BPhO Round 1 Marking- November 2019

Updated 9th Dec 2019

Thank you for taking part in marking the scripts. It is of enormous benefit to young students to be able to take part in these competitions, tackling much harder problems than they normally get, and be able to have them marked by physicists who know what they are doing. It is because of your expertise in the subject that it is possible to do this. Exams marked by non-specialists are more of a tick box exercise, which is of little value in stretching students to grasp the subject at a deeper level. The layout of the work may be annoying. That is a national problem and we are not going to change that easily, although we do our best, as do the teachers of these students.

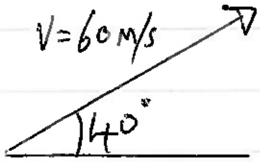
- Positive marking is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- Significant figures. This is not a test of significant figures. A leeway of ±1 sig fig is generally allowed, but we are not being strict at all in penalising for sig figs. If the published solution gives 3, allow 2 or 4 in the students answer. However, unless they have put down all of the figures on their calculator, there are no questions this year that are likely to invoke a sig fig penalty.
- Some answers can be left in fractional form.
- Units should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Some allowance may be made if it is clear that the unit has been used a line or two earlier.
- If the units are a required part of the answer for the mark, they must be there.
- Error carried forward (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks for the next one or two steps only. You do not need to spend time working through laborious arithmetic calculations if there is a possible ecf. Just make a decision as to whether they should have the single mark or not.
- You are not required to spend time deciphering scribble.
- There may be a lot of working for the answer. If they are almost there, you may give the mark even if there is a numerical mistake in the last line. Use your judgement. The ticks for the marks are not exact i.e. they are for the idea and almost getting there.
- Full marks are awarded for the correct answer, provided that there is some supporting working and it is not a "show that" question.
- You must follow the mark scheme so that we mark CONSISTENTLY. Do not make your own independent mark scheme.
- Avoid relying on your memory for the mark allocation. You need the mark scheme open beside you.

If you need advice, email Robin Hughes rh584@cam.ac.uk . I will respond promptly. You can send a phone photo or just ask a question. We want to be fair in the marking, so your good judgment should be acceptable.

1. Do not mix up students' names when entering marks.
2. Add up the marks correctly. Check your addition. For each little section, note the total in a circle, as in the mark scheme. Then note the total for the page at the bottom in two parallel lines to distinguish it.
3. Do not leave papers in any unsecured place. The loss of papers would be serious. They are confidential.

Nov 2019 Round I Section 1.

(a)



(i) $V_v = u - gt$

$= 60\sin 40 - 9.81\times 3$

$= \underline{9.14 \ m/s}$

only 1 mark if given as V_v, V_H.

$\underline{V_H = 46.0 \ m/s}$

$\therefore \ \underline{\underline{V = 46.9 \ m/s}}$ ✓ at $\theta = \tan^{-1}\dfrac{9.137}{45.96}$ ✓

$= \underline{\underline{11.2°}}$ above horizontal

[magnitude, direction]

(ii) $S_v = ut - \frac{1}{2}gt^2$

$= u\sin\theta . t - \frac{1}{2}gt^2$

$= 60 . \sin 40 . 3 - \frac{1}{2} 9.81\times 3^2$

$\underline{\underline{= 71.6 \ m}}$ ✓

$S_H = 60\cos\theta . 3$

$\underline{\underline{= 138 \ m}}$ ✓

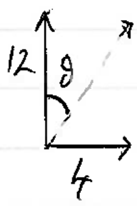
4

(b) Drone: $\vec{s} = 2t\hat{i} + 6t\hat{j}$

$\vec{v} = 2\hat{i} + 6\hat{j}$

(i) $\underline{\text{speed}} = \sqrt{2^2 + 6^2} = 2\sqrt{10} = \underline{\underline{6.3 \ m/s}}$ ✓

(ii) Bearing at $t = 2s$ is



$\tan\theta = \frac{4}{12} = \frac{1}{3}$.

$\underline{\underline{\theta = 18.4°}} = 018.4°$

(allow 18.4°). ✓

(iii) $\underline{\underline{a = 0}}$ ✓

3

(c) Areal density of paper ~ 100 g/m^2

pinhead $\approx$ 1 mm × 1 mm
$= 10^{-6}\ m^2$

$\therefore$ Mass $= 10^{-6} \times 100\ g$
$= 10^{-4}\ g$
$= \underline{\underline{10^{-7}\ kg}}$ ✓

clear working approach ✓
Just numbers – 0

(2)

$3 \times 10^{-8} \leqslant M \leqslant 0.9 \times 10^{-6}\ kg$

i.e. $0.3 \times 10^{-7} \leqslant M \leqslant 9 \times 10^{-7}\ kg$

(d) $[a]$ is m/s^2 ✓

$[b]$ is kg/s^2 ✓

(2)

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cross sectional area of a strand is A, length, l.

(e) $R_{Al} = \dfrac{\rho_{Al} \cdot l}{6A}$

$R_s = \rho_s \dfrac{\cdot l}{A}$

either correct form using resistivity. ✓

Rcable: $\dfrac{1}{R_c} = \dfrac{1}{R_{Al}} + \dfrac{1}{R_s} \Rightarrow R_c = \dfrac{R_{Al} \cdot R_s}{R_{Al} + R_s}$

correct relation (either) ✓

As the steel reduces the resistance of the cable by δR to R_c, then, $\underline{\underline{R_c + \delta R = R_{Al}}}$ ✓

So $\delta R = R_{Al} - R_c$

$= R_{Al} - \dfrac{R_A \cdot R_s}{R_{Al} + R_s}$

$= \dfrac{R_{Al}^2}{R_{Al} + R_s}$

$= \dfrac{\rho_{Al} \cdot l}{6A} \times \cancel{\dfrac{\rho_{Al} \cdot l}{6A}} \dfrac{1}{(\rho_{Al} + 6\rho_s)\cancel{\dfrac{l}{6A}}}$

$= \dfrac{\rho_{Al} \cdot l}{6A} \dfrac{1}{\left(1 + 6\frac{\rho_s}{\rho_{Al}}\right)}$

$$\therefore\ l = \frac{6A \cdot \delta R}{\rho_{Al}}\left(1 + 6 \cdot \frac{\rho_s}{\rho_{Al}}\right)$$ ✓ 'close' algebraic form for l.

$$= \frac{1.4\times10^{-4}\times6\times5\times10^{-4}}{3.2\times10^{-8}}\left(1 + 6\times\frac{20\cancel{\times10^{-8}}}{3.2\cancel{\times10^{-8}}}\right)$$

$$= 1.4\times\frac{30}{3.2}\left(1+\frac{120}{3.2}\right)$$

$$= \frac{42}{3.2}\left(1+37.5\right)$$

$$= 505\ \text{m}$$

$$= \underline{\underline{500\ \text{m}}}$$ ✓ (5)

(f) Density

Sinks when weight of Pt and wt. of K (potassium) equal weight of Hg displaced

✓ condition in words or equation

$$\rho_{Hg} V_{Hg} \cdot \cancel{g} = \rho_{Pt} \cdot V_{Pt} \cdot \cancel{g} + \rho_K \cdot V_K \cdot \cancel{g}$$

$$V_{Hg} = V_{Pt} + V_K$$ ✓

$$\therefore\ 13.6\,(V_{Pt} + 10) = 21.5\,V_{Pt} + 0.89\times10$$ ✓ correct substitution

$$V_{Pt} + 10 = \frac{21.5}{13.6}\cdot V_{Pt} + \frac{8.9}{13.6}$$

$$10 - \frac{8.9}{13.6} = V_{Pt}\left(\frac{21.5}{13.6} - 1\right)$$ (all the marks for the answer)

$$\underline{V_{Pt} = 16.1\ \text{cm}^3} = \underline{\underline{16\ \text{cm}^3}}$$ ✓ (4)

or algebraically,

$$V_{Pt} + V_K = \frac{\rho_{Pt}}{\rho_{Hg}}\cdot V_{Pt} + \frac{\rho_K}{\rho_{Hg}}\cdot V_K$$

$$0 = \left(\frac{\rho_{Pt}}{\rho_{Hg}} - 1\right) V_{Pt} + \left(\frac{\rho_K}{\rho_{Hg}} - 1\right) V_K$$

$$V_{Pt} = \frac{(\rho_{Hg} - \rho_K)}{(\rho_{Pt} - \rho_{Hg})}\cdot V_K = \left(\frac{13.6 - 0.89}{21.5 - 13.6}\right)\cdot 10 = 16.1\ \text{cm}^3 = 16\ \text{cm}^3$$

In notationless form.

$$10\times0.89 + x\times21.5 = 13.6(10 + x)$$

$$8.9 + 21.5x = 136 + 13.6x$$

$$7.9x = 127.1$$

$$\underline{\underline{x = 16.1\ \text{cm}^3}}$$

✱ Allow 4 marks if mixed up Pt and K to give 16.1 : 10 ratio i.e. $\underline{\underline{6.2\ \text{cm}^3}}$ of K.

Ice Melting

(g) 1 kg of ice has volume 1·087 litres. ✓

So there are 3·913 l of water in the bucket and hence 3·913 kg of water ✓

Heat 0.5 kg of ice + 0.5 kg of ice water up to T_{final} and cool 3·913 kg of water from 15°C to T_{final} } energy conservation idea/expression ✓

$\therefore \quad 3 \cdot 913 \times 4180 \times (15 - T_f) = 0.5 \times 3.34 \times 10^5 + 0.5 \times 4180 \times (T - 0)$

✓ correct substitution.

$3 \cdot 913 \times 4180 \times 15 - 0.5 \times 3.34 \times 10^5 = (3 \cdot 913 + 0.5)\, 4180\, T$

$T = 4.25°C$ ✓

$= 4.3°C$

5

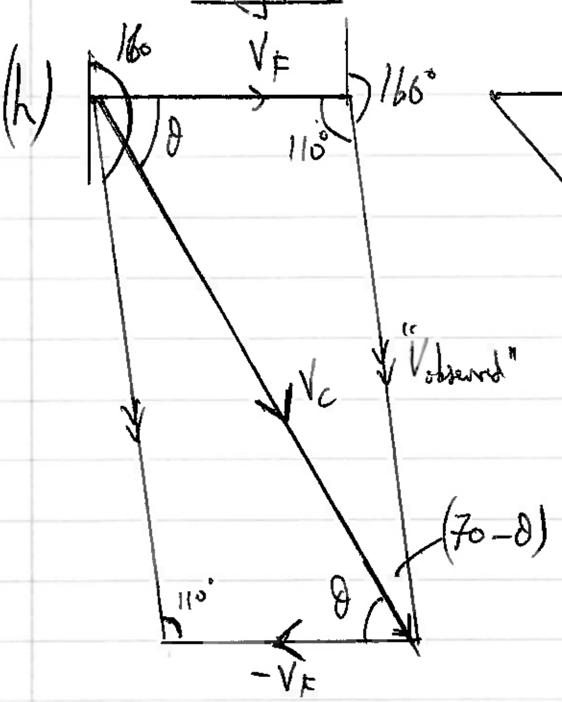
Not 4·8°C as they have missed out the ice-water warming (0.5 kg) from 0 → T_{final}

Max 3 marks then.

(h) Ferry and boat next page

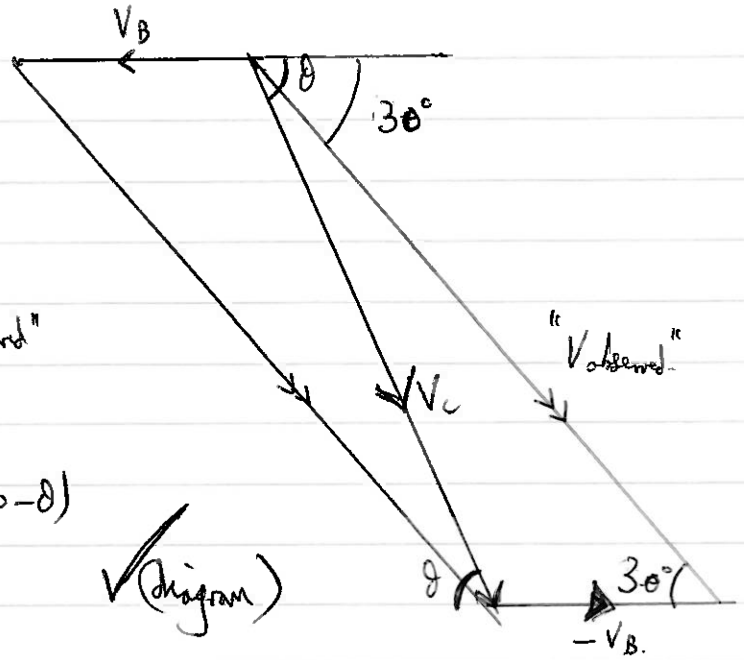
(h)

Ferry view



✓ (diagram)

Pilot Boat view



✓ (diagram)

Sine Rule

$$\frac{V_C}{\sin 110^\circ} = \frac{V_F}{\sin(70^\circ-\theta)} \quad ✓ \qquad \frac{V_c}{\sin 30^\circ} = \frac{V_B}{\sin(\theta-30)} \quad ✓$$

Dividing to eliminate V_c, ✓ (procedure) $\quad \dfrac{\sin 30}{\sin 110} = \dfrac{\sin(\theta-30)}{\sin(70-\theta)} \cdot \dfrac{V_F}{V_B}$ — 5, — 7.5

$$\frac{5}{7.5}\,\frac{\sin 110}{\sin 30} \times \sin(\theta-30) = \sin(70-\theta)$$

$$1.253\,(\sin\theta\cos 30 - \sin 30\cos\theta) = \sin 70\cos\theta - \cos 70\sin\theta$$

$$1.427\sin\theta = 1.566\cos\theta$$

$$\tan\theta = 1.097$$

$$\underline{\underline{\theta = 47.7^\circ}} \quad ✓$$

Bearing is 138°

$$\left[\tan\theta = \frac{5}{(2\sqrt{3}+3\tan 20^\circ)}\right].$$

$$V_c = \frac{0.5\times 7.5}{\sin(\theta-30^\circ)} = 12.36 = \underline{\underline{12.4\ m/s}} \quad ✓$$

7

Car accelerating.

(i) $F.V = P$

so $MaV = P$ — Zero marks after this point if they use eqns. for constant acceleration.

and $M \times V \frac{dV}{dx} \times V = P$ — $\left[a = V\frac{dV}{dx}\right]$

so $MV^2 dV = P\,dx$

Integrating $\frac{MV^3}{3} = Px + c$

At $x=0$, $V=0$ $\therefore c=0$. — constant of integration is zero specifically evaluated at $x=0$, $V=0$

So $V = \left(\frac{3Px}{m}\right)^{\frac{1}{3}}$

(4)

or $F.V = \frac{d}{dt}\left(\frac{1}{2}mV^2\right)$

and $ma\frac{dx}{dt} = \frac{d}{dt}\left(\frac{1}{2}mV^2\right) = \frac{1}{2}m\,2V\frac{dV}{dt}$

$\therefore a\frac{dx}{dt} = V\frac{dV}{dt}$

and $a = V\frac{dV}{dx}$.

etc. as above.

By dimensions.
$V = P^{\alpha}x^{\beta}M^{\gamma}$
$LT^{-1} = [ML^2T^{-3}]^{\alpha}L^{\beta}M^{\gamma}$
equate powers.
L: $1 = 2\alpha + \beta$
M: $0 = \alpha + \gamma$
T: $-1 = -3\alpha$
$\alpha = \frac{1}{3}$, $\beta = \frac{1}{3}$, $\gamma = -\frac{1}{3}$
$V = k\left(\frac{Px}{m}\right)^{\frac{1}{3}}$ But no marks as we want the constant k.

In terms of t

$E_k = Pt$

$\frac{1}{2}MV^2 = Pt$

$t = \frac{MV^2}{2P}$, $V = \sqrt{\frac{2Pt}{m}}$

$\frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$

$\int_0^x dx = \sqrt{\frac{2P}{m}}\int_0^t t^{\frac{1}{2}}dt$

$x = \sqrt{\frac{2P}{m}}\,\frac{2}{3}t^{\frac{3}{2}}$

Hence $x = \sqrt{\frac{8Pt^3}{9m}} = \sqrt{\frac{8P}{9M}\cdot\frac{M^3V^6}{8P^3}}$

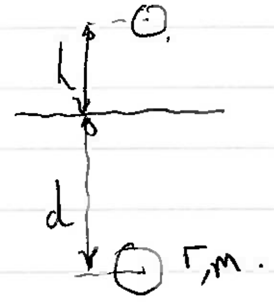
$= \frac{MV^3}{3P}$

$\therefore V = \left(\frac{3Px}{m}\right)^{\frac{1}{3}}$

(4)

Submerged ball.

(j)



Net force upwards = buoyancy force − weight

$$ma = \rho_w \cdot V \cdot g - mg$$

statement about forces.

$$a = \rho_w \frac{V \cdot g}{m} - g$$

a is constant, so eqns. of motion apply (implied)

At depth d, $v^2 = 0^2 + 2\left(\frac{\rho_w V \cdot g}{m} - g\right) \cdot d$

"$v^2 = 2gh$" at surface.

$$\therefore \quad 2gh = 2g\left(\frac{\rho_w \cdot V}{m} - 1\right)d$$

$$\frac{h}{d} = \left(\frac{\rho_w \cdot V}{m} - 1\right)$$

$$\frac{h}{d} = \left(\frac{\frac{4}{3}\pi r^3 \rho}{m} - 1\right)$$

- Lose 1 mark if $\left(1 - \frac{\frac{4}{3}\pi r^3 \rho}{m}\right)$ i.e. wrong sign. 5

Egg Timer

(k) $\frac{dm}{dt} = k\,\rho . A . g$

$$[MT^{-1}] = [ML^{-3}]^{\alpha} . [L^{2}]^{\beta} . [LT^{-2}]^{\gamma}$$

or $$kg\,s^{-1} = (kg\,m^{-3})^{\alpha} . (m^{2})^{\beta} . (m\,s^{-2})^{\gamma}$$

dimensional or unit equation. ✓

equating powers

kg	$1 = \alpha$	$\Rightarrow \alpha = 1$
s	$-1 = -2\gamma$	$\Rightarrow \gamma = \frac{1}{2}$ ✓
m	$0 = -3\alpha + 2\beta + \gamma$	$\Rightarrow \beta = \frac{5}{4}$

✓ for α and β. (1 mark for γ).

$$\frac{dm}{dt} = k\,\rho\,A^{\frac{5}{4}} . g^{\frac{1}{2}}$$

$$\frac{\Delta m}{\Delta t_M} \Big/ \frac{\Delta m}{\Delta t_E} = \sqrt{\frac{g_m}{g_E}}$$

$$\Delta t_M = \Delta t_E \sqrt{\frac{g_E}{g_M}}$$

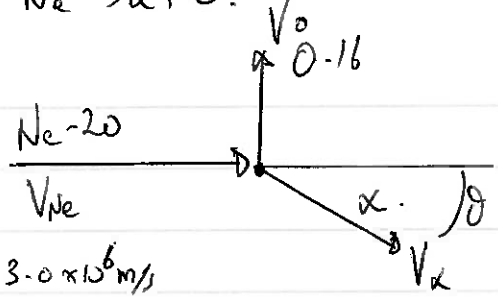
$$= \sqrt{\frac{9.8}{1.6}} \times 15$$

$$= 37.1 \text{ minutes}$$ ✓

Even if they obtain this result, they need the other parts to be correct in this question. 4

$Ne \rightarrow \alpha + O.$

(l)



$V_{Ne} = 3.0 \times 10^6 \, m/s$

Cons of mom along line of flight $\quad 20.\cancel{u} \times V_{Ne} = 4\cancel{u}.V_\alpha \cos\theta$ ① ✓

" " ⊥ line of flight $\quad 16\cancel{u} V_o = 4\cancel{u} V_\alpha \sin\theta$ ② ✓

square and add $\quad \underline{400 V_{Ne}^2 + 256 V_o^2 = 16 V_\alpha^2}$ ③ ✓

Cons of energy: $\quad \underline{\frac{1}{2}.20u.V_{Ne}^2 + E_0 = \frac{1}{2}.4u.V_\alpha^2 + \frac{1}{2}.16u.V_o^2}$ ④

either or for mark.

Substituting values.

In ③ $\quad 3.6 \times 10^{15} + 16^2 V_o^2 = 16 V_\alpha^2$ ⑤

In ④ $\quad \underline{\frac{1}{2} 20u.(3\times10^6)^2 + 62.5 \times 10^6 \times 1.6\times10^{-19} = \frac{1}{2} 4u V_\alpha^2 + \frac{1}{2} 16u V_o^2}$ ✓✓

(×2) $\quad$ So, $20u(9\times10^{12}) + 2\,10^{-12} = 4u.V_\alpha^2 + \left(V_\alpha^2 - \frac{3.6\times10^{15}}{16}\right)u$

and Substituting ⑤ ÷16

(÷u) Then $\quad 180 \times 10^{12} + \frac{2\times10^{-12}}{u} = 5V_\alpha^2 - \frac{3.6\times10^{15}}{16}$

$$(180 + 225)\times10^{12} + \frac{2\times10^{-12}}{1.66\times10^{-27}} = 5V_\alpha^2$$

$$(405 + 1205)\times10^{12} = 5V_\alpha^2$$

$$1610 \times 10^{12} = 5V_\alpha^2$$

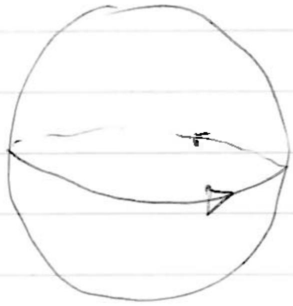
$$V_\alpha = 1.79 \times 10^7 \quad ✓$$

$$\underline{\underline{V_\alpha = 1.8 \times 10^7 \, m/s}}$$

(5)

plane flying

(m)



$$V_E = \frac{2\pi r}{T}$$

$$= \frac{2\pi \times 6.37\times10^6}{24\times3600}$$

$$= \underline{463 \text{ m/s}}$$ ✓ speed of equator.

$$W_1 = mg - m\frac{(463-250)^2}{r}$$ ✓ subtract speeds and $\frac{V^2}{r}$

$$W_2 = mg - m\frac{(463+250)^2}{r}$$ ✓ add speeds and $\frac{V^2}{r}$.

$$W_1 - W_2 = \frac{m}{r}\left[(463+250)^2 - (463-250)^2\right]$$ ✓

$$= \frac{1}{6.37\times10^6}\left[713^2 - 213^2\right]$$

$$= \frac{926\times500}{6.37\times10^6}$$

$$= 0.0727 \text{ N}$$

$$= \underline{0.073 \text{ N}}$$ ✓ full marks for the answer with any working.

or $W_2 - W_1 = \frac{m}{r}\left[(v_E+v_p)^2 - (v_E-v_p)^2\right]$

$= \frac{m}{r}4v_E v_p$

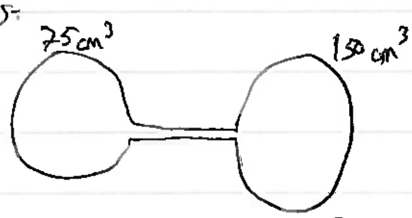
$= \frac{4(436\times250)}{6.37\times10^6}$

$= \underline{0.073\text{N}}$.

5

Glass bulbs.

(n)



Quantity of gas (n moles) is fixed ✓ idea

$$\therefore n = \frac{P_1V_1}{RT_1} + \frac{P_2V_2}{RT_2} = \frac{P}{RT}(V_1+V_2)$$

$$= \frac{0.91\times10^5\,(225\times10^{-6})}{8.31\times261}$$ ✓ values and kelvin

$$= \underline{9.44\times10^{-3} \text{ moles}}$$ ✓ result

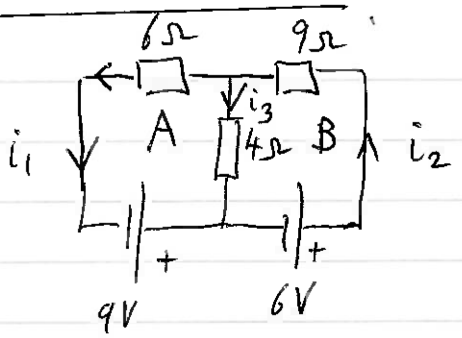
Then $$n = \frac{P}{R}\left(\frac{V_1}{T_1} + \frac{V_2}{T_2}\right) = \frac{P_{final}}{8.31}\left(\frac{75\times10^{-6}}{297} + \frac{150\times10^{-6}}{261}\right)$$ ✓ values. AND in kelvin

$$= \frac{P_{final}\times10^{-6}}{8.31}\left(\frac{75}{297} + \frac{150}{261}\right).$$

$$P_{final} = 94832 \text{ Pa}$$
$$= \underline{95000 \text{ Pa}}$$ ✓ result.

5

(o) Current in 6.0 Ω resistor.



currents with directions marked ✓

KII Loop A: $9 = -4i_3 + 6i_1$ ① ✓

B: $6 = 9i_2 + 4i_3$ ② ✓

KI $i_2 = i_1 + i_3$ ③ ✓

[consistent with diagram.]

Substitute for i_1 in ① $9 = 6i_2 - 6i_3 - 4i_3$

$9 = 6i_2 - 10i_3$

$\times \frac{3}{2} \longrightarrow \frac{3}{2}.9 = 9i_2 - 15i_3$

subtract ② $\longrightarrow \frac{3}{2}9 - 6 = -15i_3 - 4i_3$

$7.5 = -19i_3$

$i_3 = -\frac{7.5}{19} = -0.395\ \text{A}$

So $9 = 6i_1 - 4i_3$

$i_1 = \frac{47}{38} = 1.237 = \underline{1.24\ \text{A}}$ flowing to the left. ✓ (direction must be clear from arrows on diagram)

5

(p) Photoelectric Effect.

$E_\gamma = \frac{hc}{\lambda} = 1.326 \times 10^{-18}\ \text{J}$

$= 8.2875\ \text{eV}$ ✓

Max ke of emitted electrons is $8.2875 - 4.5\ \text{eV}$

$= 3.7875\ \text{eV}$ ✓

So $V_{max} = 3.7875\ \text{V} = 3.8\ \text{V}$ ✓

$V_{max} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$

$\therefore Q = 4\pi\varepsilon_0\, r\, V_{max} = 8.42 \times 10^{-13}\ \text{C}$.

$= \underline{5.3 \times 10^{6}\ \text{electrons}}$. ✓

4

(q) Three conducting spheres

$k = \frac{1}{4\pi\varepsilon_0}$ $V = k\frac{Q}{r}$

$\frac{1}{3}R$ ① $\frac{1}{2}R$ ② R ③

$Q = \frac{r V}{k}$

$$Q_{total} = Q_1 + Q_3$$

$$= \frac{1}{3}\frac{RV}{k} + \frac{RV}{k}$$

$$Q_t = \frac{4}{3}\frac{RV}{k}$$

Then 3 spheres are connected and reach potential V'

So $Q_t = \frac{R}{3}\frac{V'}{k} + \frac{R}{2}\frac{V'}{k} + \frac{RV'}{k}$

$$= \frac{RV'}{k}\left(\frac{1}{3} + \frac{1}{2} + 1\right)$$

$$= \frac{RV'}{k}\cdot\frac{11}{6}$$

$\therefore \frac{4}{3}\frac{RV}{k} = \frac{11}{6}\frac{RV'}{k} \Rightarrow V' = \frac{8}{11}V$ (Full marks for this result)

Hence $\frac{Q_2}{Q_{total}} = \frac{\frac{R}{2}\frac{V'}{k}}{\frac{4}{3}\frac{RV}{k}}$

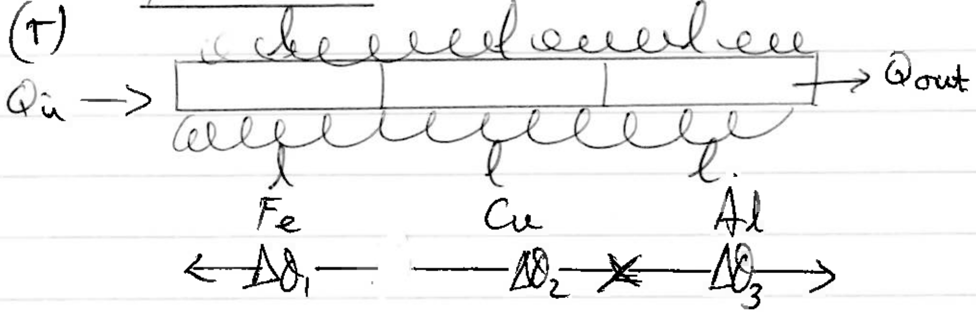
$$= \frac{\frac{R}{2}\frac{8}{11}V}{\frac{4}{3}RV} = \frac{8\cdot 3}{2\cdot 11\cdot 4}$$

$$= \frac{3}{11}$$

5

Thermal conduction

(T)



$$k_1 A \frac{\Delta\theta_1}{l} = k_2 A \frac{\Delta\theta_2}{l} = k_3 A \frac{\Delta\theta_3}{l}$$

So $\quad k_1 \Delta\theta_1 = k_2 \Delta\theta_2 = k_3 \Delta\theta_3$ ✓ either form

and $\quad \Delta\theta_1 + \Delta\theta_2 + \Delta\theta_3 = 100$ ✓

Now we can write $\quad \Delta\theta_1 = \frac{k_2}{k_1} \Delta\theta_2$

and $\quad \Delta\theta_2 = \frac{k_3}{k_2} \cdot \Delta\theta_3$

Hence $\quad \Delta\theta_1 = \frac{\cancel{k_2}}{k_1} \cdot \frac{k_3}{\cancel{k_2}} \cdot \Delta\theta_3$

So that $\quad \left(\frac{k_3}{k_1} + \frac{k_3}{k_2} + 1\right) \Delta\theta_3 = 100$ ✓ or equivalent step

Thus $\quad \left(\frac{240}{60} + \frac{240}{400} + 1\right) \Delta\theta_3 = 100$

$$\Delta\theta_3 = \frac{100}{5.6} = \underline{17.86^\circ C}$$

$$\Delta\theta_1 = \frac{k_3}{k_1} \Delta\theta_3 = 4\Delta\theta_3 = \underline{71.43^\circ C}$$

$$\Delta\theta_2 = \frac{k_3}{k_2} \Delta\theta_3 = \frac{240}{400} \Delta\theta_3 = 10.7^\circ C$$

$\therefore$ Cu/Al junction is $\underline{17.9^\circ C}$ ✓

Fe/Cu junction is $17.86 + 10.7 = \underline{28.6^\circ C}$ ✓

5

<u>bicycle pump</u>

(s)

displacement node + antinodes.

N A

$\frac{\lambda}{4}$ fundamental note in tube (ignore end effect).

$\therefore \frac{\lambda}{4} = \frac{330}{4\times512} = \underline{0.161\ m}$ ✓

Boyle's Law $P_1V_1 = P_2V_2$

$10^5 \times 25\,cm = P_2 \times 16.1\,cm$

$\underline{P_2 = 1.55\times10^5\ Pa}$ ✓

$\therefore F = (P_{inside} - P_{at}) \times area$ ✓ $(P_{in}-P_{at})$.

$= (1.55 - 1.0)\times10^5\times A$

$\underline{F = 5.5\times10^4\ A}$ newtons. ✓

(4)

<u>cooling</u>

(t) $\frac{d\theta}{dt} = -k(\theta-\theta_0)$ ✓ $\longrightarrow \frac{\Delta\theta}{\Delta t} = k(\theta-\theta_0)$ ✓ (need $\theta-\theta_0$) ✓

$\frac{0.5}{1} = k(9.95)$

integrate. $\ln(\theta-\theta_0)\Big|_{\theta_i}^{\theta_f} = -kt\Big|_0^t$ ✓

So, $k = \frac{0.5}{9.95}$ ✓

$\ln\frac{(\theta_f-\theta_0)}{(\theta_i-\theta_0)} = -kt$

$\frac{(24-23)}{t} = \frac{0.5}{9.95}(3.5)$ ✓

So $\ln\frac{(29.7-20)}{(30.2-20)} = -k.1$

Hence $\ln\left(\frac{9.7}{10.2}\right) = -k$

$\Rightarrow \underline{t = 5.7\ minutes}$ ✓

$\underline{k = 0.0503}$ ✓

So. $\ln\left(\frac{23-20}{24-20}\right) = -0.0503\,t$ ✓

$\ln\frac{3}{4} = -0.0503\,t$

$\underline{t = 5.7\ \text{Minutes}}$ ✓

(5)

Question 2

(a) $E = mgh$

$P = \frac{mgh}{t}$ ✓

$= 3380 \times 10^3 \times 9.81 \times 61$ ✓ (correct values)

$= 2.02 \times 10^9$ W ✓ (3)

or $P = \rho g h$, $F = \rho A g h$

$P = F.v$

$= \rho A v g h$

$= 3380 \times 10^3 \times 9.81 \times 61$

$= 2.02 \times 10^9$ W

(b) To keep from blocking the outlet, the water must keep moving. ✓ (idea)

$P_{wasted} = \frac{1}{2} \frac{m v^2}{t}$

$= \frac{1}{2} 3380 \times 10^3 \times 8^2$

$= 0.108 \times 10^9$ W $= 0.11 \times 10^9$ W ✓

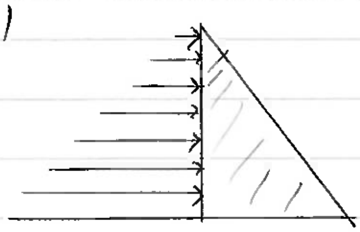
$\eta = 1 - \frac{0.108}{2.023} = 1 - 0.0535$

$= 94.7\%$

$= 95\%$ ✓

(Not 5%) (3)

(c) (i)



Resultant force $= \int_0^h P(x) \cdot W \, dx$

$= \rho g W \int_0^h x \, dx$

$= \rho g W \frac{h^2}{2}$ ← required. ✓ (1)

or P varies linearly with depth. $\Rightarrow$ average pressure $= \rho g \frac{h}{2}$.

$\therefore$ resultant force $= \rho g \frac{h}{2} . (h W)$

$= \rho g W \frac{h^2}{2}$

(ii)

$$\text{Total moment} = \sum x \cdot \delta F$$
$$= \sum (P \cdot W\,\delta x)\,x \qquad (\text{Moment about } F) \checkmark$$
$$= \int_0^h \rho g (h-x)\,.dx.W.x$$
$$= \rho g W \int_0^h (h-x)\,x.dx$$
$$= \rho g W \left[\frac{hx^2}{2} - \frac{x^3}{3}\right]_0^h$$
$$= \rho g W \left[\frac{h^3}{2} - \frac{h^3}{3}\right]$$
$$= \rho g W \frac{h^3}{6} \quad . \text{ about } F. \checkmark$$

Using the previous resultant force acting at ht H from F,

$$H \cdot \rho g W \frac{h^2}{2} = \rho g W \frac{h^3}{6}$$

gives $H = h/3$ above F. ✓ (3)

Alternatively, from the top of the dam,

$$\text{Total moment} = \int_0^h \rho g\, x\, dx \cdot W \cdot x$$
$$= \rho g W \left.\frac{x^3}{3}\right|_0^h$$
$$= \rho g W \frac{h^3}{3}.$$

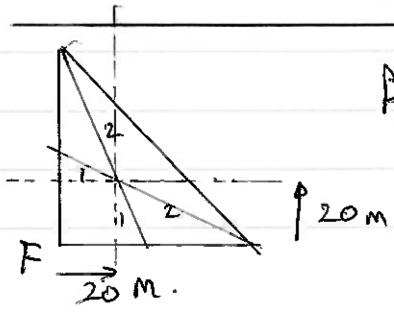
Using resultant force acting at a point H' below the top of the dam,

$$\rho g W \frac{h^3}{3} = \rho g W \frac{h^2}{2} \cdot H'$$

so $H' = \frac{2}{3}h$ below the top of the dam.

so $\frac{1}{3}h$ above F.

(iii)



By proportions, centre of mass is 20m above F and 20m to the right of F. ✓ (1)

(iv) Moment about C due to water pressure:
This is the same as the moment about F.

So, Mom about C is $\rho g W \frac{h^3}{6}$

$$= 1000 \times 9.81 \times 2700 \times \frac{60^3}{6}$$

$$= \underline{9.54 \times 10^{11} \text{ Nm}}$$

Mom. about C due to weight of dam is $\frac{1}{2} W . t . h g \rho_{concrete} \times \frac{2}{3} t$

$$= \frac{1}{2} 2700 \times 60^2 \times 9.81 \times \rho_{conc} \times 40$$

$$= 1.91 \times 10^9 \rho_{concrete}$$

③

(vi) $\therefore \quad \rho_{concrete} = \dfrac{9.54 \times 10^{11}}{\frac{1}{2} 2700 \times 60^2 \times 9.81 \times 40}$

$$= \underline{500 \text{ kg/m}^3}$$

i.e. in symbols $\rho g W \frac{h^3}{6} = \frac{1}{2} W t h g \rho_{conc} \cdot \frac{2}{3} t$

$$\rho \frac{h^2}{6} \cdot \frac{3}{t^2} = \rho_{concrete}$$

$t = h$ so $\rho_{concrete} = \frac{1}{2} \rho_{water} = 500 \text{ kg/m}^3$.

(v) $\mu . \frac{1}{2} W t h g \rho_{conc.} = \rho g W \frac{h^2}{2}$.

$$\mu t \rho_{conc} = \rho h$$

$$\rho_{conc} = \frac{\rho h}{\mu t} = \frac{1000}{0.75} \times \frac{60}{60}$$

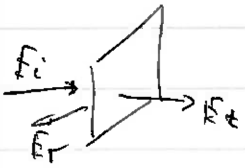
$$= \underline{1300 \text{ kg/m}^3}$$

②

(10)

(d) (i) Energy flow into boundary = energy flow out of boundary ✓ idea, diagram, statement.

$$\omega^2 Z_1 A_i^2 = \omega^2 Z_1 A_r^2 + \omega^2 Z_2 A_t^2$$



$$Z_1 (A_i^2 - A_r^2) = Z_2 A_t^2$$

(ii)

$$Z_1 (A_i - A_r)(A_i + A_r) = Z_2 A_t^2$$

Divide by eqn. 1 squared

$$(A_i + A_r)^2 = A_t^2$$

then

$$Z_1 \frac{(A_i - A_r)}{(A_i + A_r)} = Z_2$$

$$Z_1 \frac{\left(1 - \frac{A_r}{A_i}\right)}{\left(1 + \frac{A_r}{A_i}\right)} = Z_2$$

$$Z_1 - Z_1 \frac{A_r}{A_i} = Z_2 + Z_2 \frac{A_r}{A_i}$$

$$\frac{A_r}{A_i} = \frac{(Z_1 - Z_2)}{(Z_1 + Z_2)}$$

✓ for derivation (difference of squares, use of 2nd equation etc.

No mark for answer.

(iii)

$$\frac{A_t}{A_i} = 1 + \frac{A_r}{A_i}$$

$$= \frac{Z_1 + Z_2 + Z_1 - Z_2}{(Z_1 + Z_2)}$$

$$= \frac{2Z_1}{(Z_1 + Z_2)}$$ ✓

(iv) If $Z_1 = Z_2$, there is no identifiable boundary, so $A_r = 0$ and $A_t = A_i \Rightarrow \frac{A_r}{A_i} = 0$ AND $\frac{A_t}{A_i} = 1$ ✓

(v) $Z_2 > Z_1$ $\quad \frac{A_r}{A_i}$ has a negative value corresponding to a phase change on reflection. ✓

5

No mark for this "Show that"

(vi) $$R = \left(\frac{A_r}{A_i}\right)^2 = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}$$

$$T = \frac{Z_2 A_t^2}{Z_1 A_i^2} = \frac{\cancel{Z}_2}{Z_1} \frac{4 . Z_1^2}{(Z_1 + Z_2)^2}$$

$$= \frac{4 Z_1 Z_2}{(Z_1 + Z_2)^2}$$

✓

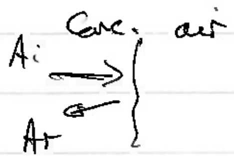
(vii) And $$R + T = \frac{(Z_1 - Z_2)^2 + 4 Z_1 Z_2}{(Z_1 + Z_2)^2}$$

$$= \frac{(Z_1 + Z_2)^2}{(Z_1 + Z_2)^2}$$

$$= \underline{1}$$

✓

(viii)



$$R_{(i)} = \frac{(2400 \times 3700 - 1 \cdot 2 \times 330)^2}{(2400 \times 3700 + 1 \cdot 2 \times 330)^2}$$

$= 0 \cdot 9998$

$= 100\%$

✓

$$R_{(ii)} = \left(\frac{2400 \times 3700 - 1000 \times 1480}{2400 \times 3700 + 1000 \times 1480}\right)^2$$

$= 0 \cdot 51$

$= \underline{51\%}$

✓

4

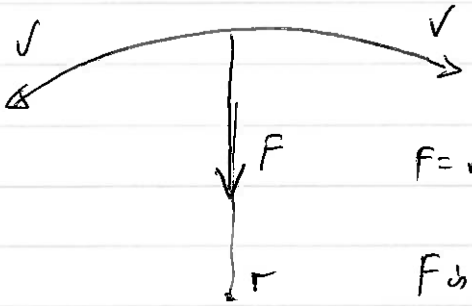
(9)

$\boxed{25}$

Question 3

(a) The velocity is constantly changing and by NII this means a force is acting.

or direction of motion Not just direction

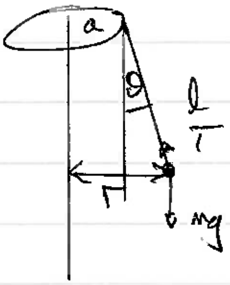


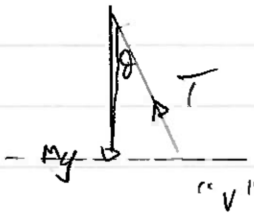
$F = \frac{mv^2}{r} = mr\omega^2$

F is towards the centre of the circular path.

2

(b)





"v" "ω"

$T\sin\theta = \frac{mv^2}{r} = mr\omega^2$

$= \frac{mv^2}{(a + l\sin\theta)} = m\omega^2(a + l\sin\theta)$

and $T\cos\theta = mg$

So $\tan\theta = \frac{v^2}{g(a + l\sin\theta)} = \frac{\omega^2}{g}(a + l\sin\theta)$

$\omega^2 = \frac{g\tan\theta}{(a + l\sin\theta)}$

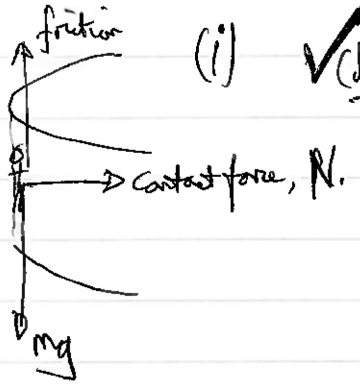
substituting. $\omega^2 = \frac{9.81 \times \frac{1}{\sqrt{3}}}{(6 + 4 \times 0.5)}$

$\omega = 0.84$ rad/s.

either or.

2

(c) (i) ✓ (diagram)



(ii) $N = mr\omega^2$

Resolve V: $f = mg$

so $\mu N = mg$

$\mu m r \omega^2 = mg$ ✓

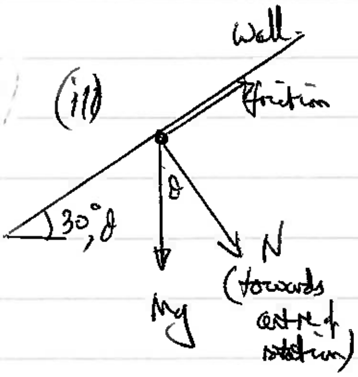
$\omega^2 = \dfrac{g}{\mu r}$

$\omega = \sqrt{\dfrac{9.81}{0.4 \times 5}}$

$\omega = 2.215$

$\omega = 2.2 \text{ rad/s}$ ✓ (3)

(iii) ✓ diagram



(iv) Resolve parallel to wall, $mg\cos 60 = f$ ①

Perpendicular to wall $mg\sin 60 + N = mr\omega^2$ ✓

and $f = 0.4 N$

$\therefore$ from ① $0.4N = mg\cos 60$

$N = 2.5 mg\cos 60$

Hence ② $mg\sin 60 + 2.5 mg\cos 60 = mr\omega^2$

$g\dfrac{\sqrt{3}}{2} + 2.5\dfrac{g}{2} = r\omega^2$

$2.12g = r\omega^2$

$\omega = 2.04 \text{ rad/s}$ [shown].

$= 2.0 \text{ rad/s}$ ✓ (3)

Alternative using $\theta = 30°$

Resolve parallel to wall - $mg\sin\theta = f$ ③

$\perp$ to wall $mg\cos\theta + N = mr\omega^2$ ④

$f = \mu N$ ⑤

so $mg\sin\theta = \mu N$ ③ and ⑤

Thus $mg\left(\cos\theta + \frac{1}{\mu}\sin\theta\right) = mr\omega^2$ from ④, So $\omega^2 = \frac{g}{r\mu}(\mu\cos\theta + \sin\theta)$ $\qquad \theta = 30°$.

$= \frac{9.81}{5 \times 0.4}\left(0.4\frac{\sqrt{3}}{2} + \frac{1}{2}\right)$

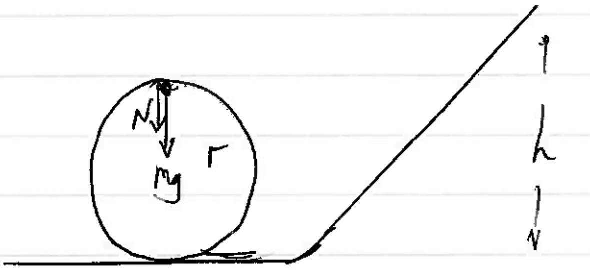
$= 4.15$

$\omega = 2.04$ rad/s.

$= \underline{2.0 \text{ rad/s}}$.

(d)

(i)



To remain in contact at the top of the loop. $N + mg = \frac{mv^2}{r}$

When $V = V_0$, minimum speed and $N = 0$

$\therefore mg = \frac{mV_0^2}{r}$

So $\frac{1}{2}mV_0^2 = \frac{1}{2}mgr$

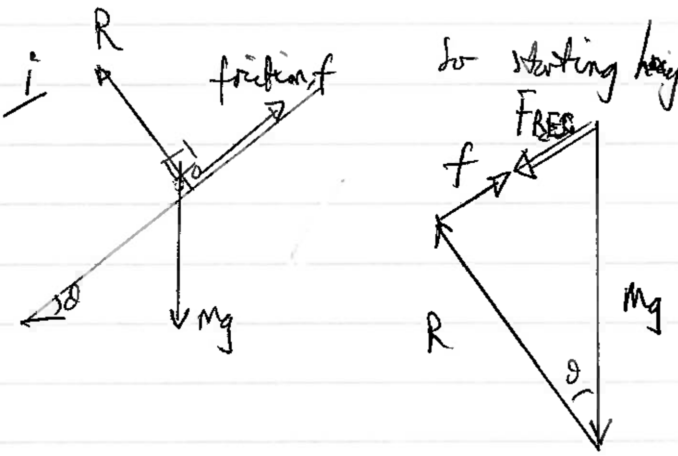
So energy at top of loop is $2mgr + \frac{1}{2}mgr$

$= \frac{5}{2}mgr$

So starting height is $h = \frac{5}{2}r = 50\,m$ ✓

(2)

(ii) i



✓ diagram

(continues overleaf)

ii WD by friction $= f.l$
$= \mu R . l$
$= \mu . l . mg\cos\theta$ ✓

Notation: final speed with friction, v_f
" " without " , v_0

$$\therefore \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_0^2} = \frac{mgl(\sin\theta - \mu\cos\theta)}{mgl\sin\theta}$$

$$\frac{v_f^2}{v_0^2} = 1 - \mu\cot\theta$$

✓ a result showing no 'm' dependence, only μ and θ this (May not be equation)

$$= 1 - 0.05 . \frac{1}{\tan 40^\circ}$$
$$= 0.9404$$
$$\frac{v_f}{v_0} = 0.96975$$

So, Loss in speed $= 0.03025$
$= 3.0\ \%$ ✓

4

(iii) There is a normal reaction force R, on the straight track. As soon as the carriage starts on the curve, there is a centripetal force added, of $\frac{mv^2}{r}$

Using energy cons. $\frac{1}{2}mv_0^2 = mgl\sin\theta$
$v_0^2 = 2gl\sin\theta$ ✓

$$\therefore \frac{mv_0^2}{r} = \frac{m}{l/2} . 2lg\sin\theta$$
$$= 4mg\sin\theta$$
$$= 4 \times 60 \times 9.81 \times \sin 40^\circ$$
$$= 1513\ N$$
$$= 1500\ N$$ ✓

2

8

(e) (i) Both bars fall through height h when A hits the ground.

So $\frac{1}{2}mv^2 = mgh \Rightarrow v = \underline{\sqrt{2gh}}$ ✓

(ii) B compresses the spring by ("not for" ...) an amount x_m

So Loss of gpe is $mgh + mgx_m$.

So for B, ($\frac{1}{2}mv^2$) $= mg(h + x_m) = E$ ✓

The loss of gpe is stored in the TWO springs.

$$\therefore \quad mg(h + x_m) = 2 \times \frac{1}{2}kx_m^2$$

(iii) B starts to rise again and will stretch the springs by x_e when A starts to leave the ground.

So $\underline{kx_e = \frac{1}{2}mg}$ for a spring. ✓ note factor of $\frac{1}{2}$.

(iv) Energy in the springs is $2 \times \frac{1}{2}kx_e^2$ ✓

whilst B has risen by $(x_e + x_m)$ from its lowest point

$\therefore$ E_{min} for lift A is gpe gained by B + pe in springs ✓

$$E_{min} = kx_e^2 + mg(x_e + x_m)$$ ✓

and this came from the compression of the springs by B $(2 \times \frac{1}{2}kx_m^2)$

$$\therefore \quad kx_e^2 + mg(x_e + x_m) = (mgh + mg\,x_m)$$

(initial energy of B)

thus $\quad kx_e^2 + mgx_e = mgh$ ✓

But we know already that $kx_e = \frac{mg}{2}$

Hence $\quad k\left(\frac{mg}{2k}\right)^2 + mg \cdot \frac{mg}{2k} = mgh$

$$\frac{mg}{4k} + \frac{mg}{2k} = h$$

$$\underline{h = \frac{3mg}{4k}}$$ ✓

(7)

Question 4

(a) (i) on the surface (on both faces). ✓

(ii)  ✓ (field lines to left AND right)

(iii) The denser (or closer together) the field lines the stronger E. ✓

(iv) $F = q E_s$ ✓

(4)

(b) (i) i $\sigma = \frac{Q}{4\pi r^2}$

$E = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2}$ } both. ✓

ii Hence $E = \frac{\sigma}{\varepsilon_0}$ ✓

and E does not depend upon r.

iii Half the field emerges on one face and half on the other

$\therefore$ $E = \frac{1}{2}\frac{\sigma}{\varepsilon_0}$ ✓

(ii) i $E_s = \frac{E_{sphere}}{2}$. ✓

ii The field strength in the hole would be cancelled by the $\frac{E_{sphere}}{2}$ of E_s.

$\therefore$ field through hole is $\frac{E_{sphere}}{2}$. ✓

iii Force on δA is field strength × charge

$\frac{E_{sphere}}{2} \times \delta q$

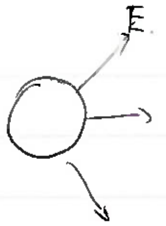
$= \frac{1}{2} \cdot \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \times \sigma \, \delta A$ ✓

iv Pressure on surface is $\frac{F}{\delta A} = \frac{1}{2} \frac{1}{\varepsilon_0} \cdot \frac{Q}{4\pi r^2} \cdot \sigma$

$= \frac{\sigma^2}{2\varepsilon_0}$ ✓

(7)

(c) (i)



$$V_{sphere} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$$

$$E_{sphere} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$$

$$= \frac{V_{sphere}}{r}$$

$$\therefore r = \frac{V_{sphere}}{E_{sphere}} = \frac{7000}{3\times10^6}$$

$$= \underline{2.3 \text{ mm}}$$ ✓

(ii)

$$Q = V_{sphere} \times 4\pi\varepsilon_0 \times r$$
$$= 1.82\times10^{-9}\,C$$
$$= \underline{1.8\,nC}$$ ✓

$$\sigma = \varepsilon_0 E$$
$$= 8.85\times10^{-12}\times3\times10^6$$
$$= \underline{2.7\times10^{-5}\,C\,m^{-2}} \quad (2.655\,C/m^2)$$ ✓

(iii) Energy: $mgh + qV = \frac{1}{2}mv^2$ ✓

$$2gh + 2\frac{q}{m}.V = v^2$$

$$2gh + \frac{2qV}{\frac{4}{3}\pi r^3\rho} = v^2$$

ecf allowed for r and Q from above

$$2\times9.81\times0.1 + \frac{2\times1.8\times10^{-9}\times7000}{\frac{4}{3}.\pi(2.33\times10^{-3})^3\times10^3} = v^2$$

(only 1 mark allowed)

$$v = 1.56$$
$$= \underline{1.6\,m/s}$$

$[v = 0.692\,m/s$ if mgh left out$]$ ✓✓

(iv)

$$I = \frac{Q}{t} = \frac{Nq}{t} = \underbrace{\left(\frac{64\times10^{-6}}{3600}\right)}_{\text{volume rate}} \div \underbrace{\left(\frac{4}{3}\pi(2.33\times10^{-3})^3\right)}_{\text{drop vol.}} \times \underbrace{\left(1.8\times10^{-9}\right)}_{q}$$

allow ecf   allow ecf

$$= 6.03\times10^{-10}\,A$$
$$= \underline{6.0\times10^{-10}\,A}$$ ✓

(7)

(d)

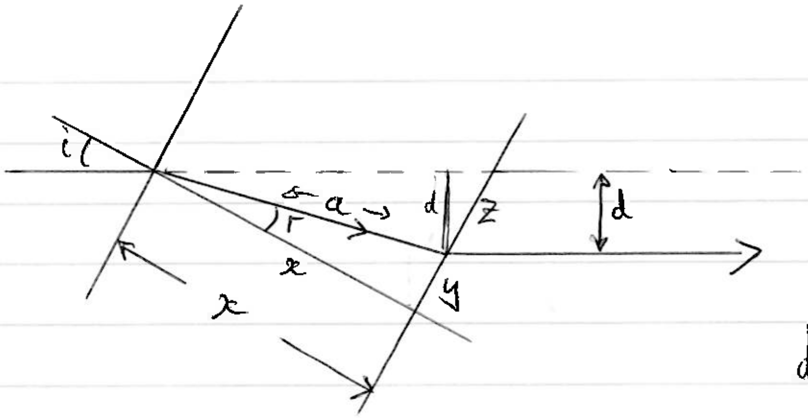


diagram ✓.

$$n = \frac{\sin i}{\sin r}$$

$$\frac{x}{a} = \cos r$$

understandable working. ✓

$$\frac{d}{a} = \sin(i-r)$$

Hence $d = a\sin(i-r) \approx a(i-r)$

$$a = \frac{x}{\cos r} \approx x$$

$$r = \frac{i}{n}$$ ✓

So $d = x\left(i - \frac{i}{n}\right) = \underline{\frac{xi}{n}(n-1)}$ ✓

$$\frac{dd}{dt} = \frac{di}{dt} \cdot \frac{x}{n}(n-1)$$ ✓

$$= 2\pi f \cdot \frac{x}{n}(n-1)$$ ✓

$$= 2\pi \cdot 200 \times 5\times10^{-2}\left(\frac{0.5}{1.5}\right)$$

$$= \pi \frac{2}{3} \cdot 10$$

$$= \frac{20\pi}{3} = \underline{\underline{21\ \text{ms}^{-1}}}$$ ✓

(7)

Question 5

(a) If it has a speed greater than the escape velocity, it is not a captive object. (comment)

$$\frac{1}{2}\cancel{m}v_e^2 = \frac{G\cancel{m}M_S}{r}$$

$$v_e^2 = \frac{2\times 6.67\times10^{-11}\times 1.99\times10^{30}}{38.1\times10^9}$$

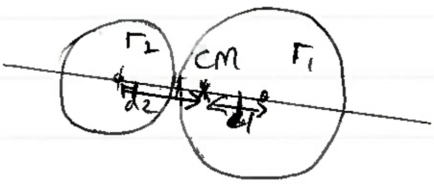
$$= \frac{6.67\times 3.98}{38.1}\times10^{10}$$

$$v_e = 8.35\times10^4\ m/s$$

$$= 83.5\ km/s$$

which is less than the speed of the object. Comment required
So not bound to the Sun. 4

(b) (i)



CM position given by

$$M_1 d_1 = M_2 d_2$$

OR take moments about centre of r_2, say.

$$\frac{4}{3}\pi r_1^3\rho\, d_1 = \frac{4}{3}\pi r_2^3\rho \cdot d_2$$

$$d_2\left(\frac{4}{3}\pi\rho\left(r_1^3+r_2^3\right)\right) = \frac{4}{3}\pi\rho r_2^3\times(r_1+r_2)$$

So $r_1^3 d_1 = r_2^3 d_2$

or $d_1 = \left(\frac{r_2}{r_1}\right)^3 d_2$

Then $d_2 = \frac{r_2^3(r_1+r_2)}{(r_1^3+r_2^3)}$

But $d_1 + d_2 = r_1 + r_2$

So $d_1 = \left(\frac{r_2}{r_1}\right)^3(r_1+r_2-d_1)$

Hence $d_1\left(1+\left(\frac{r_2}{r_1}\right)^3\right) = \left(\frac{r_2}{r_1}\right)^3(r_1+r_2)$

So $d_1(r_1^3+r_2^3) = r_2^3(r_1+r_2)$

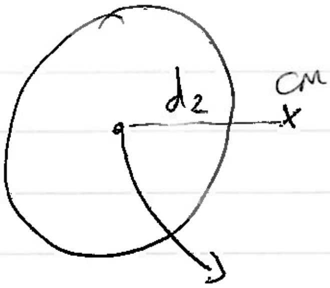
and $d_1 = \frac{r_2^3.(r_1+r_2)}{(r_1^3+r_2^3)}$

$r_1 = 9750\ m$
$r_2 = 7100\ m$

$d_1 = 4694\ m = 4700\ m$
$= 4.7\ km$ 2

[OK if d_2 calculated instead. $d_2 = 12150\ m$ ✓]

(ii)



Lobe 2 orbits about the CM and is attracted by lobe I.
Centripetal force produced by gravity

$$M_2 d_2 \omega^2 = \frac{G M_1 M_2}{(r_1+r_2)^2} \quad ✓$$

$$\therefore \quad M_1 = \frac{d_2 (r_1+r_2)^2 \omega^2}{G}$$

$$= \frac{d_2}{G}(r_1+r_2)^2 \cdot \frac{4\pi^2}{T^2}$$

Now, $d_2 = r_1 + r_2 - d_1$

$= 16850 - 4700$

$= \underline{12150 \text{ m}}$ ✓

$$\therefore \quad M_1 = \frac{12150 \times 16850^2 \times 4\pi^2}{6.67\times10^{-11} \times (15\times3600)^2}$$

$= \underline{7.00 \times 10^{14} \text{ kg}}$ ✓

$= \frac{4}{3}\pi r_1^3 \rho.$

So $\rho = \frac{7.00\times10^{14}}{9750^3} \cdot \frac{3}{4\pi}$

$= \underline{180 \text{ kg/m}^3}$ ✓ [very low density for rocky material].

4

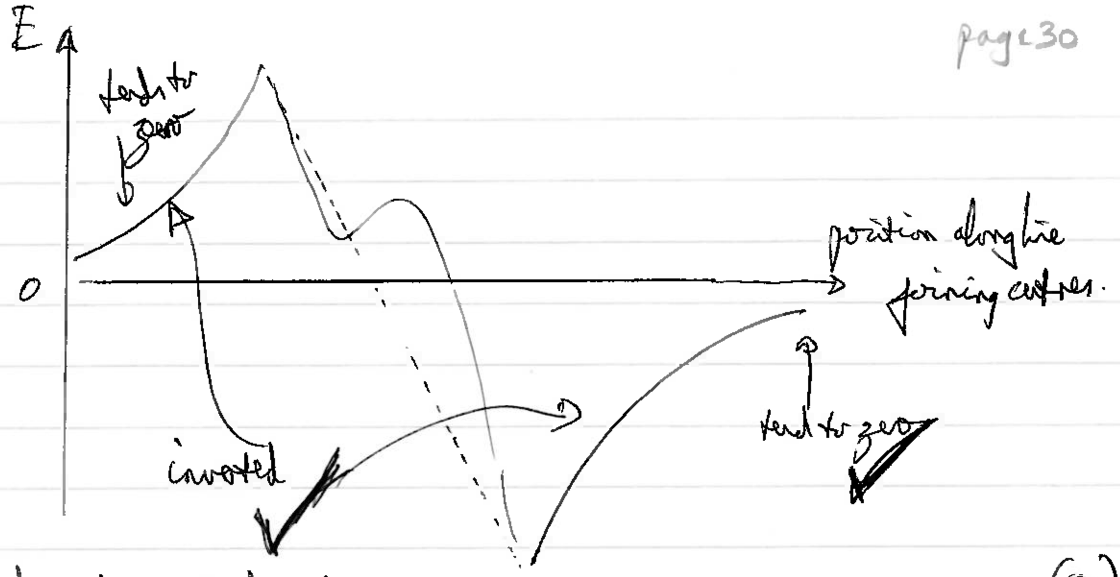
Moredirectly, using $M_1 = d_2 \frac{(r_1+r_2)^2}{G} \cdot \omega^2$ with $d_2 = \frac{r_1^3 (r_1+r_2)}{(r_1^3+r_2^3)}$

we have $\frac{4}{3}\pi\rho \cancel{r_1^3} = \cancel{r_1^3} \frac{(r_1+r_2)^2}{(r_1^3+r_2^3)} \cdot \frac{4\pi^2}{T^2 G}.$

$$\rho = \frac{(r_1+r_2)^2}{(r_1^3+r_2^3)} \cdot \frac{3\pi}{T^2 G} = \frac{3.724 \times 3\pi}{(15\times3600)^2\times6.67\times10^{-11}}$$

$= \underline{180 \text{ kg/m}^3}$

(iii)



(This graph can be upside down.)

(2)

(iv) $\text{Grav. p.e. lost} = GM_1M_2\left(\frac{1}{(r_1+r_2)} - \frac{1}{\infty}\right)$ ✓

$$= 6.67\times10^{-11} \times \frac{3.06\times10^{16}\times1.18\times10^{16}}{16850}$$

$$= \underline{1.4(3)\times10^{18}\ J}$$ ✓

(v) Cons. of mom. with CM stationary, ✓

$$M_1V_1 = M_2V_2$$

$$V_1 = \frac{M_2}{M_1}.V_2$$

$$\therefore KE = \tfrac{1}{2}M_1V_1^2 + \tfrac{1}{2}M_2V_2^2$$

$$= \tfrac{1}{2}M_1\frac{M_2^2}{M_1^2}.V_2^2 + \tfrac{1}{2}M_2V_2^2$$

$$= \tfrac{1}{2}M_2V_2^2\left(\frac{M_2}{M_1}+1\right)$$ ✓

$$= \frac{M_2}{2M_1}(M_1+M_2).V_2^2$$

$= 1.43\times10^{18}\ J$ total energy ✓

Substituting, $\underline{V_2 = 13.2\ m/s} = \underline{13\ m/s}$ ✓

$$\underline{V_1 = 5.1\ m/s}$$

$$\underline{\underline{V_1+V_2 = 18\ m/s}}$$ ✓

(7)

Do not allow adding $KE_1 + KE_2 = \frac{1}{2}(M_1+M_2)V_2$ for V.

(vi) No. of moles is $\frac{4.24 \times 10^{16}}{56} \times 10^{3}$ ← g → kg ✓

$= 7.57 \times 10^{17}$ moles.

$= 7.6 \times 10^{17}$ moles. ✓

$$E = M \int_4^{T} C \, dT$$ ✓

$$= M \frac{12\pi^4 R}{5\theta_D^3} \int_4^T T^3 \, dT$$

✓ use of moles here.

So, $1.43 \times 10^{18} = 7.57 \times 10^{17} \times \frac{12\pi^4 \times 8.31}{5 \times 464^3} \left[\frac{T^4}{4} - \frac{4^4}{4} \right]$ ✓

$T = 25\,K$ ✓

6

Question 6

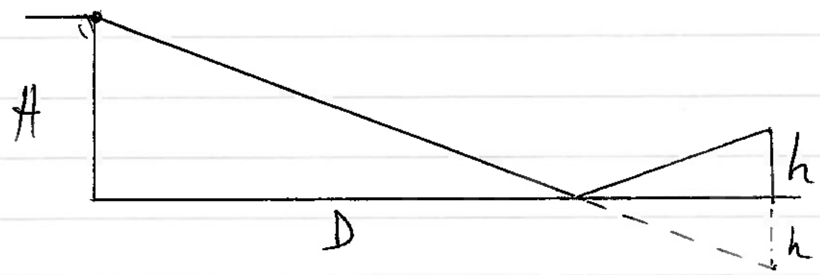
(a)

- Constant frequency [not same frequency].
- Same type of waves
- Same polarisation
- Coherent
- Waves crossing [do not accept same medium].

4 for 2 marks ✓✓
3 for 1 mark ✓

(2)

(b) (i)



path difference is $\sqrt{(H+h)^2+D^2} - \sqrt{(H-h)^2+D^2}$ ✓

$$= D\left(1+\left(\frac{H+h}{D}\right)^2\right)^{\frac{1}{2}} - D\left(1+\left(\frac{H-h}{D}\right)^2\right)^{\frac{1}{2}}$$

✓ expansion

$$\approx D\left(1+\frac{1}{2}\left(\frac{H+h}{D}\right)^2 - 1 - \frac{1}{2}\left(\frac{H-h}{D}\right)^2\right)$$

$$= \frac{D}{2}\left(\left(\frac{H+h}{D}\right)^2 - \left(\frac{H-h}{D}\right)^2\right)$$

$$= \frac{D}{2D^2}\left(H^2+h^2+2hH - H^2 - h^2 + 2hH\right)$$

$$= \frac{1}{2D}4hH$$

$$= \frac{2hH}{D}$$ ✓

And p.d. for a maximum when there is a phase change of π is $\frac{\lambda}{2}$. ✓

$$\therefore \frac{\lambda}{2} = \frac{2hH}{D}$$

$$h = \frac{\lambda D}{4H}$$

(no mark for the answer)

(4).

(ii) i $D = 1500m$

$h = 20\,m$

$H = 80\,m$

$f = 70 \times 10^6 Hz$

$\therefore \lambda = 4.286\,m$

For a minimum, $pd = \lambda$

$\therefore \lambda = \frac{2hH}{D}$

h, D constant

$\therefore H' = \frac{\lambda D}{2h} = \frac{4.29 \times 1500}{2 \times 20}$

$= 161\,m$

So the water level needs to fall by 81 m ✓

ii If H changes by 5m;

then pd with $H = 80m$ is $\frac{2hH}{D} = \frac{2 \times 20 \times 80}{1500}$

$= \frac{32}{15} = \underline{\underline{2.13m}}$

p.d. with $H = 85m$ is $\frac{2 \times 20 \times 85}{1500}$

$2.13m$ pd is $\simeq \frac{\lambda}{2}$ [for 70.3MHz]. $= \underline{2.27m}$ ✓

So a maximum, and 2.27 m is only 6% different, so little change in signal strength. ✓

iii For $h = 0.5m$

$pd = \frac{2 \times 0.5 \times 80}{1500} = \frac{8}{150} = 0.053m$

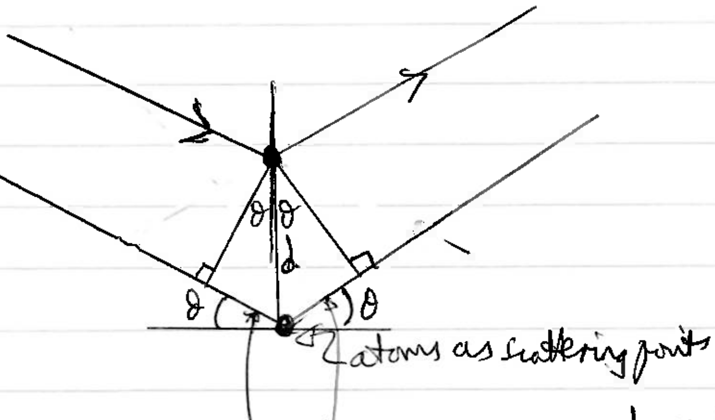
$\simeq \frac{1}{80}\lambda$

So the π phase shift produces destructive interference and hence a minimum. ✓

4

(c)

(i)



✓ DIAGRAM

path difference $= d \sin\theta + d \sin\theta$
$= 2d \sin\theta$ ✓ clearly shown where the pd is.

and for constructive interference the pd is $n\lambda$ ✓ statement

$\therefore \quad n\lambda = 2d\sin\theta$ (No mark for answer) (3)

(ii) 1 mole of salt has a volume of $\frac{58.4}{2.17} \left(\frac{cm^3}{g}\right)\left(\frac{g}{mol}\right)$

$= 26.9 \times 10^{-6}\ m^3$ ✓

This contains N_A salt "molecules", so $2N_A$ atoms ✓ factor 2

$\therefore \quad d = \sqrt[3]{\frac{26.9 \times 10^{-6}}{2 \times 6.02 \times 10^{23}}}$

$= 2.8(2) \times 10^{-10}\ m$ ✓ (3)

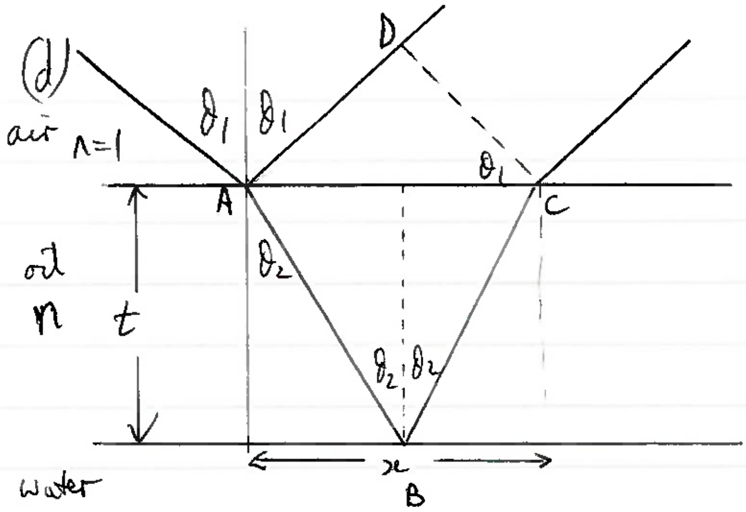
(iii) substituting in $n\lambda = 2d\sin\theta$

$n\lambda = 2 \times 2.82 \times 10^{-10} \sin 25$ ✓

$n = 1 \quad \lambda = 2.4 \times 10^{-10}\ m$ ✓ assume $n = 1$

$(n = 2 \quad \lambda = 1.2 \times 10^{-10}\ m)$

(etc.) (2)



Correct refraction ✓
AD marked or stated for pd ✓
ABC marked or stated for pd ✓

path difference = ABC − AD ✓ Statement.

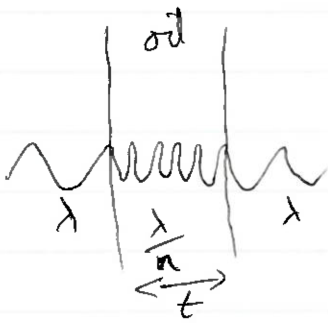
$$\sin\theta_1 = n\sin\theta_2$$

$$\tan\theta_2 = \frac{x}{2}\cdot\frac{1}{t}$$

$$\therefore \quad x = 2t\frac{\sin\theta_2}{\cos\theta_2}$$

$$\text{path in air (AD)} = x\sin\theta_1 = 2t\frac{\sin\theta_2}{\cos\theta_2}\cdot\sin\theta_1$$

✓ factor n.



optical path = nt.

so $\dfrac{t}{\lambda/n} = \dfrac{nt}{\lambda}$.

$$\text{Optical path in oil (ABC)} = \frac{2nt}{\cos\theta_2}$$

$$\therefore \; pd = \frac{2t}{\cos\theta_2}\left(n - \sin\theta_2\sin\theta_1\right)$$

$$= \frac{2t}{\sqrt{1-\sin^2\theta_2}}\left(n - \frac{\sin^2\theta_1}{n}\right)$$

$$= \frac{2tn}{\sqrt{1-\frac{\sin^2\theta_1}{n^2}}}\left(1 - \frac{\sin^2\theta_1}{n^2}\right)$$

$$= 2tn\left(1 - \frac{\sin^2\theta_1}{n^2}\right)^{\frac{1}{2}}$$

$$= \underline{2t\sqrt{n^2 - \sin^2\theta_1}}$$

π phase change at air/oil interface
No phase change at oil/water interface.
$\therefore$ p.d. for constructive interference is $\frac{\lambda}{2}$ ✓

So $\frac{\lambda}{2} = 2t\sqrt{n^2 - \sin^2\theta_1}$

$$\frac{410 \times 10^{-9}}{2} = 2t\sqrt{1.5^2 - \sin^2 30}$$

$$\underline{t = 7.9 \times 10^{-8}\ m}$$ ✓

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