BPhO British Physics Olympiad

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BPhO Round 1

Section 1

10th November 2023

This question paper must not be taken out of the exam room

Instructions

Time: **1 hour 20 minutes** for this section.

Questions: Students may attempt any parts of *Section 1*, but are not expected to complete all parts.

Working: Working, calculations, explanations and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be clear.
Marks are given for intermediate steps if they can be seen: underline or circle them so that the marker can find them.

Marks: A **maximum of 50 marks** can be awarded for *Section 1*. There is a total of ≈ **83** marks allocated to the problems of Question 1 which makes up the whole of *Section 1*.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode". Code may not be written for any of the BPhO competitions.

Solutions: **1.** Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY** (pages will be scanned). **2.** Students should write their **name** and their **school/college** clearly on every answer sheet. **3.** Number each question clearly. **4. Number your pages** at the top. **5.** Write "END" at the end of your script. **6.** Fill in the Front Cover Sheet your teacher will give you - **just one for the two sections**.

Sitting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes. ***Section 1* should be collected in after 1 hour 20 minutes** and only then should *Section 2* be given out.

b. *Section 1* and *Section 2* may be sat in two separate sessions of 1 hour 20 minutes each. *Section 1* must be collected in after the first session and *Section 2* only handed out at the beginning of the second session.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_e	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_n	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Mass of Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_E	$6.37 \times 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_w	$4180\,\mathrm{J\,kg^{-1}\,^\circ C^{-1}}$

$$T_{(\mathrm{K})} = T_{(^\circ\mathrm{C})} + 273$$

$$\text{Volume of a sphere} = \tfrac{4}{3}\pi r^3$$

$$\begin{aligned}
e^x &\approx 1 + x + \ldots && \text{for } x \ll 1 \\
(1+x)^n &\approx 1 + nx && \text{for } x \ll 1 \\
\frac{1}{(1+x)^n} &\approx 1 - nx && \text{for } x \ll 1 \\
\tan\theta &\approx \sin\theta \approx \theta && \text{for } \theta \ll 1 \\
\cos\theta &\approx 1 - \frac{\theta^2}{2} && \text{for } \theta \ll 1
\end{aligned}$$

Section 1 — 50 marks maximum

Question 1

a) A skier of mass m moves from rest down a slope of angle θ to the horizontal. The skier experiences a constant resistive force, F_r. At time t they have travelled a distance d down the slope. Obtain an expression for F_r in terms of the quantities given, and the acceleration due to gravity g.

[3]

b) A car of mass 1200 kg tows a caravan of mass 600 kg. The resistive forces acting on the car and caravan are proportional to their individual weights and their sum equals 900 N. The vehicle accelerates at $2.0\,\mathrm{m\,s^{-2}}$ on a flat road.

(i) What force is exerted by the engine?

(ii) What is the tension in the tow bar?

(iii) What is the power being converted by the engine after 5 s?

[3]

c) The distance between village A and village B is 50 km. Helen and Robert decided to cycle from A to B.
Robert left A at 8.00 am, with a speed of $12\,\mathrm{km\,h^{-1}}$ and had a break of 30 minutes.
Helen left at 8.30 am, did not have a break and reached B an hour before Robert. What was Helen's speed?

[3]

d) A uniform plank of mass m stands on a smooth floor and leans against a smooth wall at an angle α to the horizontal. It is held in place by a horizontal string attached to the bottom of the ladder and to the bottom of the wall, as shown in **Fig. 1**.
What is the tension, T, in the string in terms of m, g and α?

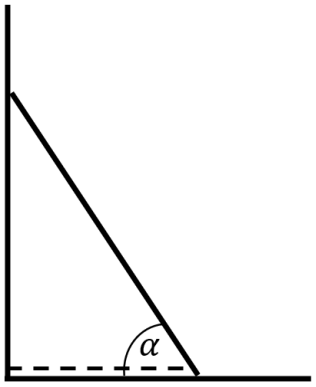

Figure 1: Plank on a smooth floor leaning against a smooth wall held in place by a light string (dotted line).

[3]

e) An object falls under gravity. The ratio of the distance fallen by the object in the last second of its fall to the distance covered in the last but one second of its fall is 3 : 2.

(i) Find the height from which the object fell, and

(ii) The speed at which it hit the ground.

[3]

f) A mass m is suspended from a horizontal rod by two identical wires of negligible weight, each at angle $\theta = 30°$ to the vertical when attached to points A and B on the rod as shown in **Fig. 2**. The tension in each wire is T.

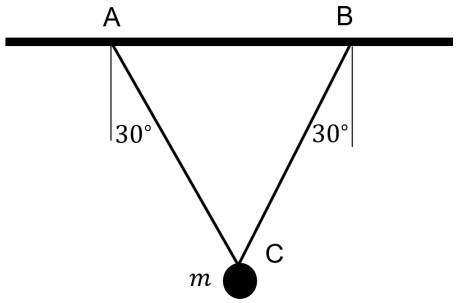

Figure 2: A mass suspended from two wires.

(i) Obtain an expression for T, the tension in each wire, in terms of m and g.

(ii) The wire BC is now cut. At the same instant the tension in wire AC, T_{AC}, will change as the system is now in motion. What is the ratio $\dfrac{T_{\text{AC}}}{T}$?

[3]

g) The resistance of a copper wire 1 m long with a mass of 1 g is 0.15 Ω. Find the length of a wire of the same material with a mass of 1000 kg and a resistance of 6000 Ω.

[3]

h) A glass block, with a reflecting lower surface, is shaped as a rectangular slab but whose left and right sides are curved in the shape of quarter circles of radius R, as shown in **Fig. 3**. The base A ray of light enters horizontally from the left and passes out through the right side of the block at the same height above the base. The length of the top plane surface is ℓ.

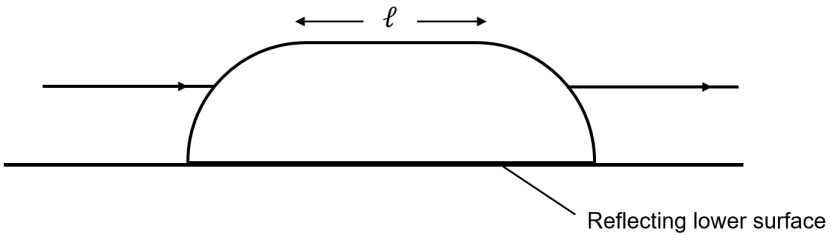

Figure 3: Light passing through a glass block.

If the depth of the block is 2.0 cm and the refractive index $n = 1.46$, what would be the minimum value of ℓ to satisfy this situation?

[3]

i) An 8 W beam of light is shone on a surface at normal incidence. The surface reflects 50% of the incident light and absorbs the other 50%.

(i) What is the average force exerted on the surface by the radiation?

Hint: the momentum of a photon, p, is given by its energy, E, divided by the speed of light, c. i.e. $p = \dfrac{E}{c}$

(ii) The average wavelength of the light is 600 nm and the beam covers an area of 12 cm^2 when it is incident on the surface. Calculate the volume density of photons in the beam.

[4]

j) A short pulse of 10^8 neutrons is fired through a vacuum at a target. If the bunch of neutrons is travelling at a speed $v = 2200\ \text{m s}^{-1}$ and the half life of a neutron is 880 s, how many neutrons will decay whilst travelling a distance of 11 m towards the target?

[4]

k) A smoothly hinged rod of mass m rests, at angle θ to the vertical, on a smooth cylinder of radius r which sits on the floor. The hinge is set on a vertical wall at a height of $3r$ above the floor, and the lower end of the rod is at a height r above the ground when the cylinder is in the corner of the wall and the floor, as shown in **Fig. 4**.

(i) Determine the value of $\tan\theta$.

It may help to know the identity, $\tan\theta = \dfrac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}$.

(ii) What is the length ℓ of the rod in terms of r? Give your answer as a fraction.

(iii) A light horizontal thread is attached to the rightmost point on the surface of the cylinder and it is pulled slowly to the right until the tension in the thread reduces to zero. What is the minimum amount of work that needs to be done by the thread?

(iv) As the cylinder is pulled away from the wall, the angle θ increases from its minimum value θ_0, which is illustrated in **Fig. 4**, to its maximum value θ_{max}, before decreasing again. What is the ratio $\dfrac{\cos\theta_0}{\cos\theta_{\text{max}}}$?

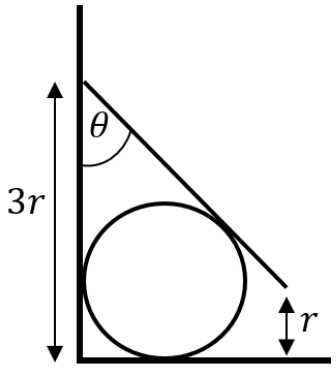

Figure 4: A hinged rod resting on a cylinder.

[4]

l) An object of mass m_1 slides down the smooth sloping surface of a wedge of mass m_2, as shown in **Fig. 5**. The angle of the slope is $\theta = 30°$ to the horizontal. The wedge sits on a smooth horizontal surface.

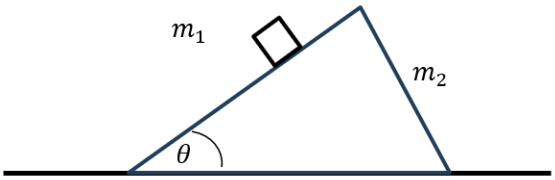

Figure 5: A block of mass m_1 sliding down the smooth face of a wedge of mass m_2 that sits on a smooth horizontal surface.

(i) Mark on the forces on two free-body diagrams.

The block's acceleration can be resolved into two components; one is down the slope, which would be the case if the wedge was fixed, and a second horizontal component so that it remains in contact with the accelerating wedge.

(ii) Resolve the forces on the sliding object normal to the slope.

(iii) Hence or otherwise, obtain an expression for the acceleration of the wedge in terms of m_1, m_2 and g.

[4]

m) Two ships, A and B spot each other at the moment ship B is due south of A with a range of 2.0 nautical miles. Ship A is travelling on a bearing 090° at 5.0 knots, and ship B on a bearing of 030° at 20 knots. What is their distance of closest approach?
1 knot = 1 nautical mile per hour.
A bearing is an angle measured clockwise from 0° as due north.

[5]

n) A steel tube of 4.0 cm internal diameter and 30 cm long, with a wall 0.25 cm thick, is covered externally and lined internally with copper tubes 0.20 cm thick. The three concentric tubes are firmly connected. This compound tube is placed under tension by a load and the stress produced equals $6.2 \times 10^7\ \mathrm{N\,m^{-2}}$. Determine

(i) The extension of the tube

(ii) The stress in the copper tubes

(iii) The load carried by the compound tube

Young's Moduli: Steel 200 GPa
Copper 110 GPa

[5]

o) A thin wire of mass $m = 6.0\,\text{g}$ and length $L = 1.2\,\text{m}$ is stretched to a tension T between two fixed supports. The wire is excited so that the third harmonic standing wave is formed of maximum amplitude A_{max}. In such a wire, the total KE and elastic PE are equal. The tension is $T = 45\,\text{N}$ and $A_{\text{max}} = 1.8\,\text{cm}$.

(i) Sketch a graph of the maximum kinetic energy of the particles in the wire against their position along the length of the wire in the range 0 to L. Draw a line on your graph to show the the total energy of the particles along the wire from 0 to L.

(ii) Calculate the frequency of vibration of the wire.

(iii) Calculate the total energy due to the vibration of the wire.

Hint: The speed of a wave v in a stretched wire is given by $v = \sqrt{\frac{T}{\mu}}$ where μ is the mass per unit length of the wire.

[5]

p) A cell of emf ε with an internal resistance r is connected across two resistors R_1 and R_2 in parallel, as in **Fig. 6**. Resistor R_2 is variable, whilst R_1 is fixed.

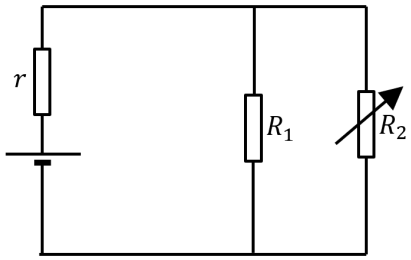

Figure 6: A circuit with two resistors connected in parallel with a cell.

Obtain expressions for

(i) the current I flowing through the cell in terms of ε, R_1, R_2, r,

(ii) the current I_2 flowing through R_2 in terms of I, R_1, R_2,

(iii) the power P_2 converted in R_2 in terms of ε, R_1, R_2, r.

(iv) By considering the term $\frac{1}{P_2}$ or otherwise, determine an expression for R_2 in terms if R_1 and r such that the power P_2 converted in R_2 is a maximum.

[5]

q) Three capacitors, C_1, C_2 and C_3 are shown connected to a pair of cells of emfs ε_1 and ε_2 as shown in **Fig. 7**. Obtain an expression in terms of the capacitor values and the emfs for the potential across C_3.
It is important that you write down your initial equations clearly.

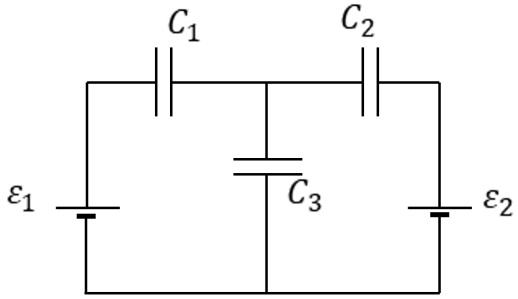

Figure 7: Three capacitors and two cells in a circuit.

[5]

r) A uniform rod, of length $2a$, floats partly immersed in a liquid, being supported by a string fastened to one of its ends, the other end of the string being attached to a fixed point A, as in **Fig. 8**. The density of the liquid is a factor $4/3$ times that of the rod. Determine

(i) the fraction of the rod's length that will be submerged, and

(ii) the tension, T in the string.

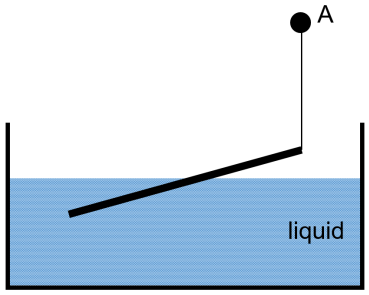

Figure 8: A rod in a liquid.

[6]

s) Two balls A and B of masses and volumes m_A, V_A and m_B, V_A respectively are attached to the ends of a light thread. The thread is hung over a freely moving pulley above a beaker of water. The pulley is lowered so that B is fully submerged and ball A floats on the water. $V_A = 1000\,\text{cm}^3, m_A = 500\text{g}, V_B = 50\,\text{cm}^3$ and $m_B = 390\,\text{g}$.

(i) Sketch a diagram showing the forces on each object in the system.

(ii) What fraction f of the volume of A is submerged below the water surface?

Suppose that a short section of the thread is replaced by a spring of negligible mass of spring constant $k = 100\,\text{N m}^{-1}$.

(iii) By how much is the spring extended?

Density of water $\rho = 1.0\,\text{g cm}^{-3}$

[6]

t) A vertical wooden pole with a circular cross section of diameter 30 cm has its footing embedded in a concrete block. A loose steel guy wire, A, is fixed to the ground a distance $b = 2.4$ m from the base of the pole, and is attached to the pole at a height $h = 12$ m above the ground. It is subject to a pull to the right by tension P in a cable, attached at the same height, as shown in **Fig. 9**. This causes the wooden pole to bend a small amount to the right in the arc of a circle of radius r with $r \gg h$. The loose steel guy wire, A, then becomes tight and prevents the pole from bending any further. The bent pole stretches on one side and compresses on the other, with a neutral line down the centre of the pole. The pole has a maximum stress of 4000 N m^{-2} at one side when bent.

(i) Calculate radius r and the angle of the circular arc formed by the bent pole when the neutral line remains unstressed.

(ii) Draw a diagram to show how this angle can be used to calculate the length of the guy wire, A, when it is tight.

(iii) Calculate an approximate value for the length of slack in the guy wire when the pole is unloaded and straight and vertical.

You may assume that the bend is small enough so that the height of the pole when bent is not changed significantly.
Young's modulus for wood is 14 GPa.

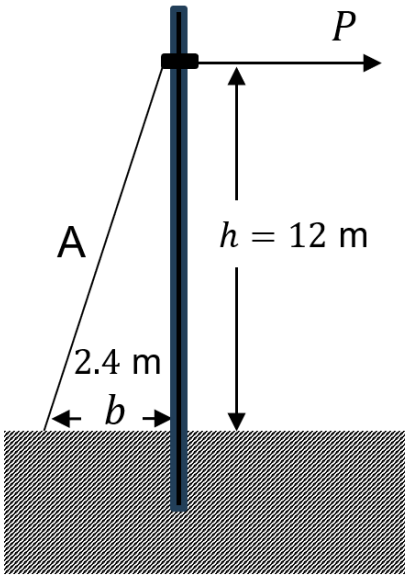

Figure 9: A pole with a guy wire A attached that will bend to the right by the tension in the cable P.

[6]

END OF SECTION 1