# BPhO Round 1 Marking- 2020

Version 4 13/12 2020

Thank you for taking part in marking the scripts. It is of enormous benefit to young students to be able to take part in these competitions, tackling much harder problems than they normally get, and be able to have them marked by physicists who know what they are doing. It is because of your expertise in the subject that it is possible to do this. Exams marked by non-specialists are more of a tick box exercise, which is of little value in stretching students to grasp the subject at a deeper level. The layout of the work may be annoying. That is a national problem and we are not going to change that easily, although we do our best, as do the teachers of these students.

- We need **ACCURACY** and **CONSISTENCY**. Care is required. The mark scheme has marks allocated to make the marking easier. The mark allocation is different to the paper. **USE THE MARK SCHEME ALLOCATIONS.**
- You do not need to spend time working through laborious arithmetic calculations. The marks are set out for steps achieved.
- **Positive marking** is the aim. Marks should be awarded for good physics, even if the reasoning does not follow the mark scheme. Alternative routes to the answers can be allowed.
- **Significant figures**. This is not a test of significant figures. A leeway of ±1 sig. fig. is generally allowed, but we are not being strict at all in penalising for sig figs. If the published solution gives 3, allow 2 or 4 in the students answer, or even 1 after a long numerical calculation. However, unless they have written down all of the figures on their calculator display, there are no questions this year that are likely to invoke a sig fig penalty.
- Some answers can be left in fractional form.
- **Units** should be given for the final answer. It may be that the unit is given a little earlier and that it does not appear on the very last line. Allowance may be made if it is clear that the unit has been used a line or two earlier.
- If the units are a required part of the answer for the mark, they must be there.
- **Error carried forward** (ecf) is allowed provided ridiculous results do not start appearing. A mark is lost for the initial mistake, but then they can carry on (if it is possible) to gain some of the subsequent marks for the next one or two steps only. Just make a decision as to whether they should have the single mark or not.
- You are not required to spend time deciphering **scribble**.
- There may be a lot of working for the answer. If they are almost there, you may give the mark even if there is a numerical mistake in the last line. Use your judgement. The ticks for the marks are not exact i.e. they are for the idea and almost getting there.
- **Full marks are awarded for the correct answer, provided that there is some supporting working** and it is not a "show that" question. Look out for suspicious results with **insignificant working**.
- You must follow the mark scheme so that we mark **CONSISTENTLY**. Do not make your own independent mark scheme.
- Avoid relying on your memory for the mark allocation. You need the mark scheme open beside you.

**If you need advice, email Robin Hughes [redacted]. I will respond promptly. You can send a phone photo or just ask a question.**

1. **Do not mix up students' names when entering marks**.
2. **Add up the marks correctly**. Check your addition. For each little section, note the total in a circle, as in the mark scheme. Then note the total for the page at the bottom in two parallel lines to distinguish it.
3. Do not leave "pdf papers" in any unsecured place. They are confidential.
4. You can write on the answers if it helps you keep track. Ticks are for marks so do not tick everything. Ticks are counted up when checking.
5. Updates to the mark scheme are inevitable and will be sent out. If you see a mistake, email me.

# SECTION I  WORKING IS REQUIRED

(a) Stone in free fall. $Mgh = \frac{1}{2}mv^2$ or $V^2 = u^2 + 2as$ ✓

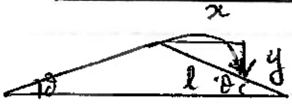
$$h = \frac{v^2}{2g} = \frac{330^2}{2 \times 9.8} = 5560\ \text{m} = 5.6\ \text{km}$$ ✓

(Any answer 5–6 km acceptable) or 5.5 km

or 5.4 km ($g = 10$)

$\boxed{2}$

(b)



(i) Horizontally: $u\cos\theta . t = x$ ✓

and $x = l\cos\theta$

So $t = l/u$. ✓

(ii) "$s = ut + \frac{1}{2}gt^2$" ($\uparrow$ +ve)

$-y = u\sin\theta . t - \frac{1}{2}gt^2$ ✓

$y = l\sin\theta$

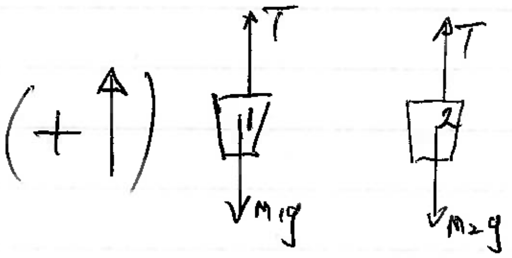
$\therefore -l\sin\theta = \cancel{u}\sin\theta . \frac{l}{\cancel{u}} - \frac{1}{2}g\frac{l^2}{u^2}$

$2l\sin\theta = \frac{gl^2}{2u^2}$

$l = \frac{4u^2\sin\theta}{g}$ ✓

$\boxed{4}$

(c)



~~forces~~ diagram ✓

$N II$. on $M_1$ : $T - M_1g = M_1a$

on $M_2$ : $T - M_2g = -M_2a$ eqns ✓

Subtract:

$-M_1g + M_2g = M_1a + M_2a$

$a = \frac{(M_2 - M_1)g}{(M_1 + M_2)}$ ✓ (correct signs)

"$v^2 = u^2 + 2ah$"

$V^2 = 2h\frac{(M_2 - M_1)g}{(M_1 + M_2)}$

$V = \sqrt{\frac{2h(M_2 - M_1)g}{(M_1 + M_2)}}$ either ✓

Bucket 1 has this speed. $\Delta h = \frac{V^2}{2g} = \frac{h(M_2 - M_1)}{(M_1 + M_2)}$ ✓

$\boxed{5}$

(2)

(No mark for speed of player X.)

(d) Speed of player Y is $\sqrt{3^2+4^2} = \underline{5\ m/s}$ (needed later)

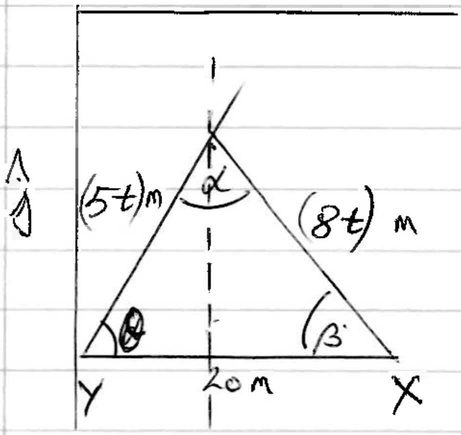



Diagram ✓

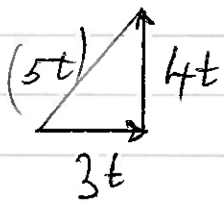
Pythagoras: ✓

$$(8t)^2 = (4t)^2 + (20-3t)^2$$

Gives: $64t^2 = 16t^2 + 400 + 9t^2 - 120t$

So, $39t^2 + 120t - 400 = 0$

Then, $t = \dfrac{-120 \pm \sqrt{120^2 + 4 \cdot 400 \cdot 39}}{2 \cdot 39}$




$$t = \frac{1}{39}\left(-60 + 80\sqrt{3}\right)$$

$$= \underline{2 \cdot 0(145)\ s}$$ ✓

Sine Rule: $\dfrac{8t}{\sin\theta} = \dfrac{5t}{\sin\beta}$ ✓

and given $\tan\theta = \frac{4}{3}$ $[3\hat{i}+4\hat{j}]$ $\Rightarrow \cos\theta = \frac{3}{5}$, $\sin\theta = \frac{4}{5}$.

$\therefore \sin\beta = \frac{5}{8}\sin\theta = \frac{5}{8}\cdot\frac{4}{5} = \frac{1}{2}$

$\underline{\beta = 30^\circ}$ ✓

displacement of Y is • 6 m E and 8 m N
or • 10 m at $\theta = 53^\circ$
or • $(6\hat{i} + 8\hat{j})$
} for the mark. ✓

Or // If $\sin\beta$ has been found, then sine rule gives

$$\frac{5t}{\sin\beta} = \frac{20}{\sin\alpha}$$

$$\frac{10t}{\frac{1}{2}} = \frac{20}{\sin\alpha}.$$

$$t = \frac{2}{\sin\alpha} = 2 \cdot 0\ s$$

$\alpha = 180 - \beta - \theta$
$= 180 - 30 - 53 \cdot 13$
$\alpha = \underline{96 \cdot 87^\circ}$

$\boxed{6}$

| |
|---|
| If $\underline{X}$ is taken $\underline{\text{West}}$ of $\underline{Y}$ instead, ✓<br>$t = 5 \cdot 09 = \underline{5 \cdot 1\ s}$<br>and X travels at $\underline{44 \cdot 4^\circ\ N\ \theta\ E}$. ✓ Allow the 3 marks<br>Y is at $(15 \cdot 3\hat{i} + 20 \cdot 4\hat{j})$ ✓ |

(e) Ball of Wire

$$\rho_{res.} = \frac{RA}{l}$$

$$\text{Weight of water displaced} = 4.60 - 4.08 = 0.52\,N$$

$$\text{Mass of water} = \frac{0.52}{9.81}\ kg$$

$$\text{Vol of wire} = \text{Vol of water} = \frac{0.52}{9.81}\frac{1}{10^3}\ m^3$$

$$\text{Vol of wire} = Al.$$

$$\therefore\ l = \frac{0.52}{9.81}\frac{1}{10^3}\frac{1}{\pi(0.7\times10^{-3})^2} = \underline{34.43\,m}$$

$$\text{So, } \rho_{res.} = \frac{0.478\times\pi\times(0.7\times10^{-3})^2}{34.43} = \underline{2.14\times10^{-8}\,\Omega m}$$

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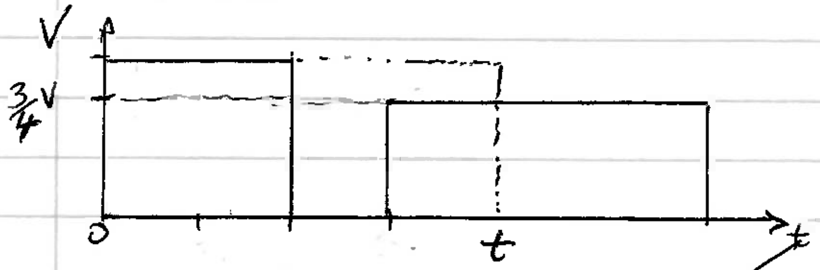
(f) Train



Use units of time of hours
$t$ is the time of arrival if speed $V$ maintained.
$S$ = distance travelled in km.
$V$ in km/h.

Direct ① $S = V.t$

Half hour stop ② $S = V.1 + \frac{3}{4}V(t\ \ 1\ \ -\frac{1}{2})$. Arrives at $(t+1\frac{1}{2})$ travels for $(t+1)$

$$S = V + \frac{3}{4}Vt$$

$$\text{So, } Vt = V + \frac{3Vt}{4} \longrightarrow \frac{Vt}{4} = V \text{ so } \underline{\underline{t = 4 \text{ hours}}}$$

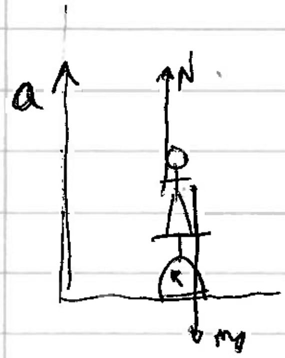
The Delayed half hour stop ③ Travels 45km at speed V initial run, but now travels the 45km at $\frac{3V}{4}$ causing it to arrive $\frac{1}{2}$ hour late (+ half hour of zero speed delay)

$$\therefore \tfrac{1}{2}\text{hour} = \frac{45}{(3V/4)} - \frac{45}{V} \Rightarrow \frac{V}{2} = 60 - 45 \Rightarrow V = 30\,km/h$$

and $\underline{\underline{S = 120\,km}}$

4

## (9) Emmy in the lift




On Emmy, $N - mg = ma$

Scales measures $N. = ma + mg$

But shows a value of $M(t) = \frac{N}{g}$

$$\therefore M(t) = \frac{m(a+g)}{g}$$

$$= m\left(\frac{a}{g} + 1\right)$$

and we are given the behaviour of the lift, such that

$$M(t) = 60\left(1 + \frac{t}{10} - \frac{t^2}{100}\right) = m\left(\frac{a(t)}{g} + 1\right)$$

When $t = 0$, $a = g$ so $m(0) = 60$ kg ✓

and $$\frac{a(t)}{g} = \frac{t}{10} - \frac{t^2}{100}$$ ✓

So $$V(t) = g\int\left(\frac{t}{10} - \frac{t^2}{100}\right) dt$$

$$V(t) = g\left(\frac{t^2}{20} - \frac{t^3}{300}\right) + c$$

At $t = 0$, $V = 0$ so $c = 0$.

$$V(10) = g\left(\frac{10^2}{20} - \frac{10^3}{300}\right) = g\frac{5}{3} = 16.4 \text{ m/s}$$ ✓

$$S = \int_0^{10} V\, dt$$

$$= g\int_0^{10}\left(\frac{t^2}{20} - \frac{t^3}{300}\right) dt = g\left[\frac{t^3}{60} - \frac{t^4}{1200}\right]_0^{10}$$

$$= g\left[\frac{100}{6} - \frac{100}{12}\right] = 81.8 \text{ m}$$ ✓

Final distance, $S = 18.25$ m

$u = 16.35$ m/s

$$0 = u^2 - 2as \longrightarrow a = \frac{u^2}{2s} = \frac{16.35^2}{36.50} = 7.32 \text{ m/s}^2$$ ✓

$$\text{Mass shown} = \frac{g-a}{g} \times 60 = 15.2 \text{ kg}$$ ✓ $\boxed{6}$

(h) Mixing Liquids

The linear variation of $n$ with volumes is expressed as

$$n = \frac{V_a n_a + V_b n_b}{V_a + V_b}$$ ✓✓

so if $V_b = 0$ $n = n_a$

and if $V_a = 0$ $n = n_b$

and if $V_a = V_b$ $n = \frac{n_a + n_b}{2}$

etc.

For $n_g = ?$

$$n_g = \frac{100 \times 1.15 + 64 \times 1.52}{100 + 64}$$

$$n_g = 1.29$$ ✓

$\boxed{3}$

This follows from. $n = x n_1 + (1-x) n_2$

$$x = \frac{V_1}{V} = \frac{V_1}{V_1 + V_2}$$

$$1 - x = \frac{V_1 + V_2 - V_1}{V_1 + V_2} = \frac{V_2}{V_1 + V_2}.$$

(i)(i) (i) Three resistors in series $R_2 = R_4 = \infty$

(ii) " " = parallel $R_2 = R_4 = 0$

(iii) Two identical resistors in parallel. $R_2 = R_3$ and $R_4 = R_1$

✓✓ for ~~all three~~. One mark for two correct.

(ii) $V_{3\Omega} = 2 \times 3 = 6V$ ✓

So $6V$ across $4\Omega$ resistor.

$I_{4\Omega} = 1.5A$

Hence KI means $\frac{1}{2}A$ through $5\Omega$

Hence $V_{5\Omega} = 2.5V$

Hence $\mathcal{E} = 6 + 2.5$
$= 8.5V$ ✓

$\boxed{4}$

## Resistor combinations

(j) (i)

$$R_1 = \frac{R_1 R_2}{R_1 + R_2} + R_3.$$

$$R_1^2 + \cancel{R_1 R_2} = \cancel{R_1 R_2} + R_3 R_1 + R_3 R_2$$

$$R_3 = \frac{R_1^2}{(R_1 + R_2)}$$

Simplified expression not required.

(ii)

$$R_{AB} = \frac{R_3 (R_1 + R_2)}{R_3 + (R_1 + R_2)}$$

and $R_{AB} = R_1$ (given)

Hence.

$$\cancel{R_1 R_3} + R_1 R_1 + R_1 R_2 = \cancel{R_3 R_1} + R_3 R_2.$$

then $$\left(\frac{R_1}{R_2}\right)^2 + \frac{R_1}{R_2} - \frac{R_3}{R_2} = 0.$$

$$\left[\frac{R_3}{R_2} = 6\right]$$

$$x^2 + x - 6 = 0.$$

$$x = -\frac{1}{2} \pm \frac{5}{2}.$$

$$x = 2 = \frac{R_1}{R_2}$$

$\boxed{4}$

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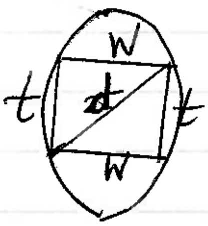
(k) **Beam**




Pythagoras. $W^2 + t^2 = d^2$

$$W = \sqrt{d^2 - t^2}$$

and $S = k t^3 . W$

$$= k t^3 \sqrt{d^2 - t^2}$$ — ✓ as a single variable

$$\frac{dS}{dt} = k t^3 . \frac{1}{2} \frac{(-2t)}{\sqrt{d^2 - t^2}} + k 3t^2 \sqrt{d^2 - t^2}$$

When $\frac{dS}{dt} = 0$

$$\frac{t^2}{\sqrt{d^2 - t^2}} = 3\sqrt{d^2 - t^2}$$

$$t^2 = 3(d^2 - t^2)$$

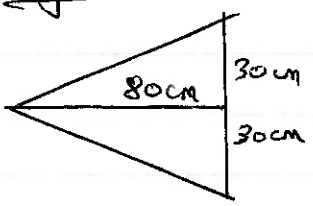
$$t = \sqrt{\frac{3}{4}}\, d.$$

and $W = \sqrt{d^2 - t^2} = \sqrt{d^2 - \frac{3}{4} d^2} = \frac{d}{2}$

$W = 10$ cm. $t = \frac{\sqrt{3}}{2} \cdot 20 = 17 \cdot 3$ cm

$\boxed{5}$

(l) Light



$$n\lambda = (a+b)\sin\theta$$

$$1 \cdot \lambda = \frac{1}{6\times10^{5}} \cdot \sin\theta$$ ✓

$$\tan\theta = \frac{30}{80}$$ ✓

$$\theta = 20.56^\circ$$

$$\lambda = 585\,\text{nm}$$ ✓ $\boxed{3}$

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(m) Meteorite crater

$$d = k\,E^{\alpha}\rho^{\beta}g^{\gamma}$$

Dims. $[L] = [ML^{2}T^{-2}]^{\alpha}[ML^{-3}]^{\beta}[LT^{-2}]^{\gamma}$

equate powers of $[L]$: $1 = 2\alpha - 3\beta + \gamma$ ✓

$[M]$: $0 = \alpha + \beta$ ✓

$[T]$: $0 = -2\alpha - 2\gamma$ ✓

$$\alpha = -\beta$$
$$\alpha = -\gamma$$
$$\alpha = \tfrac{1}{4}$$

$$d = k\left(\frac{E}{\rho g}\right)^{\frac{1}{4}}$$ ✓

(ii)

$$1200^{4} = 1 \cdot \frac{1}{2}\frac{m(15\times10^{3})^{2}}{3000\times9.81}$$

$$m = 5.4\times10^{8}\ \text{kg}$$ ✓
$$= 540\ \text{Mt}$$

$$V = m/\rho$$

$$\frac{4}{3}\pi\frac{d^{3}}{8} = \frac{5.4\times10^{8}}{8000}$$

$$d = 51\ \text{m.}$$
$$\simeq 50\ \text{m}$$ ✓

$\boxed{6}$

petrol engine

(n) consumption rate $= \dfrac{5\cdot 3}{100}\dfrac{l}{km}$ at $100\ km/h$

$= \dfrac{5\cdot 3}{100}\times 100\ \dfrac{l}{km}\cdot\dfrac{km}{h} = 5\cdot 3\ \dfrac{l}{h}$ ✓

$= 5\cdot 3\times 30\ \dfrac{MJ}{l}\cdot\dfrac{l}{h} = \dfrac{5\cdot 3\times 30\times 10^6}{3600}\ \dfrac{J}{s}$

power into cooling system = 23% of consumption rate $= \dfrac{5\cdot 3\times 30\times 10^6\times 0\cdot 23}{3600}\ W$ ✓ (23%)

$= 1\cdot 02\times 10^4\ W$

This is equated to $\dfrac{mc\Delta\theta}{t} = \dfrac{m}{t}\times 4180\times(40-16)$ ✓

Hence $\dfrac{dm}{dt} = \dfrac{5\cdot 3\times 30\times 10^6\times 0\cdot 23}{3600\times 4180\times 24}\ \dfrac{kg}{s} = \dfrac{5\cdot 3\times 6\cdot 9}{3\cdot 6\times 4\cdot 18\times 24}$

$= \underline{\underline{0\cdot 10\ \dfrac{kg}{s}}} = 360\ \dfrac{kg}{h}$ ✓

· If range of temp rise taken as 40°C then result is $0{,}06\ \dfrac{kg}{s}$.

$\boxed{4}$

(o) Black body radiation

$\dfrac{q_{1000}}{q_{500}} = \left(\dfrac{1273}{773}\right)^4 = 7\cdot 36$ ✓ $\boxed{KELVIN}$ ✓

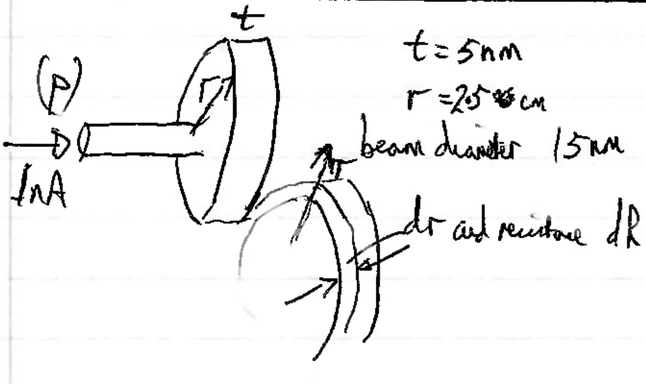
$T_{1000}\times\lambda_{m_{1000}} = T_{500}\cdot\lambda_{m_{500}}$ ✓

$1273\times\lambda_{m_{1000}} = 773\times 3750$

$\underline{\underline{\lambda_{max_{1000}} = 2280\ nm}}$ ✓

$\boxed{14}$

(p)




$t = 5nm$

$r = 2{,}5\ cm$

$dR = \rho\dfrac{l}{A} = \rho\dfrac{dr}{2\pi r t}$ ✓

$R = \dfrac{\rho}{2\pi t}\displaystyle\int_{r_1}^{r_2}\dfrac{dr}{r}$ ✓

$= \dfrac{\rho}{2\pi t}\ln\dfrac{r_2}{r_1}$

$= \dfrac{2\cdot 8\times 10^{-8}}{2\pi\times 5\times 10^{-9}}\ln\dfrac{2\cdot 5\times 10^{-2}}{7\cdot 5\times 10^{-9}}$ ✓

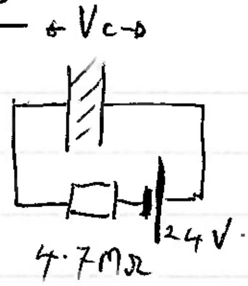
$= \dfrac{2\cdot 8}{\pi}\ln\dfrac{10^7}{3} = \underline{\underline{13\cdot 4\ \Omega}}$ ✓

$\Delta V = IR = 1\times 10^{-9}\times 13\cdot 4 = \underline{\underline{13\cdot 4\ nV}}$ ✓ $\boxed{5}$

10

Capacitor

(q)




$$R_{dielectric} = \frac{\rho l}{A} = \frac{1.5\times10^{12}\times0.82\times10^{-6}}{0.603}$$
$$= 2.04\times10^{6}\,\Omega$$ ✓

$$\therefore V_c = \frac{2.04\times10^{6}\times24}{(2.04+4.7)\times10^{6}}$$
$$= 7.26\text{ V}$$
$$= \underline{7.3\text{ V}}$$ ✓

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85\times10^{-12}\times0.603}{0.82\times10^{-6}}$$
$$= 6.51\,\mu F$$ ✓

$$Q = VC$$
$$= 7.26\times6.51\times10^{-6}$$
$$= 4.7\times10^{-5}\text{ C}$$
$$= \underline{\underline{47\mu C}}.$$ ✓

Discharge. $Q = q_0 e^{-t/RC}$ ✓

$$\frac{1}{2} = e^{-t/RC}$$
$$-\ln 2 = -\frac{t}{2.04\times10^{6}\times6.51\times10^{-6}}$$
$$\ln 2 = \frac{t}{13.28}$$
$$\underline{\underline{t = 9.2\text{ s}}}$$ ✓

$\boxed{6}$

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(r) gas expansion

Isothermal; $pV = k$

$$p\frac{dV}{dt} + V\frac{dp}{dt} = 0.$$ ✓

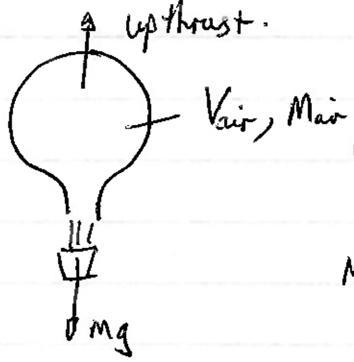
$$\frac{dp}{dt} = \frac{p}{V}\cdot\frac{dV}{dt} = \frac{k}{V^2}\frac{dV}{dt}$$ ✓

$$= \frac{380}{0.17^2}\times0.005$$
$$= 65.7 = \underline{\underline{66\text{ Pa s}^{-1}}}$$ ✓

$\boxed{3}$

(11)

(5) hot air balloon.




N II. "$Mg$ = upthrust"

= weight of all air displaced ✓

$$Mg + M_{HOT}\,g = M_{COLD}\,g$$

$$Mg = (\rho_{COLD} \cdot V - \rho_{HOT} \cdot V)g$$

$$Mg = Vg(\rho_C - \rho_H) \quad ✓$$

Gas Law $V \propto T$ of kelvin

so $\frac{V_1}{T_1} = \frac{V_2}{T_2}$ and for a ~~p~~

[$PV = nRT$

$P = \frac{n}{V} R \cdot T$

and $P$ is fixed in this question ].

so $\rho_C T_C = \rho_H T_H$ ✓

Hence $Mg = Vg\rho_C \left(1 - \frac{\rho_H}{\rho_C}\right)$

$= Vg\rho_C \left(1 - \frac{T_C}{T_H}\right)$ ✓

then, $\frac{T_C}{T_H} = 1 - \frac{M}{V\rho_C}$

$$\frac{288}{T_H} = 1 - \frac{240}{1.23 \times 1000}$$

$$T_H = 350\ \text{K}$$

$$= \underline{\underline{77°C}} \quad ✓$$

$\boxed{5}$

# QUESTION 2

(a) (i) $\frac{hc}{\lambda} = W + KE$ , $\frac{hc}{\lambda} = W + \frac{1}{2}mv^2$ ✓

(ii) $I = 0.1\,pA \longrightarrow \frac{N}{t} = \frac{0.1\times10^{-12}}{1.6\times10^{-19}} = \underline{\underline{625{,}000\ s^{-1}}}$ ✓

(iii) $W = \frac{hc}{\lambda} - eV$

$= \frac{6.63\times10^{-34}\times3\times10^{8}}{280\times10^{-9}} - 0.82\times1.6\times10^{-19}$

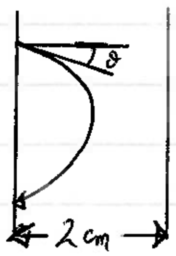
$= \underline{5.79\times10^{-19}\ J}$

$= \underline{\underline{3.62\,eV}}$ ✓

(iv) $\frac{1}{2}mv^2 = eV_{stopping}$

$v = \sqrt{\frac{2eV_s}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times0.82}{9.11\times10^{-31}}}$

$= \underline{5.37\times10^{5}\ m/s}$ ✓

(v) 


$\theta = 20°$

field between plates $= \frac{V}{d}$

force on electron $= eE = e\frac{V}{d}$

acceleration of electron $= \frac{e}{m}\frac{V}{d}$

$a = \frac{1.6\times10^{-19}}{9.11\times10^{-31}} \times \frac{0.82}{0.02}$

$= 7.20\times10^{12}\ m/s^2$

time of flight $\Delta t = 2\times\frac{u\cos\theta}{a}$

$= 2\times\frac{5.37\times10^{5}\times\cos(20)}{7.2\times10^{12}}$

{In gravity, $R = \frac{u^2\sin(2\theta)}{g}$}

$= \underline{1.40\times10^{-7}s}$

So $\Delta x = u\sin\theta\,.\,\Delta t$

$= 5.37\times10^{5}\times\sin20\times1.4\times10^{-7}s$

$= \underline{2.57\times10^{-2}\ m}$

$= \underline{2.6\ cm.}$ ✓

$\boxed{5}$

(b) (i) $P_{out} = \frac{P_{in}}{4.5^n}$

$6.6 \times 10^{-11} = 1.2 \times 10^{-3} \times 4.5^{-n}$

$n \ln 4.5 = \ln\left(\frac{1.2 \times 10^{-3}}{6.6 \times 10^{-11}}\right)$

$n = 11.1$

So integer value is $\underline{\underline{n = 12}}$ } either

since $n=11$ would not be sufficient.

(ii) So $P_{out} = \frac{1.2 \times 10^{-3}}{4.5^{12}} = \underline{\underline{1.74 \times 10^{-11}\ W}}$

(iii) $\frac{N}{t} = \frac{P_{out}}{hf} = \frac{P_{out}\lambda}{hc}$

$= \frac{1.74 \times 10^{-11} \times 585 \times 10^{-9}}{6.63 \times 10^{-34} \times 3 \times 10^{8}}$

$= \underline{\underline{5.1 \times 10^{7}}}$ photons/s

(iv) Time of travel $= \frac{300}{3 \times 10^8} = 10^{-6}$s

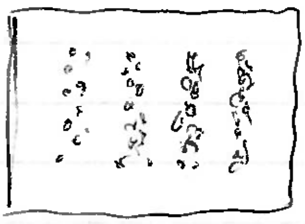
$\therefore N_{pipe} = 5.12 \times 10^{7} \times 10^{-6}$

$= \underline{51}$ photons

$\therefore \Delta x = \frac{300}{51.2} = 5.86$ m

$= \underline{\underline{6\ m}}$

(v)



interference pattern of spots.

(vi) If $n = 1.2$, same number of photons entering and leaving per second.

time of travel, $T = 1.2 \times 10^{-6}$s

$\therefore N = 1.2 \times 10^{-6} \times 51.2 \times 10^{6}$

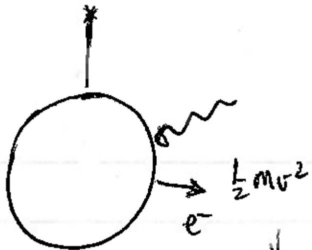
$= 61.4$ in the pipe.

$\therefore \Delta x = 4.9$m

Answer: the average distance between photons is less.

8

(C). (i)




When charged, Max KE of electron is equal to electrostatic p.e. of charge and sphere.

So, $KE_{max} = \frac{hc}{\lambda} - W = eV$

$\frac{6.63\times10^{-34}\times 3\times10^{8}}{150\times10^{-9}} - 4.7\times1.6\times10^{-19} = 1.6\times10^{-19}\,V$

↑ used this wavelength.

$V = 3.59 = 3.6\,V$

(ii) and $V_{sphere} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{Q}{r}$

$Q = 4\pi\varepsilon_0 \cdot r \cdot V_{sphere}$

$= \frac{0.08}{9\times10^{9}} \times 3.59$ allow e.c.f.

$Q = 3.2\times10^{-11}\,C$

$N_{lost} = \frac{3.2\times10^{-11}}{1.6\times10^{-19}}$

$= 2.0\times10^{8}$ electrons.

(iii) The light arrives in photons/lumps/packets of energy so the required energy to free an electron can arrive much earlier than the time required to accumulate enough "average energy" arrival, as if it was a wave intensity.

(iv) This is a capacitor discharge situation

$C = \frac{Q}{V} = 4\pi\varepsilon_0 r$

$Q = Q_0 e^{-t/RC}$

$\ln\frac{1}{2} = -t/RC$

$R = \frac{1}{C}\frac{t}{\ln 2} = \frac{1}{4\pi\varepsilon_0 r}\cdot\frac{20}{\ln 2} = \frac{9\times10^{9}\times20}{0.08\times0.693} = 3.25\times10^{12}\,\Omega$

$= 3\times10^{12}\,\Omega$

$\boxed{4}$

(15)

(d) (i) Charge on leaf at 220 V is $220 \times 6.4\times10^{-12}$

$$N_e = \frac{220\times6.4\times10^{-12}}{1.6\times10^{-19}} = \frac{1.408\times10^{-9}}{1.6\times10^{-19}}$$

$$= \underline{8.8\times10^{9}}$$ ✓

(ii) $$\text{Activity} = \frac{\Delta N}{\Delta t} = \frac{8.8\times10^{9}}{85}$$

$$= \underline{\underline{1.04\times10^{8}}}\ \text{Bq}.\ (s^{-1})$$ ✓

(iii) $\Delta N = (-)\lambda N \Delta t$

$$N = \frac{3.62\times10^{-3}\,g}{(226+80)\ g/mol} \times 6.02\times10^{23}\ Mol^{-1}$$

$$= 7.12\times10^{18}\ \text{"atoms" of RaBr.}$$ ✓

Hence $$t_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.693\times N}{(\Delta N/\Delta t)}$$

$$= \frac{0.693\times7.12\times10^{18}}{1.04\times10^{8}}$$

$$= \underline{4.7\times10^{10}\ s}\ (= 1500\ \text{years})$$ ✓

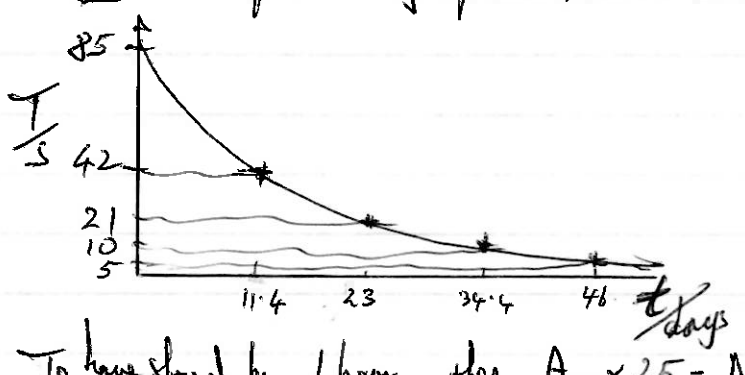
(iv) $$I = 1.04\times10^{8}\times1.6\times10^{-19}\ \left(= \frac{\Delta N}{\Delta t}\times e \times \Delta t\right).$$

$$= 1.66\times10^{-11}\,A$$

$$= \underline{\underline{17\ pA}}.$$ ✓

(v) No. It requires a high potential, never achieved in the p.e effect ✓
[Some comment needed].

(vi)




✓

(vii) To have slowed by 1 hour, then $A_{25}\times25 = N_{24}$ (counts expected in correct 24 h).

So $A_{24}\times24 = A_{25}\times25$

$\frac{A_{25}}{A_{24}} = \frac{24}{25}$ and $\frac{A_{25}}{A_{24}} = e^{-\lambda t}$ so $\frac{24}{25} = e^{-\lambda t}$, $t = \frac{t_{\frac{1}{2}}}{0.693}\times\ln\frac{25}{24}$

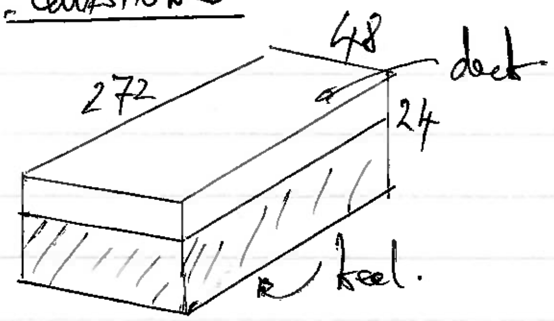
$= \underline{2.77\times10^{9}\,s}$
$= \underline{88\ years}$ ✓ 8

25 TOTAL

16

QUESTION 3

(a)




(i) Archimedes. Weight of water = weight of oil + weight of tanker

$$V_W \times 1025 \times g = 180 \times 10^6 \times g$$

$$V_W = 175610\ m^3$$ ✓

depth of submersion, $d = \frac{175610}{272 \times 48}$

$$= \underline{13.45\ m}$$

So, height of deck above water level is $24 - 13.45$

$$\underline{\underline{h = 10.55\ m}}$$ ✓

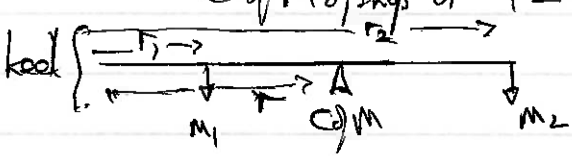
(ii) Depth of oil in tank

$$= \frac{158 \times 10^6}{900 \times 272 \times 48}$$

$$= 13.45\ m.$$

C of M of oil is $\frac{13.45}{2} = \underline{6.72}$ m above the keel. ✓

C of M of ship is 12 m above the keel.




$$m_1 r_1 + m_2 r_2 = r(m_1 + m_2)$$

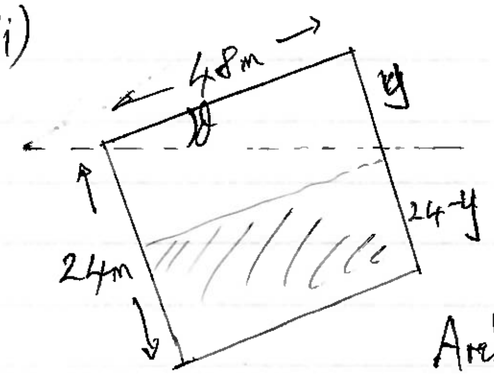
$$158 \times 10^6 \times 6.72 + 22 \times 10^6 \times 12 = r \times 180 \times 10^6$$

$$r = \frac{158 \times 6.72 + 22 \times 12}{180}$$

$$r = 7.37\ m$$

$\underline{\underline{r = 7.4\ m}}$ above the keel. ✓

(iii)




Archimedes: wt of water displaced = wt of oil + ship

trapezium: $\frac{1}{2}(24+24-y)\times 48\times 272\times 1025\times \cancel{g} = 180\times 10^6\, \cancel{g}$ ✓ (method)

$$\left(24-\frac{y}{2}\right) = 13.45$$

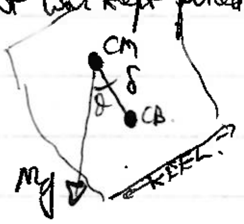
$$y = 21.1\text{ m}$$

$$\text{So } \tan\theta = \frac{21.1}{48}$$

$$\theta = 23.7°$$

$$\underline{\underline{\theta = 24°}}$$ ✓

(iv) As the ship tilts, the centre of buoyancy moves to a new position.
The centre of mass of the tanker is fixed.
[If the buoyancy point was kept fixed (it should not as it changes) then
we would have




with $\delta = 7.37 - 6.72$ m (measured from the keel)
$= 0.65$m

So the moment $= Mg\,\delta\sin\theta$
$= 180\times10^6\times 9.81\times 0.65\times\sin 23.7°$
$= 4.6\times10^8$ Nm

To calculate the new buoyancy point. P.T.O.

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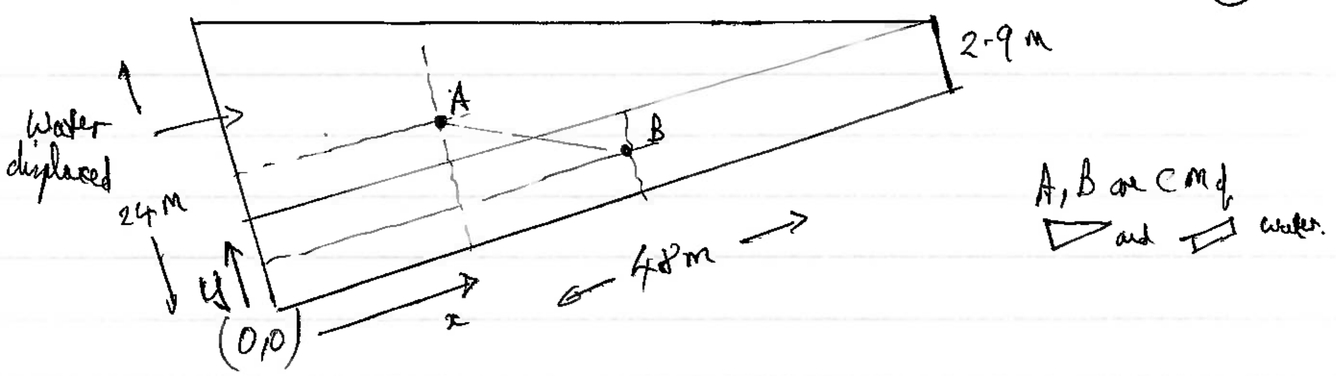



C of M of water is at (X, Y)

(X) $24\times(48\times2\cdot9)+16\times(\tfrac{1}{2}\times21\cdot1\times48)=X\times\{\tfrac{1}{2}(24+2\cdot9)\times48\}$

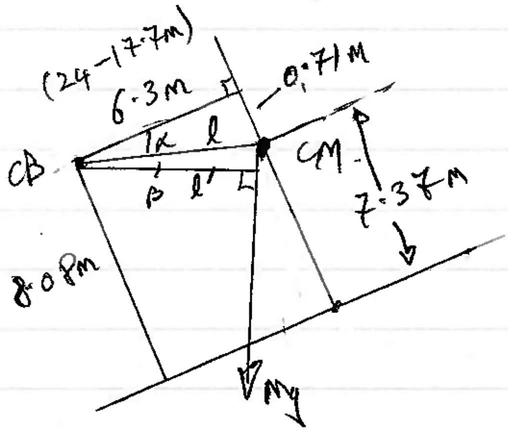
$24\times2\cdot9 + 8\times21\cdot1 = X\times\frac{26\cdot9}{2}$

$\underline{X = 17\cdot7\,m.}$ ✓

[Not needed for the calculation] (Y) $\frac{2\cdot9}{2}\times(48\times2\cdot9)+9\cdot9\times(\tfrac{1}{2}21\cdot1\times48) = Y\times[\tfrac{1}{2}(24+2\cdot9)\times48]$

$2\cdot9^2 + 9\cdot9\times21\cdot1 = Y\times26\cdot9$

$Y = 8\cdot08\ m$ }.




$\alpha = 6\cdot43^\circ = \tan^{-1}\frac{0\cdot71}{6\cdot3}.$

$\beta = 23\cdot7 - 6\cdot43$

$= 16\cdot6^\circ$

$l = 6\cdot34\,m$ (Pythagoras)

$l' = 6\cdot34\cos\beta.$

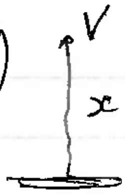
$\therefore$ Moment $= 6\cdot34\cos16\cdot6^\circ\times Mg$

$= 1\cdot07\times10^{10}\ Nm$

$\underline{\underline{= 1\cdot1\times10^{10}\ Nm}}$ ✓

$\boxed{8}$

(b) (i)




Force to lift length $x$ is weight + rate of change of momentum

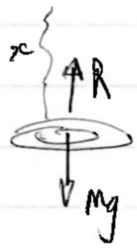
$$F = \lambda x . g + \frac{\Delta (mv)}{\Delta t}$$
$$(m)$$

$$= \lambda x g + \frac{\Delta (\lambda x)}{\Delta t} . v$$

$$\underline{\underline{F = \lambda x g + \lambda v^2}}$$

$$P = F . V$$
$$\underline{\underline{P = \lambda x g v + \lambda v^3}}$$

(ii)

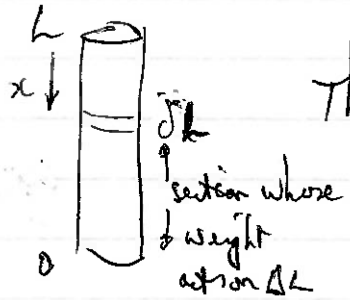



N II "R = mg"

$$\underline{\underline{R = (L - x) \lambda g}}$$

$\boxed{6}$

(c)




The extension of the ~~rod~~ chain is small.

$$E = \frac{F}{A} . \frac{L}{\delta L}$$

$$F = Mg \frac{(L - x)}{L}$$

$$\Delta L = \int_0^L \delta L$$

$$= \int_0^L \frac{F(x)}{AE} . dx$$

$$\Delta L = \int_0^L \frac{Mg}{EA} \frac{(L - x)}{L} dx = \frac{Mg}{EAL} \left[ Lx - \frac{x^2}{2} \right]_0^L$$

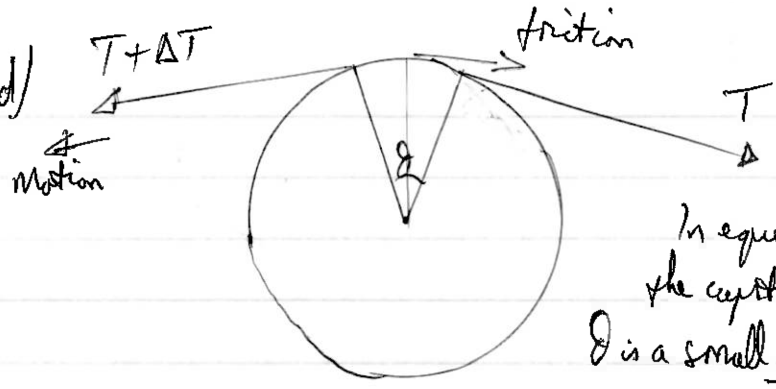
$$\underline{\underline{e = \frac{1}{2} \frac{Mg . L}{EA}}}$$

or $$\underline{\underline{e = \frac{1}{2} \frac{\lambda g}{EA} L^2}}$$

$\boxed{5}$

(the same extension as in the chain being pulled with a force equal to half its weight).

(d)




In equilibrium, the rope runs over the capstan.

$\theta$ is a small angle.

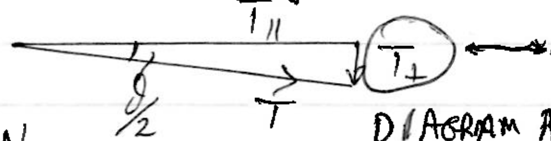



DIAGRAM REQUIRED

$\delta f$, frictional force $= \mu . N$

$= \mu 2 T_{\perp}$ since rope is also on the other side.

$= \mu 2 T \sin \theta/2$

$\delta f \simeq \mu T \theta.$

i.e. $\delta T = \mu T \, \delta\theta$ for $\theta \to \delta\theta$.

(iii)

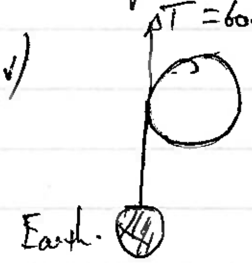
$$\int_{T_R}^{T_L} \frac{dT}{T} = \mu \int_0^{\theta} d\theta$$

$$\ln T \Big|_{T_R}^{T_L} = \mu\theta$$

$$\underline{T_L = T_R e^{\mu\theta}}$$

(The force on the left is large to pull against $T_R$ and friction)

(iv)




$$T_{large} = T_{small} \cdot e^{0.3 \times 2\pi n}$$

"n turns"

$$6 \times 10^{25} = 600 \times e^{0.6\pi n}.$$

$$\ln 10^{23} = 0.6 \pi n$$

$$\underline{\underline{n = 28}}$$

6

25 TOTAL

# QUESTION 4

(a) (i)

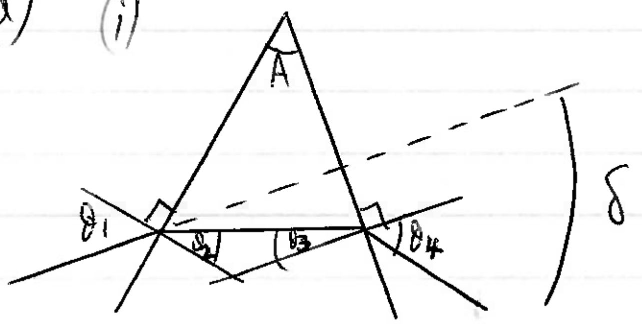



Diagram with $\theta_1 \to \theta_4$ marked ✓

DIAGRAM REQUIRED ✓

(ii) $\delta = \theta_1 - \theta_2 + \theta_4 - \theta_3.$

$= \theta_1 + \theta_4 - (\theta_2 + \theta_3)$

But $\theta_2 + \theta_3 = A$

so, $\delta = \theta_1 + \theta_4 - A$ ✓

(iii) If $\theta_1 = \theta_4$, then $\theta_1 = \dfrac{\delta + A}{2}.$

$\theta_2 = A/2$ ✓ for either.

then $n = \dfrac{\sin\left(\frac{\delta + A}{2}\right)}{\sin\left(A/2\right)}$ ✓

(iv) Red light: $\sin\left(\frac{\delta_r}{2} + 30°\right) = \dfrac{1.604}{2}$

$\delta_r = 46.64°.$

Blue light: $\sin\left(\frac{\delta_b}{2} + 30\right) = \dfrac{1.620}{2}$

$\delta_b = 48.19°$

either ✓

$\Delta(\delta_b - \delta_r) = 1.55°$

$= 1.6°$ ✓

(v) Linear separation $= f \times \Delta(\text{radians})$

$= 30 \times 0.027$

$= 0.8(1)$ cm ✓

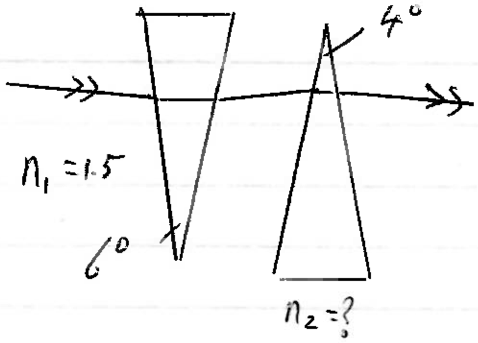
(vi) $A$ is small. so $(\theta_2 + \theta_3)$ is small

so $\theta_1, \theta_4$ are small.

so $\delta$ is small

So $n \approx \left(\frac{\delta + A}{2}\right) / \frac{A}{2} \approx \frac{\delta}{A} + 1$ ✓ $\left(\therefore \delta = A(n-1)\right)$ Not for this result.

(vii)




Minus

$$\delta_{total} = 0 = \delta_1 - \delta_2$$ ✓

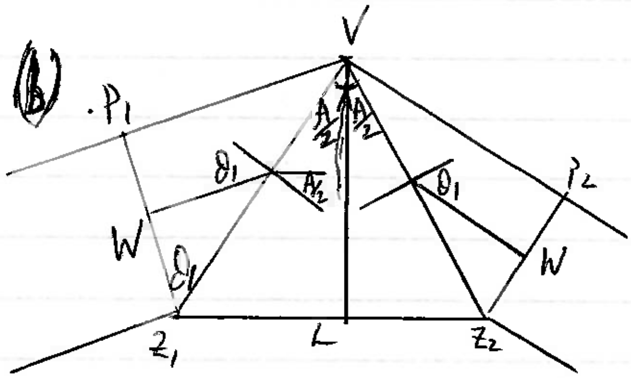
$$0 = A_1(n_1 - 1) - A_2(n_2 - 1)$$

$$6(1.5-1) = 4(n_2 - 1)$$

$$\underline{\underline{n_2 = 1.75}}$$ ✓

$\boxed{10}$

(b)




Can mark on angles $\frac{A}{2}$ and $\theta_1$

$$\left[\cos\theta_1 = \frac{W}{VZ_1}\right].$$

$$t_1 = 2.\frac{P_1 V}{c}$$

$$\left[\tan\theta_1 = \frac{P_1 V}{W}.\right]$$

$$= 2.\frac{W}{c}\tan\theta_1$$

$$= 2.\frac{W}{c}\frac{\sin\theta_1}{W}.VZ_1$$

$$\left[\sin\frac{A}{2} = \frac{L}{2}\Big/VZ_1\right].$$

$$= \frac{2}{c}\ VZ_1.\sin\theta_1$$

$$= \frac{2}{c}.\frac{L}{2}.\frac{\sin\theta_1}{\sin(A/2)}$$

$$= \frac{L}{c}\frac{\sin\theta_1}{\sin(A/2)}$$ ✓

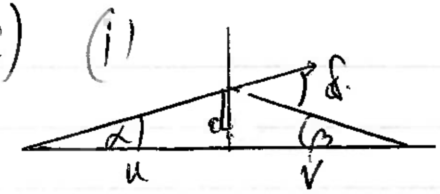
And $$t_2 = \frac{nL}{c}$$ ✓

$$\therefore \frac{t_1}{t_2} = \frac{1}{n}\frac{\sin\theta_1}{\sin(A/2)}$$ ✓

[agrees with $n = \frac{\sin\left(\frac{A+\delta}{2}\right)}{\sin\left(\frac{A}{2}\right)}$ when $t_1 = t_2$]

$\boxed{3}$

(c) (i)




$\delta = \alpha + \beta$

and we know that

$\delta = A(n-1)$

for small $\alpha, \beta$,

$$\left.\begin{aligned}\alpha &= d/u \\ \beta &= d/v\end{aligned}\right\} \checkmark$$

so $\alpha + \beta = A(n-1)$

becomes $\dfrac{d}{u} + \dfrac{d}{v} = A(n-1)$

then $\dfrac{1}{u} + \dfrac{1}{v} = \dfrac{A}{d}(n-1)$ ✓

(ii)

$$\frac{1}{\infty} + \frac{1}{40} = \frac{A}{2\cdot 5}(1\cdot 5 - 1)$$

$$\frac{1}{40} = \frac{A}{2\cdot 5} \times 0\cdot 5$$

$A = \frac{1}{8}$ radian

$A = 7^\circ$ ✓

$\boxed{3}$

(d) . Time of travel

$$\left.\begin{aligned} t_b &= \frac{60 \times 1\cdot 35}{3\times 10^8} = 2\cdot 70 \times 10^{-7}\,s \\ t_r &= \frac{60 \times 1\cdot 33}{3\times 10^8} = 2\cdot 66 \times 10^{-7}\,s \end{aligned}\right) \checkmark$$

$\Delta t = 0\cdot 04 \times 10^{-7}\,s$ ✓

when the red light goes off and the blue light begins, is time $\dfrac{1}{2f_0} = \Delta t$


$\left(\dfrac{T}{2} = \Delta t\right)$.

$f_0 = \dfrac{1}{2\Delta t}$

$f_0 = 1\cdot 25 \times 10^8\,Hz$ ✓

$\boxed{3}$

(e) From layer to layer,

$$n_4 \sin\alpha_4 = n_3 \sin\alpha_3$$

then $n_3 \sin\alpha_3 = n_2 \sin\alpha_2$

etc. $n_2 \sin\alpha_2 = n_1 \sin\alpha_1$

$n_1 \sin\alpha_1 = n_0 \sin\alpha_4$

idea.

At the top of the atmosphere $n_{top} = 1$

So $n_{top} \sin\alpha_{top} = n_0 \sin\alpha_0$

i.e. $1.\sin\alpha = n_0 \sin\alpha_0$

But we are given $\alpha = \alpha_0 + r$

then $n_0 \sin\alpha_0 = \sin(\alpha_0 + r)$

$= \sin\alpha_0 \cos r + \sin r \cos\alpha_0$

$n_0 \sin\alpha_0 \approx \sin\alpha_0 + r\cos\alpha_0$

$\div \cos\alpha_0$ $\quad n_0 \tan\alpha_0 = \tan\alpha_0 + r$

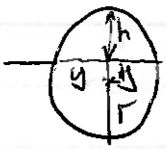
Hence $\underline{\underline{r = (n-1)\tan\alpha_0}}$

$\boxed{6}$

$\boxed{25}$ TOTAL.

# QUESTION 5

(a)




$$y^2 = h(2r-h)$$
$$9^2 = 4(2r-4)$$
$$\frac{81}{4} + 4 = 2r$$

$$r = \frac{97}{8} = 12.125\,m$$ ✓

NII $$\frac{mv^2}{r} = mg$$ ✓

$$v = \sqrt{rg}$$
$$v = 10.9\,m/s$$ ✓

$\boxed{3}$

(b) (i) $$\frac{mv^2}{r} = \frac{GmM_g}{r^2}$$

$$\left(\frac{2\pi r}{T}\right)^2 \frac{1}{r} = \frac{GM_g}{r^2}$$

$$T^2 = \frac{4\pi^2}{GM_g} \cdot r^3$$

(ii) $$M_g = \frac{4\pi^2 \cdot r^3}{GT^2}$$ ✓

$$= \frac{4\pi^2 \times (3\times10^4 \times 9.46\times10^{15})^3}{6.67\times10^{-11} \times (200\times10^6 \times 3.16\times10^7)^2}$$ ✓

(iii) $$= 3.4\times10^{41}\ kg$$
$$= 1.7\times10^{11}\ M_\odot$$

$\hat{=}\ 2\times10^{11}$ stars like the Sun. ✓

$\boxed{3}$

(c) (i) In a circular orbit there is an unbalanced / resultant force ✓ acting on the satellite.

So $Ma = \frac{mv^2}{r} = \frac{GmM_E}{r^2}$

$$v = \sqrt{\frac{GM_E}{r}}$$ ✓

and using $g = \frac{GM_E}{r^2}$ or otherwise, $v = \sqrt{rg}$

(ii) $V = \sqrt{6.37 \times 10^{6} \times 9.81}$
$= 7900 \text{ m/s}$
$= \underline{7.9 \text{ km/s}}$ ✓

(iii) $T = \dfrac{2\pi r}{V} = \dfrac{2\pi \times 6.37 \times 10^{6}}{7900}$
$= \underline{5060 \text{ s}}$. ✓

(iv) If we obtain an expression for the period in terms of $R$,

$$T^2 = \frac{4\pi^2}{GM_E} \cdot R^3 \quad \text{So} \quad T = \sqrt{\frac{4\pi^2}{GM_E}} \cdot R^{\frac{3}{2}}$$

then $$T' = \sqrt{\frac{4\pi^2}{GM_E}} (R+h)^{3/2}$$

$$\approx \sqrt{\frac{4\pi^2}{GM_E}} R^3 \left(1 + \frac{h}{R}\right)^{3/2}$$

$$T' \approx T \left(1 + \frac{3h}{2R}\right) + O\left(\frac{h^2}{R^2}\right)$$

$$\frac{T' - T}{T} = \frac{3h}{2R}$$

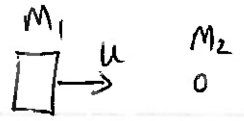
$$\Delta T = T \cdot \frac{3}{2}\left(\frac{200}{6370}\right) = 5060 \times \frac{3}{2}\left(\frac{200}{6370}\right)$$

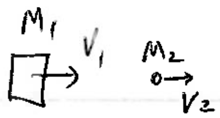
$$\Delta T = 238 \text{ s}$$

$$T' = 5060 + 240 = \underline{\underline{5300 \text{ s}}}$$ ✓

(v)


✓

(vi) Before 


After 


Momentum : $M_1 u = M_1 V_1 + M_2 V_2$ ① — ✓

KE conserved: $\frac{1}{2} M_1 u^2 = \frac{1}{2} M_1 V_1^2 + \frac{1}{2} M_2 V_2^2$ ② — ✓

Loss of KE of $M_1$ is $\frac{1}{2} M_1 u^2 - \frac{1}{2} M_1 V_1^2 = \frac{1}{2} M_2 V_2^2$ ③

From ① and ② $M_1(u - V_1) = M_2 V_2$

and $M_1(u - V_1)(u + V_1) = M_2 V_2^2$

dividing $u + V_1 = V_2$

For $M_1 \gg M_2$ $u \simeq V_1$

$\therefore$ KE lost $\simeq \frac{1}{2} M_2 4 u^2$

$= \underline{\underline{2 M_2 u^2}}$

[ using $u + V_1 = V_2$

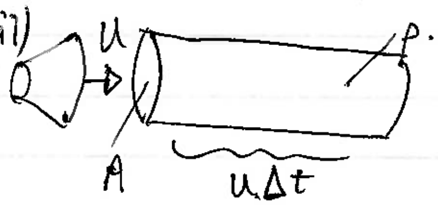
and from ① $u - V_1 = \frac{M_2}{M_1} V_2$

Subtract $2V_1 = V_2\left(1 - \frac{M_2}{M_1}\right)$

$2V_1 \approx V_2$

so $u + V_1 \approx 2V_1$

$u \simeq V_1$. ]

(vii) 


In time $\Delta t$, $M_2 = \rho A u \Delta t$ ✓

So KE lost in $\Delta t$ is $\Delta k = 2\rho A u \Delta t . u^2$

$= 2\rho A u^3 \Delta t$

So $\underline{\underline{\frac{\Delta k}{\Delta t} = 2\rho A u^3}}$ ✓

(viii) $\frac{dk}{dt} = \frac{1}{2} m \frac{d(v^2)}{dt} = \frac{1}{2} m 2v \frac{dv}{dt} = mv\frac{dv}{dt}$ ✓

$\therefore \frac{\Delta k}{k} = \frac{\Delta k}{\frac{1}{2}mv^2} = \frac{mv\Delta v}{\frac{1}{2}mv^2} = 2v\frac{\Delta v}{v^2} = 2\frac{\Delta v}{v}$

So $\frac{\Delta v}{v} = \frac{\Delta k}{mv^2} = \frac{10^4}{20 \times (7900)^2} = \underline{\underline{8 \times 10^{-6}}}$ ✓

(ix) $E = \frac{1}{2}mv^2 + -\frac{GMm}{r}$ and $\frac{mv^2}{r} = \frac{GMm}{r^2}$

$= -\frac{1}{2}\frac{GMm}{r}$ $\qquad \frac{1}{2}mv^2 = \frac{1}{2}\frac{GMm}{r}$

$= -\frac{1}{2}mv^2$

So if E becomes less then $v^2$, v increases. ✓ with some reason

(x) So we are looking at small changes.
Using the result from (vii),

$$\frac{\Delta k}{\Delta t} = 2\rho A u^3$$

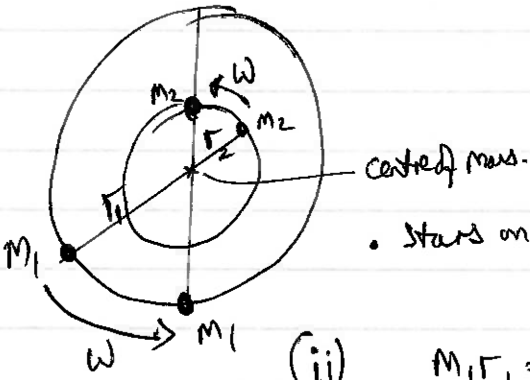
$$\frac{10^4}{5060} = 2\rho(2\cdot4) \times 7900^3$$

$\approx 5000s \Rightarrow 5060$

$$\rho = 8\cdot4 \times 10^{-13}\ kg/m^3$$

✓ $\boxed{14}$

(d)




• Stars on opposite sides (diameter) of C of M.
DIAGRAM REQUIRED ✓

(ii) $M_1 r_1 = m_2 r_2$ about C of M.

$m_2 = k m_1$

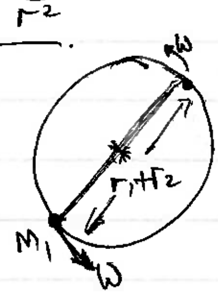
$\frac{r_1}{r_2} = k$. ✓

$\cancel{M_1} r_1 \omega^2 = \frac{G\cancel{M_1}m_2}{r^2}$

$\cancel{M_2} r_2 \omega^2 = \frac{G m_1 \cancel{M_2}}{r^2}$

$r\omega^2 = \frac{GM}{r^2}$

(iii) When the stars explodes, the angular velocity of each star, ω remains the same, The centre of mass moves.
• Stars at same separation and with same ω as before.
• Centre of mass now halfway between them. ✓



(iv) Since for an orbiting pair of stars, $r\omega^2 = \frac{GM}{r^2}$

where $r$ is the separation

$M$ is the total mass, $M = M_1 + M_2$

$= M_1 + kM_1$

$= M_1(1+k)$.

then $\omega^2 = \frac{GM_1(1+k)}{r^3}$ [and $\omega$ does not change and $r$ does not change].

The energy of the post-explosion system, $E$.

$$E = 2 \times \frac{1}{2} M_1 \left(\frac{r}{2}\right)^2 \omega^2 - \frac{GM_1^2}{r}$$

$$= M_1 \frac{r^2}{4} \cdot \frac{GM_1(1+k)}{r^3} - \frac{GM_1^2}{r}$$

$$E = \frac{GM_1^2}{r}\left(\frac{1+k}{4} - 1\right)$$

For a bound system, $E < 0$,

so $\frac{1+k}{4} < 1$

$$\underline{\underline{k < 3}}$$

$\boxed{5}$

$\boxed{25}$ TOTAL

## QUESTION 6

(a) $\dfrac{a}{\sqrt{2}}$ ✓ $\boxed{1}$

(b) The potential energy is $6 \times \dfrac{1}{4\pi\varepsilon_0} \dfrac{e}{6} \times \dfrac{e}{a/\sqrt{2}}$

$$= \frac{\sqrt{2}\, e^2}{4\pi\varepsilon_0\, a} \checkmark$$

$$= \frac{\sqrt{2} \times (1.6\times10^{-19})^2 \times 9\times10^{9}}{0.564\times10^{-9}}$$

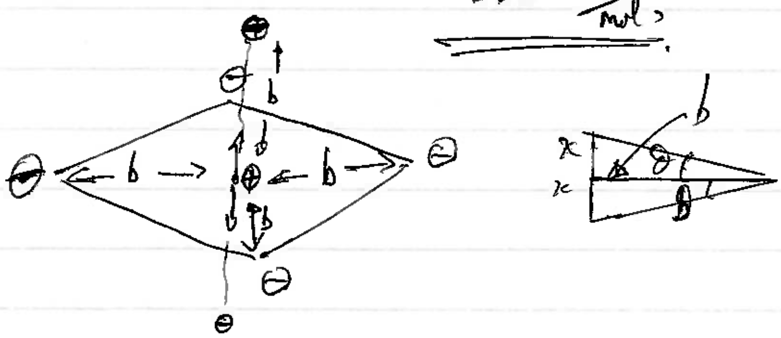
$$= \underline{5.8\times10^{-19}\,\text{J}} \checkmark$$

per mole, energy $= 5.78\times10^{-19} \times 6.02\times10^{23}$

$$= 348\ \text{kJ/mol}$$

$$= \underline{350\ \text{kJ/mol}}. \checkmark \quad \boxed{3}.$$

(c)



(A) Force on $Na^+$ due to charges out of the plane.

$$F_0 = -\frac{1}{4\pi\varepsilon_0} e^2 \left[\frac{1}{(b-x)^2} - \frac{1}{(b+x)^2}\right]. \checkmark$$

$$= -\frac{e^2}{4\pi\varepsilon_0}\left[\frac{(b+x)^2-(b-x)^2}{(b-x)^2(b+x)^2}\right]$$

$$= -\frac{e^2}{4\pi\varepsilon_0}\left[\frac{2b\cdot 2x}{(b^2-x^2)^2}\right]$$

$$= -\frac{4e^2bx}{4\pi\varepsilon_0} \cdot \frac{1}{(b^2-x^2)^2}$$

$$\simeq -\frac{4e^2bx}{4\pi\varepsilon_0 b^4}\left[1+\frac{2x^2}{b^2}\right]$$

To leading order $\quad F_0 = -\dfrac{4e^2}{4\pi\varepsilon_0}\dfrac{x}{b^3}$ ✓

(B) Force on $Na^+$ due to the four charges in the plane.

$$F_i = -4 \times \frac{1}{4\pi\varepsilon_0} \frac{e^2}{(x^2+b^2)} \cdot \sin\theta$$

and $\sin\theta = \dfrac{x}{(x^2+b^2)^{\frac{1}{2}}}$.

$$\therefore \quad F_i = \frac{-4e^2}{4\pi\varepsilon_0} \cdot \frac{x}{(x^2+b^2)^{\frac{3}{2}}}$$

$$= -\frac{4e^2}{4\pi\varepsilon_0} \cdot \frac{x}{b^3}\left[1 - \frac{3}{2}\frac{x^2}{b^2} + \cdots\right]$$

To leading order, $F_i = -\dfrac{4e^2}{4\pi\varepsilon_0} \dfrac{x}{b^3}$ (the same as $F_0$)

$$\therefore \quad F_{\text{restoring}} = F_0 + F_i = -\frac{8e^2}{4\pi\varepsilon_0}\frac{x}{b^3}$$

and using $b = \dfrac{a}{\sqrt{2}}$

$$M\ddot{x} = -\frac{8e^2}{4\pi\varepsilon_0} \cdot \frac{x \cdot 2\sqrt{2}}{a^3}$$

So $\omega^2 = \dfrac{16\sqrt{2}\, e^2}{4\pi\varepsilon_0 a^3 M}$

$$f = \frac{\omega}{2\pi} = \frac{4e}{2\pi}\sqrt{\frac{\sqrt{2}}{4\pi\varepsilon_0 a^3 \times 23 u}}$$

$$= 2 \times \frac{1.6\times10^{-19}}{\pi}\sqrt{\frac{\sqrt{2}\times 9\times10^9}{(0.564\times10^{-9})^3}\cdot\frac{1}{23\times1.66\times10^{-27}}}$$

$$= 4.39\times10^{12}\ \text{Hz}$$

$$= 4.4\times10^{12}\ \text{Hz}$$

$\boxed{6}$

(d) (i) For diffraction, $n\lambda = a\sin\theta_n$

de Broglie wave, $\lambda = h/p$

KE. $eV = \dfrac{p^2}{2m}$ $\quad p = \sqrt{2meV}$

So $n\dfrac{h}{\sqrt{2meV}} = a\sin\theta_n$

$$\sin\theta_n = \frac{nh}{a\sqrt{2meV}}$$

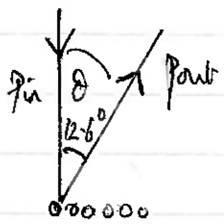
(ii) $$\sin\theta_n = \frac{n \times 6.63\times10^{-34}}{0.514\times10^{-9}\sqrt{2\times9.11\times10^{-31}\times1.6\times10^{-19}\times100}}$$

$$= n \times 0.218$$

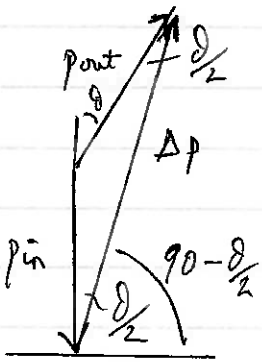
$n=1$ $\theta_1 = 12.6° = \underline{13°}$ ✓

$n=2$ $\theta_2 = 25.8° = \underline{26°}$ ✓

(iii) <u>Picture</u>




<u>Momenta</u>




Vector sketch ✓
$\Delta P$, angles marked ✓

(iv) $90-\frac{\theta}{2} = 90-6.3 = 83.7° = \underline{84°}$ ✓

Magnitude: Sine rule $\frac{\Delta p}{\sin 167.4°} = \frac{p_{in}}{\sin 6.3°}$

$p_{in} = \sqrt{2meV}$
$= 5.4\times10^{-24}\ kg\,m\,s^{-1}$.

So $\Delta p = \frac{\sin 167.4}{\sin 6.3} \times 5.4\times10^{-24}\ kg\,m\,s^{-1}$

$\Delta p = 1.07\times10^{-23}$

$\underline{\Delta p = 1.1\times10^{-23}\ kg\,m\,s^{-1}}$ ✓

$\boxed{8}$

(e) energy per atom, $E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}$

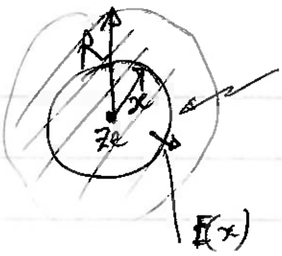
So energy per mole is $E_m = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{r}.N_A$. ✓

Here $r = \frac{9\times10^9 \times (1.6\times10^{-19})^2\times6.02\times10^{23}}{496\times10^3}$

$\underline{r = 2.8\times10^{-10}\,m}$ ✓

$\boxed{2}$

(d)




$\sigma$ (i) $E(x) = E_+ + E_-$

$$= \frac{1}{4\pi\varepsilon_0}\cdot\frac{Ze}{x^2} + \frac{1}{4\pi\varepsilon_0}\left[\frac{\frac{4}{3}\pi x^3(-\sigma)}{x^2}\right] \quad \text{(where } \tfrac{4}{3}\pi x^3(-\sigma) = Q(x)\text{)}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{Ze}{x^2} + - \frac{x^3(Ze)}{R^3 x^2}\right]$$

$$= \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{x^2} - \frac{x}{R^3}\right].$$

$$\sigma = \frac{Ze}{\frac{4}{3}\pi R^3}$$

$$Q(x) = -\frac{4}{3}\pi x^3 \sigma$$

$$= -\frac{4}{3}\pi\, \frac{x^3 . (Ze)}{\frac{4}{3}\pi R^3}$$

$$Q(x) = -\frac{x^3(Ze)}{R^3}$$

check: when $x = R$, $E = 0$.

(ii) $V = -\int_R^x E . dx$

$$= -\frac{Ze}{4\pi\varepsilon_0}\int_R^x \left[\frac{1}{x^2} - \frac{x}{R^3}\right] dx$$

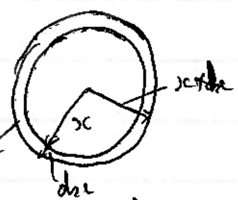
$$= \frac{-Ze}{4\pi\varepsilon_0}\left[-\frac{1}{x} - \frac{x^2}{2R^3}\right]_R^x$$

$$= +\frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{x} + \frac{x^2}{2R^3} - \frac{1}{R} - \frac{R^2}{2R^3}\right]$$

$$= +\frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{x} + \frac{x^2}{2R^3} - \frac{3}{2R}\right].$$

N.B. When $x = R$ $V = 0$ as expected as there is no field outside the atom.

(iii) $W = \int_0^{Ze} V . dq = \int_0^R \frac{Ze}{4\pi\varepsilon_0}\left[\frac{1}{x} + \frac{x^2}{2R^3} - \frac{3}{2R}\right] . 4\pi x^2 \sigma . dx$




$\text{Vol} = 4\pi x^2 . dx$

$dQ = 4\pi x^2 . dx . \sigma.$

$$= \frac{Ze}{4\pi\varepsilon_0} . 4\pi\sigma \int_0^R \left[x + \frac{x^4}{2R^3} - \frac{3x^2}{2R}\right] dx$$

$$= \frac{Ze . \sigma}{\varepsilon_0}\left[\frac{x^2}{2} + \frac{x^5}{5.2R^3} - \frac{3 . x^3}{3.2R}\right]_0^R.$$

$$= \frac{Ze\sigma}{\varepsilon_0}\left[\frac{R^2}{2} + \frac{R^2}{10} - \frac{R^2}{2}\right].$$

$$= \frac{Ze\sigma R^2}{\varepsilon_0 \times 10} = \frac{Ze R^2}{10\varepsilon_0} . \frac{Ze}{\frac{4}{3}\pi R^3} = \frac{(Ze)^2}{4\pi\varepsilon_0} . \frac{3}{10R}$$

$$\text{Energy/mol} = \frac{(11 \times 1.6\times10^{-19})^2}{3\times10^{-10}} \times 0.3 \times 9\times10^9 \times 6.02\times10^{23} = 1.7\ \text{MJ/mol}$$

$\boxed{5}$

$\boxed{25}$ TOTAL