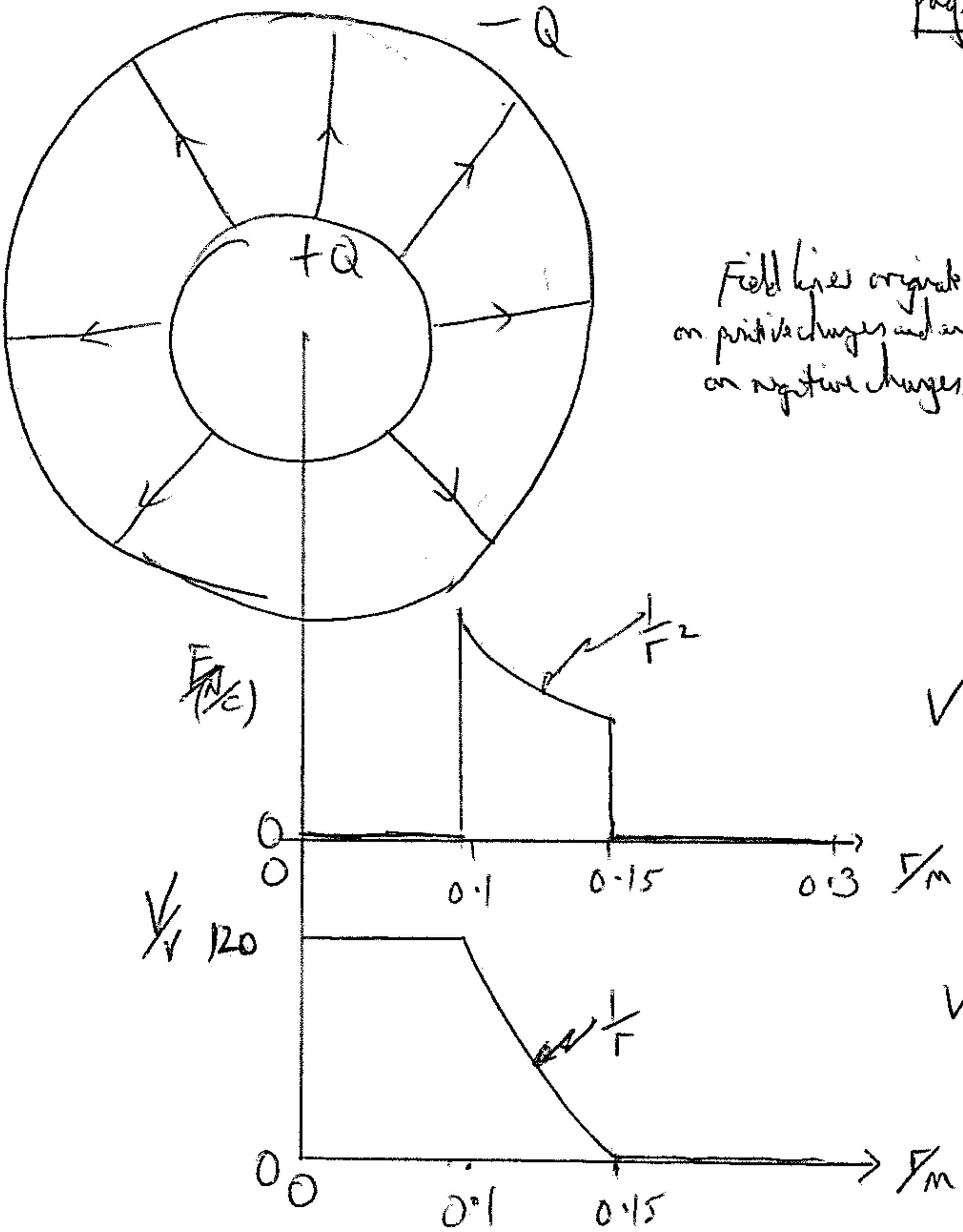
$$q_3 = \frac{180}{11}\,\mu C = 16\tfrac{4}{11}\,\mu C$$


A
F
$-7\frac{7}{11}\mu C$
$+7\frac{7}{11}\mu C$
$+4\frac{4}{11}\mu C$
$-4\frac{4}{11}\mu C$
$+16\frac{4}{11}$
$-16\frac{4}{11}\mu C$
$1\mu F$
$+12\mu C$
$2\mu F$
$+12\mu C$
$3\mu F$
$-24\mu C$
$V=\frac{Q}{C}$
$7\frac{7}{11}V$
$2\frac{2}{11}V$
$5\frac{5}{11}V$
Total 0V


V
0
$5\frac{5}{11}$
$-7\frac{7}{11}$
c_1
c_2
c_3

9

(d)



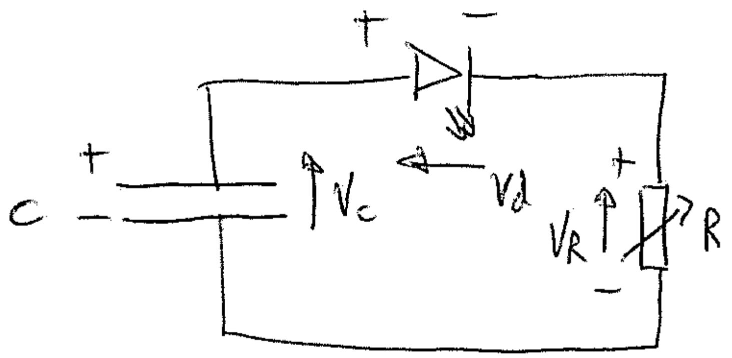
Field lines originate on positive charges and end on negative charges.

(iii)
$$V_i = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{0.1}$$
$$V_o = \frac{1}{4\pi\varepsilon_0} \frac{-Q}{0.15}$$

(iv)
$$V_i - V_o = 120\ \text{V} = \frac{1}{4\pi\varepsilon_0} \cdot Q \left(\frac{1}{0.1} - \frac{(-1)}{0.15} \right)$$
$$120 = 8.99 \times 10^9 . Q \left(\frac{0.25}{0.015} \right)$$
$$\underline{Q = 8.0 \times 10^{-10}\ \text{C}}$$

(4)

(e)



$$V_c = V_d + V_R$$ Kirchhoff II. ✓

$$\left(V_0 - \frac{Q}{C}\right) = V_d + IR$$ charge Q flows, reducing V_c from V_0

$$V_0 - \frac{1}{C}\int_0^t I\,dt = V_d + IR$$

I is constant (stated in question)

$$\therefore \quad V_0 - \frac{I}{C}\int_0^t dt = V_d + IR$$ ✓

$$V_0 - \frac{I}{C}t = V_d + IR$$

$$R = -\frac{t}{C} + \frac{(V_0 - V_d)}{I}$$

So R varies linearly with time — it decreases to zero } equation or statement ✓

When $R = 0$,

$$t = \frac{C(V_0 - V_d)}{I}$$

$$= 0.4\times10^{-6}\frac{(5.0 - 2.7)}{0.8}$$

$$= \frac{2.3\times10^{-6}}{2}$$

$$= \underline{1.15\times10^{-6}\ s}$$ ✓

(4)

END OF QUESTION 6.