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BPhO Round 1

Section 2

11th November 2022

<u>This question paper must not be taken out of the exam room</u>

<u>Instructions</u>

1 hour 20 minutes

Questions: Only **two questions** out of the four questions in *Section 2* should be attempted.

> ***Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.***

Working: Working, calculations, explanations of the physics and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be brief but clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified. Marks are given for intermediate steps if they can be seen: underline or circle them so that the marker can find them.

Marks: Students are recommended to spend about 40 minutes on each question. Each question in *Section 2* is out of 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode". Code may not be written for any of the BPhO competitions.

Solutions: 1. Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY (pages will be scanned)**. 2. Students should write their **name** and their **school/college** clearly on every answer sheet. 3. Number each question clearly. 4. Number your pages at the top. 5. Write "END" at the end of your script. 6. Fill in the Front Cover Sheet your teacher will give you - **just one for the two sections**.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes . If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_{p}	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_{w}	$4180\,\mathrm{J\,kg^{-1}\,^{\circ}C^{-1}}$

$$T_{(\mathrm{K})} = T_{(^{\circ}\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\pi r^3$$

$$e^x \approx 1 + x + \dots \qquad x \ll 1$$

$$(1+x)^n \approx 1 + nx \qquad x \ll 1$$

$$\frac{1}{(1+x)^n} \approx 1 - nx \qquad x \ll 1$$

$$\tan\theta \approx \sin\theta \approx \theta \qquad \text{for } \theta \ll 1$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} \qquad \text{for } \theta \ll 1$$

Section 2 — attempt two questions only

Question 2

This question is about Centre of Mass and Centre of Gravity.

Centre of mass has many applications in systems, and can enable otherwise complex calculations to be simplified, often in collisions and rotational dynamics.

a) Two weights with masses m_1 and m_2 are attached to the ends of a light rod at distances x_1 and x_2 respectively, measured from a point that is along the line of the rod but beyond the end. A normal force, F, applied to some point on the rod will hold it in equilibrium.

(i) What is meant by the the word *equilibrium*?

(ii) By taking moments about the origin point of x_1 and x_2, show that a single force, F, applied normal to the rod will hold it in equilibrium, if it acts at the point given by

$$X_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

(iii) If an excess force ΔF is applied to the centre of mass of the rod, what is the ratio of the kinetic energy gained of m_1 to that of m_2? **(3)**

b) (i) A $75\,\text{cm}$ long uniform plank of mass $0.24\,\text{kg}$ hangs over the edge of a table as in **Fig. 1** and supports a $0.35\,\text{kg}$ mug of tea with a diameter of $8.0\,\text{cm}$ which sits on the end of the plank with one side on the edge. A pile of textbooks, each of mass $0.48\,\text{kg}$ is used weigh the plank down on the table, with $15\,\text{cm}$ of plank resting under the books that are also $15\,\text{cm}$ wide. How many books are required to support the system?

(2)

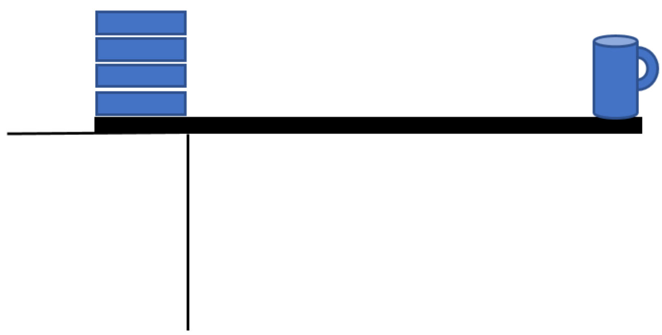


Figure 1: Cup of tea on a plank supported by books.

(ii) Two uniform, similar rods of the same material **AB** of lengths L_{AB} and **BC** of length L_{BC} respectively, are joined rigidly at B. They are suspended from A on a string and the rods hang at angles α and β to the vertical.

Obtain an expression for $\dfrac{L_{\text{AB}}}{L_{\text{BC}}}$ in terms of α and β.

(3)

(iii) A wooden stick of length L and density ρ_{w} has a rectangular cross section of side b. It floats in a liquid of density ρ_{l}. If it floats vertically, obtain an expression for the vertical difference in heights of the centres of mass of the liquid displaced and the stick, in terms of L, ρ_{w} and ρ_{l}

(1)

(iv) A light rod of length $L = 30\,\mathrm{cm}$ has masses of $0.40\,\mathrm{kg}$ and $0.90\,\mathrm{kg}$ attached to the ends of the rod. It is thrown vertically upwards with a flick of the wrist, from a height h above the ground, rotating at eight times each second, and returning to height h. It is given $110\,\mathrm{J}$ of energy initially.
How many rotations does it make before it lands?

(3)

c) A small ball of mass $\frac{1}{3}m$ travelling at speed $2v$ collides linearly and inelastically with a ball of mass $\frac{2}{3}m$ moving at speed v. A set of equal time interval photographs are taken of the collision, and are set out in **Fig. 2**.

(i) Sketch this and add to it the position of the centre of mass of the particles.

(ii) From measurements of the tracks on this paper, estimate the fraction of the KE lost in the collision.

(2)

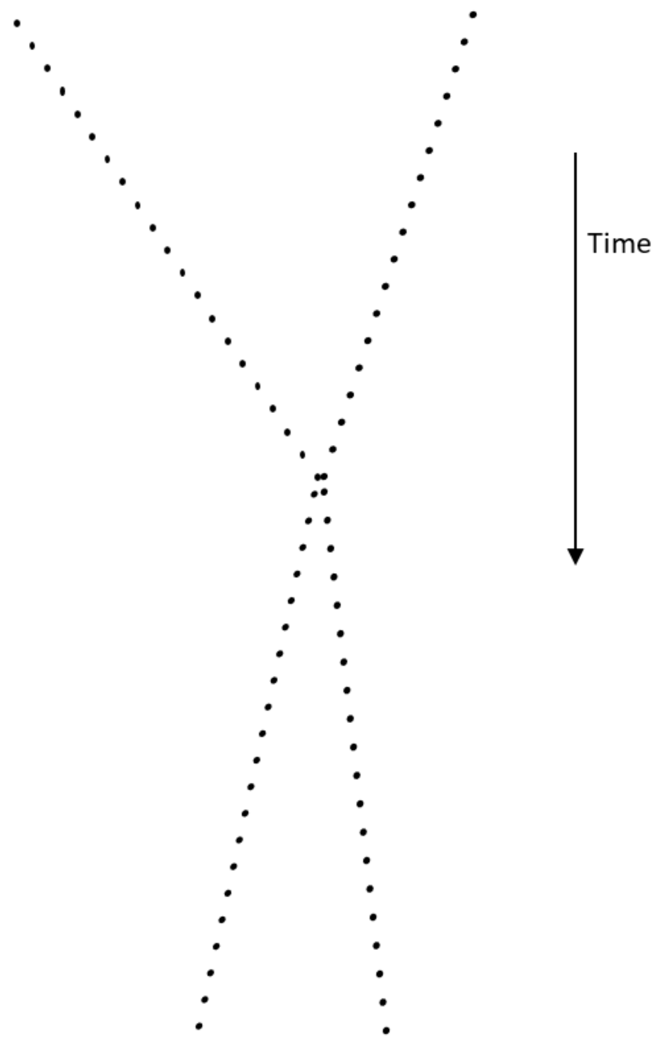


Figure 2: Collision of two particles.

d) (i) A thin uniform string of mass m and length $2a$ hangs over a light, smooth pulley of negligible radius. When displaced from equilibrium, it slides off and leaves the pulley at speed v. Obtain an expression for v in terms of a and g.

(4)

(ii) In a pendulum clock, the length of the pendulum must be kept constant, independent of temperature changes, if the clock is to run at the correct rate. In the diagram of **Fig. 3**, a triangular support for the pendulum mass is made of an isosceles triangle of rods, the horizontal rod of length L_1 with coefficient of thermal expansion α in $^\circ\text{C}^{-1}$ and two sloping rods of length L_2 of a different metal with coefficient of thermal expansion β. The pendulum mass is attached at its centre and the pendulum is supported so that it can swing in and out of the page.

If the pendulum is temperature compensated for small temperature changes ΔT, what is the required ratio of the lengths $\dfrac{L_1}{L_2}$ in terms of α and β?

(3)

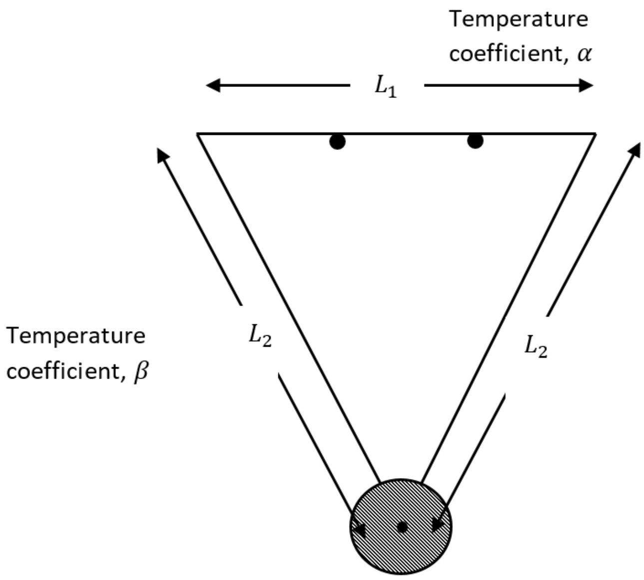


Figure 3: A temperature compensated pendulum constructed from a triangular support of rods, so that it can swing in and out of the page.

(iii) A light rod of length ℓ has two similar masses m attached to the left and right ends. It is suspended horizontally by a string in a non-uniform vertical gravitational field which increases linearly in strength from g at the left end to $1.1g$ at the right end.

i. When the rod is balanced, as a fraction of ℓ, what is the difference between the centre of mass of the system and the point of suspension (the centre of gravity)?

ii. If the string is now attached to the centre of the rod, a third mass m must also be attached to the rod in order to balance it. In terms of ℓ, how far from the left end of the rod must this mass be attached?

(4)

[25 marks]

Question 3

This question is about investigating materials with X rays and electrons.

An accelerated charge radiates electromagnetic radiation. A synchrotron electron storage ring uses this idea to produce a continuous spectrum of electromagnetic radiation as the electrons are accelerated in a circular trajectory.

a) Particular wavelengths of light can be selected using a monochromator, some of which make use of diffraction and interference from crystals. The diagram shows two rays from the continuous spectrum striking two adjacent crystal planes separated by a distance a. The crystal planes act as partially reflecting mirrors. For certain angles θ, measured to the reflecting plane, the emerging rays are monochromatic.

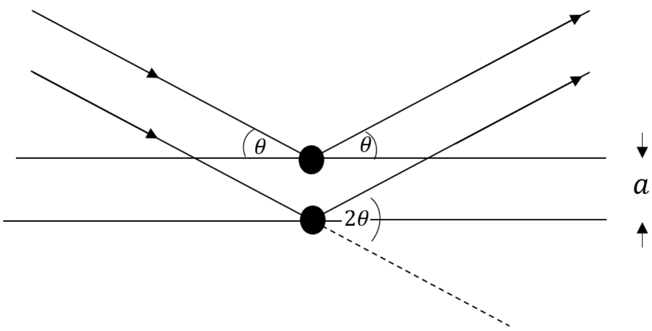


Figure 4: Reflection from two atomic surfaces in a crystal.

(i) What is the path difference between the two rays shown, in terms of a?

(ii) Determine the wavelength in terms of θ and a.

(iii) If $a = 0.31\,\text{nm}$, calculate the first order wavelength of light strongly scattered at an angle of $\theta = 15^\circ$, and

(iv) the direction and magnitude of the change in momentum of the photons of this wavelength. You may use a diagram. **(5)**

b) In practice two crystals are used such that the light enters and leaves the monochromator in the same direction. The height, h between the incoming and outgoing beams is fixed. Using a suitable approximation to first order, obtain a formula for x in terms of the wavelength λ, the crystal lattice spacing a, and the height h.

(3)

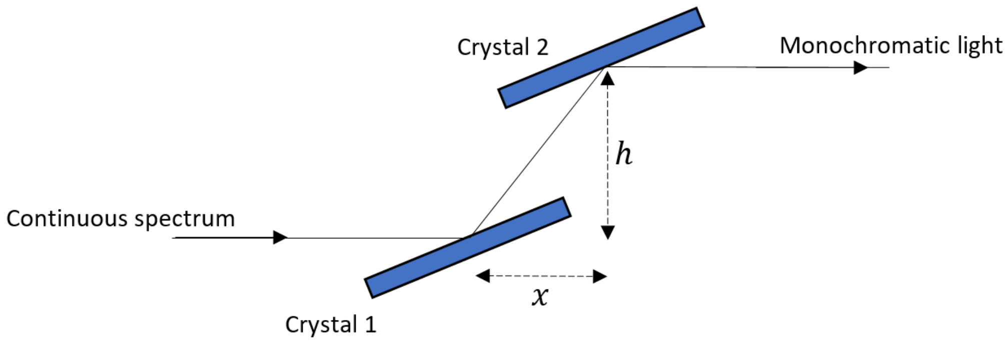


Figure 5

c) Monochromatic light of part (b) is incident on a sample of copper so that electrons are emitted via the photoelectric effect. The energy of the emitted electrons are investigated using a hemispherical electron energy analyser, shown in **Fig. 6**.

E_0, referred to as the *pass energy*, is the energy of the electron travelling from the analyser entrance to the exit slit of the electron detector along the equipotential path defined by, $R_0 = \frac{R_{\text{out}}+R_{\text{in}}}{2}$, in which R_{in} and R_{out} are the inner and outer hemisphere radii, respectively.

These charged spheres produce a radial electric field between them. Only electrons of the particular energy E_0, describe a perfectly semi circular path and are detected. The inner sphere is at a potential V_{in} and the outer one is at a potential V_{out} so as to make the electric field radial.

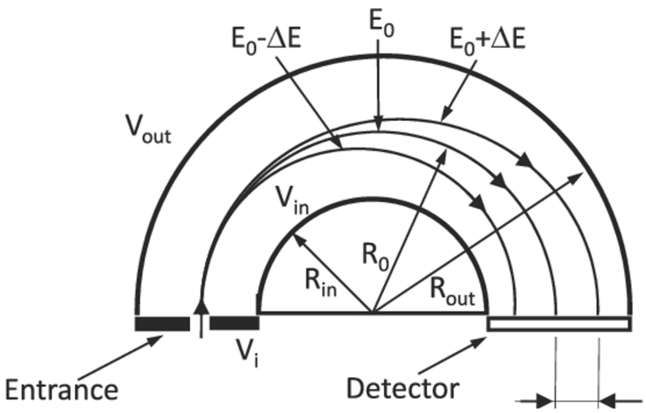


Figure 6: Hemispherical electron energy analyser.
Figure published in Progress in Materials Science 2020 X-ray photoelectron spectroscopy: Towards reliable binding energy referencing G. Greczynski, L. Hultman

(i) Draw a sketch of the electric field lines between the hemispheres.

(ii) Write down the potential V_{in} of the inner hemisphere in terms of the charge Q on the hemisphere and its radius R_{in}.

(iii) In the simplest form possible, determine an expression for the pass energy, E_0 in terms of the radii of the hemispheres R_{in} and R_{out}, the difference in the potentials (i.e. $\Delta V = V_{\text{in}} - V_{\text{out}}$), and the electronic charge e.

(iv) If the mean radius of the hemispheres is $50\,\text{mm}$ with a separation of $10\,\text{mm}$, what potential difference between the plates gives a pass energy of $5.0\,\text{eV}$?

(7)

d) In a solid such as copper, the electrons exist in very closely spaced energy levels, which can be investigated. A $5.0\,\text{eV}$ free electron is produced, for example, when an electron is in an energy level $4.0\,\text{eV}$ below the top level in the solid and is released without collision in photoelectric emission. The workfunction of copper is $4.6\,\text{eV}$.

(i) What was the photon energy used?

(ii) what wavelength is this and which part of the electromagnetic spectrum does it correspond to?

(3)

(iii) When an electron travels from the solid into the vacuum, it changes speed, similar to the behaviour of light crossing a boundary. Whilst inside the material the electron was travelling in the direction shown in **Fig. 7**, at $\theta = 50°$ to the surface normal.

i. Calculate a value for the (relative) refractive index $\mu > 1$, corresponding to the electron going from the vacuum into the copper.

ii. For the the electron leaving the copper at $\theta = 50°$, at what angle does it emerge into the vacuum?

(2)

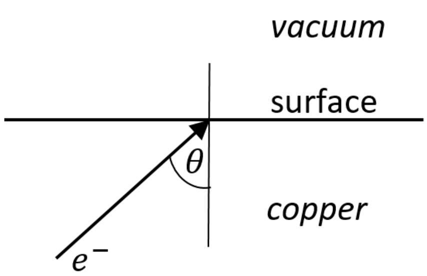


Figure 7: An electron crossing a surface from the vacuum.

e) Light of wavelength $\lambda = 640\,\mathrm{nm}$ passes through an evacuated tube of length $\ell = 8.0\,\mathrm{cm}$ with thin glass ends. The beam interferes with a second coherent beam of light on a screen, causing destructive interference, as shown in **Fig. 8**. Air has a refractive index given by $\mu_{\mathrm{air}} = 1 + 0.00029\frac{P}{P_0}$ where $P_0 = 1.0 \times 10^5\,\mathrm{Pa}$. What minimum pressure of air in the tube would cause the interference pattern to become constructive interference at the screen?

(2)

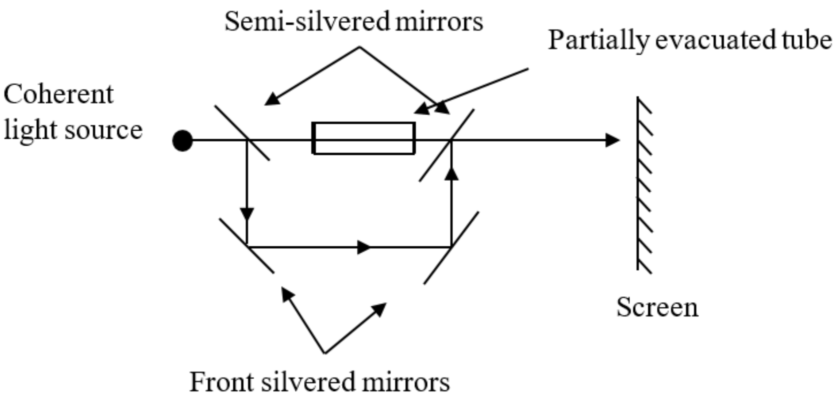


Figure 8: Interference between two beams from a coherent source of light. One beam passes through a partially evacuated tube.

f) The refractive index of a material for electrons depends upon the wavelength of the electron. So if we instead consider X rays reflecting off adjacent layers of a crystal, we

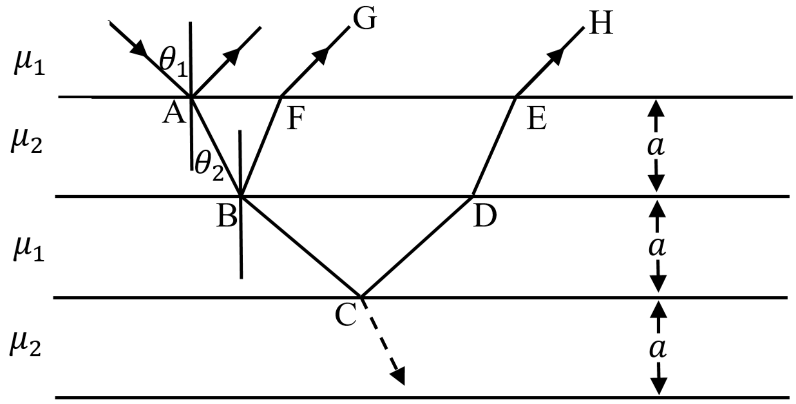


Figure 9: X ray interference between layers of atoms.

can take into account the effect of refraction within the material.
An example is illustrated in **Fig. 9** in which adjacent layers of atoms are laid down on a substrate. X rays are incident at angle θ_1 and refracted in the second layer at angle θ_r. If there are multiple layers and reflections, then these will dominate intensity of the reflection at the first surface from the vacuum. So we we can look at the path difference between the emerging rays G and H. In this very simplified model, the layers have been given the same thickness a.

(i) Determine the path length BCD.

(ii) What is the optical path difference between rays G and H?

(3)

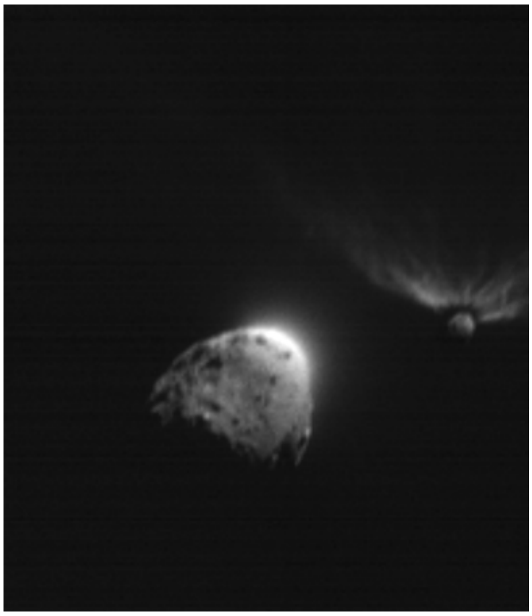
[25 marks]

Question 4

This question is about the physics of the DART mission to deflect an asteroid.

The Double Asteroid Redirection Test (DART) was a mission by NASA to measure the effectiveness of the deflection of an asteroid through a high-speed impact. Such a strategy would be a cheap way of achieving planetary defence against such asteroids should one of comparable size threaten the Earth. The DART probe successfully impacted into Dimorphos, an asteroid moon of the asteroid Didymos, on 26th September 2022 (see **Fig. 10**). The target was chosen as it would only affect the orbit of the moon and so there was no danger of it being pushed into an Earth-crossing orbit, and Dimorphos is comparable to the $140\,\mathrm{m}$ minimum size to be considered a potentially hazardous asteroid, so is like the real threats we might face.

(a)



(b)

Figure 10: Observations before and after impact.
Fig. (a): The view of the asteroid Didymos (bottom left) and its moon Dimorphos (top right) taken by DART as it was heading towards the moon for impact. Credit: NASA/Johns Hopkins APL
Fig. (b): The view of the pair taken with the probe LICIACube just after its closest approach, from a vantage point a little behind the DART impactor. There is clearly a considerable amount of material ejected from the surface of Dimorphos, as expected from a successful impact. This amplifies the effect of the impact on the orbit of Dimorphos, giving a larger change in orbital velocity than by just the probe alone. Credit: ASI/NASA

a) DART was fitted with the most up-to-date ion drive available (NEXT-C) to ensure it could accelerate up to the required impact velocity with only a fraction of the mass of fuel that would be needed for conventional rockets. It does this by applying a very small thrust for a very long time. The fuel used was singly ionised xenon-131 ions.
The accelerating potential was $1800\,\mathrm{V}$.

(i) What was the ejection speed of the xenon? Assume the xenon ions started from rest.

(ii) The current of ions was $3.52\,\mathrm{A}$. What is the thrust generated?

(iii) The probe hit Dimorphos $306\,\mathrm{days}$ after launch. If the ion drive had been run continuously for that time (and there was enough xenon to do so), what would

have been the speed at impact?
Assume linear acceleration of the probe in deep space starting from rest, and take the average mass of the probe to be $600\,\mathrm{kg}$ during the acceleration phase.

(6)

Dimorphos was discovered through dips in the brightness of Didymos as it passes in front and behind the asteroid (known as transits).

b) Radar observations of Didymos measured its diameter to be $780\,\mathrm{m}$ (assuming a spherical shape) and the centre-to-centre distance to Dimorphos to be $1.20\,\mathrm{km}$ (assuming a circular orbit). The relative depth of the dips in brightness meant that the diameter of Dimorphos was derived to be $164\,\mathrm{m}$ (again assuming a spherical shape), and the period of the orbital motion was $11\,\mathrm{h}\ 55\,\mathrm{min}$.

(i) By considering the forces acting on each of the objects in their circular orbits, calculate the total mass of the binary system.

(ii) Assuming both asteroid and moon have the same density, calculate the mass of Didymos and Dimorphos.

(3)

c) The DART probe had a mass of $570\,\mathrm{kg}$ at impact and was travelling at a speed of $6.14\,\mathrm{km\,s^{-1}}$ (relative to the rest frame of Dimorphos).

(i) By conserving momentum and assuming a head-on inelastic collision, determine the change in Dimorphos' orbital speed, Δv. Give your answer in $\mathrm{mm\,s^{-1}}$.

(ii) The impact will have made the orbit elliptical, although by considering the total energy of the orbit you can calculate the radius of an equivalent circular one.

i. obtain an expression for the total energy of a satellite, m_{s}, in orbit about a central mass, m_{c}, in terms of $G, m_{\mathrm{c}}, m_{\mathrm{s}}$ and the radius of orbit r.
Using this approach, calculate

ii. the expected decrease in orbital radius, Δr, and hence

iii. the expected decrease in orbital period, ΔT, giving your answer in minutes.

(10)

d) Less than a month after the impact, it was announced that the observed orbital period shortening was $32\,\mathrm{minutes}$. This is larger than the one in part (c) due to the momentum transfer enhancement factor, β, which quantifies how much the ejecta from the surface and deformation of Dimorphos has amplified the effect. To calculate β accurately is quite complicated and will need months of observations, but a very rough estimate can be made using the approximation $\beta \propto \Delta v$.
Treating the value in part (c) as the $\beta = 1$ case, find the observed value of Δv and hence a ballpark estimate for the value of β.

(6)

[25 marks]

Question 5

This question is about the physics involved in star formation.

The James Webb Space Telescope (JWST) has only been publicly releasing images since July 2022 and has already shown it is a more than worthy successor to the Hubble Space Telescope (HST). In **Fig. 11** is a recent side by side comparison is of the 'Pillars of Creation' in the Eagle Nebula – a famous area undergoing considerable star formation.

Figure 11: The 'Pillars of Creation' – a famous star-forming region in the Eagle Nebula – as viewed with the Hubble Space Telescope (left) and the James Webb Space Telescope (right). Since the JWST operates at infrared wavelengths it can see through the dust and reveal so many of the new stars that have been born. Credit: NASA, ESA, CSA, STScI; Joseph DePasquale (STScI), Anton M. Koekemoer (STScI), Alyssa Pagan (STScI).

Some of the new stars being born will be large and hot, so will have strong stellar winds and considerable ionising radiation that will carve out holes around them, meaning that over time the dust columns as we see them will disappear. The ionised bubble is known as a Strömgren Sphere and is created by ionising photons (those with energies greater than $13.6\,\mathrm{eV}$) ionising any hydrogen atoms in the vicinity. At the edge there is a balance between new ionisations and electrons recombining with hydrogen nuclei.
The formula for the radius of the sphere is:

$$r_\mathrm{S} = \left(\frac{3N}{4\pi\alpha}\right)^{\frac{1}{3}} n_\mathrm{H}^{-\frac{2}{3}}$$

where N is the number of ionising photons per second, α quantifies the probability that a proton and electron recombine into a hydrogen atom, and n_H is the number density of hydrogen nuclei in the sphere (assumed to be the same as the number density of electrons).

Some Solar data:
Mass of the Sun, $M_\mathrm{Sun} = 1.99 \times 10^{30}\,\mathrm{kg}$
Radius of the Sun, $R_\mathrm{Sun} = 6.96 \times 10^{8}\,\mathrm{m}$
Luminosity of the Sun (power output), $L_\mathrm{Sun} = 3.85 \times 10^{26}\,\mathrm{W}$
Effective surface temperature of the Sun, $T_\mathrm{Sun} = 5780\,\mathrm{K}$
Peak wavelength in solar spectrum, $\lambda_\mathrm{Sun} = 500\,\mathrm{nm}$
Distance from Sun to Earth $= 1.50 \times 10^{11}\,\mathrm{m}$

a) Infrared measurements by JWST suggest that in this region some O and B class stars will be formed. An example of such a star has a mass of $23M_\mathrm{Sun}$, radius of $10.0R_\mathrm{Sun}$, luminosity of $1.47 \times 10^{5}L_\mathrm{Sun}$, and effective surface temperature of $35\,800\,\mathrm{K}$.

(i) Given that the peak wavelength in a blackbody spectrum (i.e. the wavelength $\lambda_{\max}$ at which the power emitted is a maximum) is inversely proportional to the effective surface temperature, T, so $T\lambda_{\max} = \text{constant}$, verify by calculation that photons emitted at the star's peak wavelength will be ionising.

(ii) In the Pillars of Creation, $n_{\mathrm{H}} \approx 10^9\,\mathrm{m}^{-3}$ and ionisation will lead to an equilibrium temperature inside the sphere of $8000\,\mathrm{K}$ for which $\alpha = 3.1 \times 10^{-19}\,\mathrm{m}^3\,\mathrm{s}^{-1}$. Assuming that the star emits monochromatically at just the peak wavelength, estimate N and hence estimate r_{S}. Give your answer in light years (ly).

(iii) The height of the pillar on the left is $5\,\mathrm{ly}$. How many such large stars will be needed to photoevaporate the pillar (given long enough), such that it is no longer visible? Assume they are evenly spread out throughout the pillar.

(5)

As well as the ionising effects on its surroundings, a new, bright star will exert radiation pressure, and this can trigger further collapses of regions of the nebula into other stars. Assuming the region outside the Strömgren Sphere is purely neutral hydrogen, the mass that will be collapsed by an external pressure p_0 is given as:

$$M_{\text{collapse}} = \frac{1.18 k_{\mathrm{B}}^2 T_{\text{neb}}^2}{p_0^{\frac{1}{2}} G^{\frac{3}{2}} m_{\mathrm{H}}^2}$$

This is known at the Bonner-Ebert mass, where k_{B} is the Boltzmann constant, T_{neb} is the temperature of the neutral hydrogen in the nebula, and m_{H} is the mass of a hydrogen atom. The radiation pressure is given as:

$$p_{\text{rad}} = \frac{L}{4\pi r^2 c}$$

where L is the luminosity of the star (the power output), r is the distance from the star, and c is the speed of light.

b) Taking the temperature of the nebula to be $T_{\text{neb}} = 50\,\mathrm{K}$, and taking $p_0 = p_{\text{rad}}$, calculate the cloud mass that will begin to collapse just outside the Strömgren Sphere and form a new star (cluster). Give your answer in solar masses. **(2)**

Radiation pressure can also be important in some stars in keeping them in equilibrium against the gravitational forces trying to compress them. The gravitational pressure at the centre of a star is given by:

$$p_{\text{Grav}} = \int_0^{R_{\text{star}}} \frac{GM(r)\rho(r)}{r^2} dr$$

where $M(r)$ is the mass enclosed within radius r, and $\rho(r)$ is the density. Since in real stars both M and ρ are complicated functions of r, this integral is not straightforward to do and generally can only be done numerically. In equilibrium, this will be balanced by contributions from the gas pressure and blackbody radiation pressure:

$$p_{\text{tot}} = p_{\text{gas}} + p_{\text{rad}} = \frac{\rho_c k_{\mathrm{B}} T_c}{\overline{m}} + \frac{4\sigma}{3c} T_c^4$$

where ρ_c and T_c are the central density and central temperature, respectively, σ is the Stefan-Boltzmann constant, and $\overline{m}$ is the average mass of the particles. Generally, in the core of the star is a mix of fully ionised hydrogen and helium, so

$$\frac{\overline{m}}{m_{\mathrm{H}}} = \frac{2}{1 + 3X + 0.5Y}$$

where X and Y are the mass fractions of hydrogen and helium, respectively.

c) At the centre of the Sun, lots of its initial hydrogen has already been turned into helium, so $X = 0.35, Y = 0.65, \rho_c = 1.53 \times 10^5\,\mathrm{kg\,m^{-3}}$ and $T_c = 1.57 \times 10^7\,\mathrm{K}$.

(i) Estimate the pressure at the centre of the Sun, p_{tot}.

(ii) Calculate p_{rad} as a percentage of p_{tot}. Comment on your answer.

(iii) Consider a solar mass, Earth-sized white dwarf and assume it has a constant density equal to its average density. Carry out the integration and estimate p_{Grav}. How many times larger is it than the solar p_{tot}?
$R_{\mathrm{Earth}} = 6370\,\mathrm{km}$ **(10)**

The photons generated by the nuclear fusion in the core of the Sun are gamma ray photons, however by the time they reach the surface of the Sun they are mostly visible light. This is because the photons spend a very long time being absorbed and then re-emitted by particles in the plasma, and initially this is via Compton scattering, which lowers the energy of the photon with each emission, and then later via Thomson scattering, which does not affect the photon energy but does mean it continues to bounce around in the star.

Throughout all the scattering, the photons follow a "random walk", meaning that after N steps of average step size ℓ (known as the mean free path), the final displacement, d, is

$$d = \ell\sqrt{N}$$

d) The energy density of blackbody photons is $\dfrac{E}{V} = \dfrac{4\sigma}{c}T^4$ and the mean temperature of the Sun is $4.73 \times 10^6\,\mathrm{K}$.

(i) By considering the total energy stored as EM radiation in the Sun and the current luminosity, estimate the time it takes for a photon to diffuse from the core to the surface. Give your answer in years.

(ii) Hence find the mean free path, ℓ, and the number of interactions, N.

(4)

Unlike photons, neutrinos produced in the fusion reactions hardly interact at all with the dense, opaque plasma in the core. Consequently, they can tell us about the energy production happening now, which can be compared to the observed luminosity and alert us to any changes over the photon diffusion timescale. In 2018, the Borexino experiment detected, for the first time, neutrinos from the proton-proton I branch from the Sun. This branch is responsible for 92% of the Sun's energy output, and each neutrino corresponds to $13.1\,\mathrm{MeV}$ of energy provided to the star by fusion reactions. The measured neutrino flux at the position of the Earth was $(6.1 \pm 0.5) \times 10^{10}\,\mathrm{neutrinos\ cm^{-2}\,s^{-1}}$.

e) Use the photon output luminosity to predict the neutrino flux at Earth and compare it to the measured value. Comment on the implications of your answer.

(4)

[25 marks]

END OF SECTION 2