BPhO Round 1

Section 2

10th November 2023

This question paper must not be taken out of the exam room

Instructions

1 hour 20 minutes

Questions: Only **two questions** out of the four questions in *Section 2* should be attempted.

Each question contains independent parts so that later parts should be attempted even if earlier parts are incomplete.

Working: Working, calculations, explanations of the physics and **diagrams**, properly laid out, must be shown for full credit. The final answer alone is not sufficient. Writing must be brief but clear. If derivations are required, they must be mathematically supported, with any approximations stated and justified. Marks are given for intermediate steps if they can be seen: underline or circle them so that the marker can find them.

Marks: Students are recommended to spend about 40 minutes on each question. Each question in *Section 2* is out of 25, with a **maximum of 50 marks from two questions** only.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode". Code may not be written for any of the BPhO competitions.

Solutions: **1.** Answers and calculations are to be written on loose paper **ON ONE SIDE ONLY** (pages will be scanned). **2.** Students should write their **name** and their **school/college** clearly on every answer sheet. **3.** Number each question clearly. **4. Number your pages** at the top. **5.** Write "END" at the end of your script. **6.** Fill in the Front Cover Sheet your teacher will give you - **just one for the two sections**.

Setting the paper: There are two options for sitting BPhO Round 1:

a. *Section 1* and *Section 2* may be sat in one session of 2 hours 40 minutes *Section 1* should be collected in after 1 hour 20 minutes and then *Section 2* given out.

b. *Section 1* and *Section 2* may be sat in two sessions on separate occasions, with 1 hour 20 minutes . If the paper is taken in two sessions on separate occasions, *Section 1* must be collected in after the first session and *Section 2* handed out at the beginning of the second session.

Important Constants

Constant	**Symbol**	**Value**
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Mass of electron	m_{e}	$9.110 \times 10^{-31}\,\mathrm{kg}$
Mass of proton	m_{p}	$1.673 \times 10^{-27}\,\mathrm{kg}$
Mass of neutron	m_{n}	$1.675 \times 10^{-27}\,\mathrm{kg}$
atomic mass unit	u	$1.661 \times 10^{-27}\,\mathrm{kg} = 931.5\,\mathrm{MeV\,c^{-2}}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$
Earth's gravitational field strength	g	$9.81\,\mathrm{N\,kg^{-1}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Avogadro constant	N_{A}	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Gas constant	R	$8.3145\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Mass of Sun	M_{S}	$1.99 \times 10^{30}\,\mathrm{kg}$
Radius of Earth	R_{E}	$6.37 \times 10^{6}\,\mathrm{m}$
Specific heat capacity of water	c_{w}	$4180\,\mathrm{J\,kg^{-1}\,{}^{\circ}C^{-1}}$

$$T_{(\mathrm{K})} = T_{(^{\circ}\mathrm{C})} + 273$$

Volume of a sphere $= \frac{4}{3}\pi r^3$

$$e^x \approx 1 + x + \ldots \qquad x \ll 1$$

$$(1+x)^n \approx 1 + nx \qquad x \ll 1$$

$$\frac{1}{(1+x)^n} \approx 1 - nx \qquad x \ll 1$$

$$\tan\theta \approx \sin\theta \approx \theta \qquad \text{for } \theta \ll 1$$

$$\cos\theta \approx 1 - \frac{\theta^2}{2} \qquad \text{for } \theta \ll 1$$

Section 2 — attempt two questions only

You may be able to do later parts of a question even if you cannot do the earlier parts.

Question 2

This question is about wind turbines.

a) Assuming a wind direction perpendicular to the plane of the turbine blades, determine the power if all of the kinetic energy of the wind is transferred to the blades, in terms of the density of the air ρ, the length of the turbine blades r, and the wind speed v.

Figure 1: An offshore wind turbine.

b) In reality, not all of the power of the wind is extracted. The wind speed is slower behind the turbine, and the wind can be imagined to disperse over a larger area as shown in Figure 2, where A shows the area swept out by the turbine. This question will attempt a more realistic model of the energy transfers in a wind turbine. Assume the density of the air remains constant.

(i) Write down expressions for the rate of flow of air mass in region (*i*),

(ii) Write down expressions for the rate of flow of air mass in region (*ii*)

(iii) Write down expressions for the rate of flow of air mass in region (*iii*).

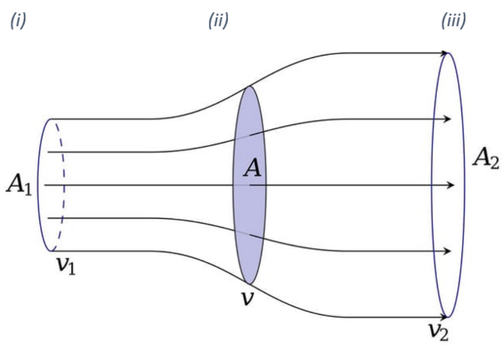


Figure 2: Flow of air through turbine.

(iv) By considering the air before entry and after entry to the turbine, determine the power output of the turbine in terms of ρ, A, v, v_1 and v_2.

(v) If we now assume that v equals the mean of v_1 and v_2, write down an expression for the power output of the turbine in terms of ρ, A, v_1 and v_2 only.

(vi) Determine the value of v_1/v_2 which gives a maximum in the output of the turbine.

(vii) Determine the maximum power output of the turbine in terms of ρ, A and the prevailing wind speed v_1.

(viii) The newest off-shore wind turbines in the North Sea have a blade length of 107 m, and the average wind speed in $10\,\mathrm{m\,s^{-1}}$. Calculate the maximum possible power output from this turbine.

c) The electrical power from the wind farm is transferred to the shore using High Voltage Direct Current (HVDC) at 320 kV with respect to earth. The cable used to transfer the power will typically consist of a 2.2 cm radius copper core, surrounded by cross-linked polyethylene (XLPE) insulation of radius 4.0 cm which itself is surrounded by a protective metal screen. The greatest length in a North Sea project will be 265 km.

The resistivity of copper is $1.68 \times 10^{-8}\,\Omega\,\mathrm{m}$ and of XLPE is $1.97 \times 10^{14}\,\Omega\,\mathrm{m}$

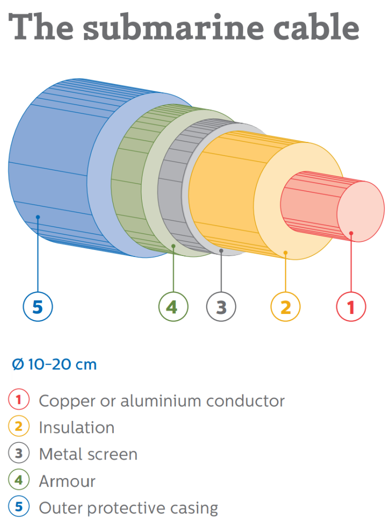


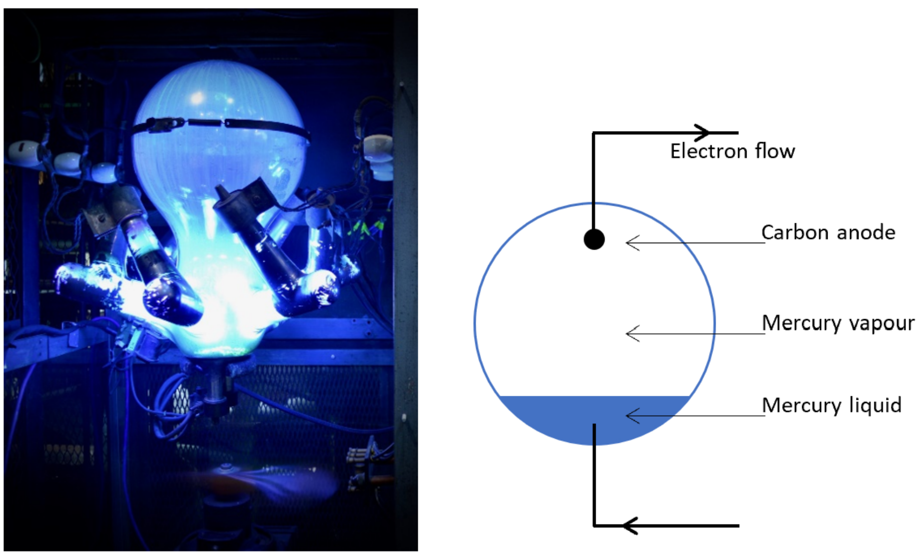
Figure 3: The structure of a high voltage cable. credit: inelfe

(i) Calculate the resistance of the copper core and the power dissipated as heat, and percentage loss in the copper, if the cable transfers 800 MW of power.

(ii) Calculate the resistance of the XLPE insulation between the copper core and the metal screen, and hence calculate the leakage current, power and percentage loss due to the leakage current, when operating at 320 kV.

(iii) The thermal conductivity of XLPE = $0.28\,\mathrm{W\,m\,K^{-1}}$. Assuming the temperature drop across all thermal barriers except the XPLE is negligible, calculate the operating temperature of the copper core at 800 MW given the ambient sea temperature is 7.0 °C.

d) High current rectification (ac to dc conversion), and inversion (dc to ac conversion) was until relatively recently achieved using a device called a mercury arc valve (see Figure 4a). In its simplest schematic form in Figure 4b:



(a) Mercury arc valve. credit: Kempton Steam Museum

(b) Schematic of mercury arc valve.

Figure 4

The principle of operation is that the mercury vapour at low pressure ($7.0\,\mathrm{mPa}$ and $40\,^\circ\mathrm{C}$) becomes ionised and the ions collide with the mercury liquid which liberates electrons. These then become available to carry the current to the carbon anode, and also via collision with atoms of mercury vapour produce mercury ions to replace those reabsorbed into the liquid phase. There is no appreciable electron emission from the carbon anode and so the current flows in one direction only in the manner of a diode.

(i) Estimate the forward bias potential of this "diode".

(ii) Inserting a grid between the anode and the mercury cathode allows the electron current to be modulated and thus an ac current produced from a dc supply.

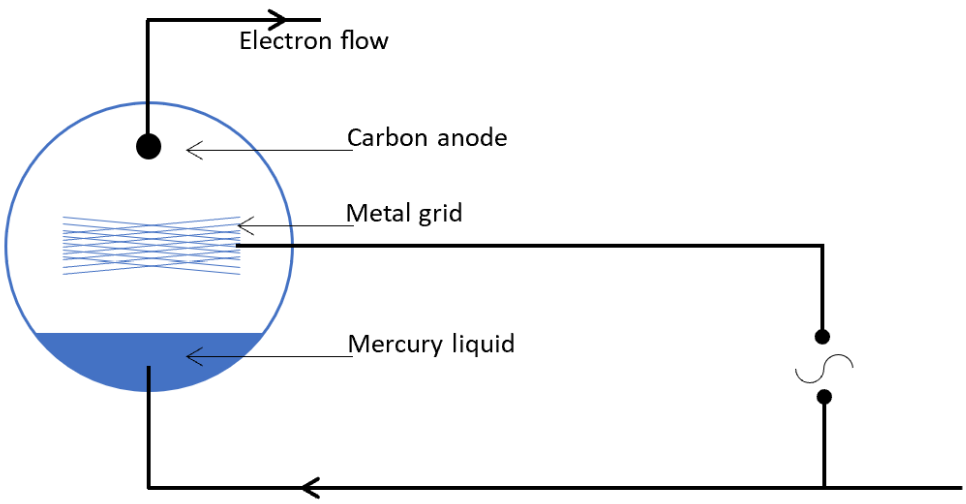


Figure 5: Mercury Arc Valve Circuit

The anode current is given by $I = a + bV_g + cV_g^2$, and $V_g = A + B\cos(\omega t)$, that is a constant bias and an oscillatory signal.
The anode current contains a second harmonic component. Determine its amplitude.

(iii) The mean value of the anode current contains a term proportional to B^2. Obtain the coefficient of this term.

(iv) If I_0 is the value of I when $B = 0$ and, when the signal is applied, I oscillates between I_{max} and I_{min}, express the ratio of the amplitudes of the second harmonic and fundamental (1^{st} harmonic) in terms of the quantity χ, which is defined as $\chi = \dfrac{I_{max} - I_0}{I_0 - I_{min}}$.

Hint: a useful identity is $\cos^2\theta - \sin^2\theta = \cos 2\theta$.

[25 marks]

Question 3

This question is about hot air balloons.

a) A hot air balloon rises when the upthrust is greater than its total weight. Calculate the volume of hot air required to lift a balloon with a combined mass of $200\,\text{kg}$ for the balloon and basket. You should include the mass of the hot air, which has a density of $0.95\,\text{kg}\,\text{m}^{-3}$ in the total weight of the balloon. The density of the cold air surrounding the balloon is $1.23\,\text{kg}\,\text{m}^{-3}$.

b) In 1783, the Montgolfier brothers' hot air balloon freely ascended, but remained at a low height. The balloon had a volume of about $2000\,\text{m}^3$, and a lifting capacity including its own weight of $830\,\text{kg}$. The air inside the balloon was heated by a fire in a basket below.
Estimate the temperature of the air in the balloon.

Air density at ground level is $1.23\,\text{kg}\,\text{m}^{-3}$.
Surface atmospheric pressure is $1.01 \times 10^5\,\text{Pa}$.
The molar mass of air $0.029\,\text{kg}\,\text{mol}^{-1}$.

Figure 6: credit: Wikipedia

c) Charles' and the Robert brothers' balloon flew not long after but was filled with hydrogen. The balloon had a capacity of $380\,\text{m}^3$ and was filled on the ground at a sea level pressure of $1.01 \times 10^5\,\text{Pa}$, and temperature of $288\,\text{K}$.

(i) Calculate the mass of hydrogen in the balloon.

(ii) The balloon was constructed of a single lamina of rubberised silk with a thickness $0.43\,\text{mm}$. The silk cloth has a mass per unit area of $66.5\,\text{g}\,\text{m}^{-2}$ and a density of $1.30\,\text{g}\,\text{cm}^{-3}$. Rubber has a density of $1.34\,\text{g}\,\text{cm}^{-3}$. Assuming the balloon is spherical, calculate the mass of the balloon to three significant figures.

(iii) Taking the mass of the two passengers, gondola and rigging to be $270\,\text{kg}$, calculate the initial acceleration of the balloon at sea level.

Figure 7: credit: Wikipedia

The density of air at sea level is $1.23\,\text{kg}\,\text{m}^{-3}$.

d) In an unrelated example, a balloon of mass m_1 with a small amount of ballast m_2 descends with constant velocity v due to a drag force F_d acting on the balloon. The drag force is proportional to the speed of the balloon through the air. What is the mass of ballast m_2, expressed in terms of F_d, that must be jettisoned from the balloon so that it rises with the same velocity v?

e) (i) Atmospheric pressure is due to the weight of air above.

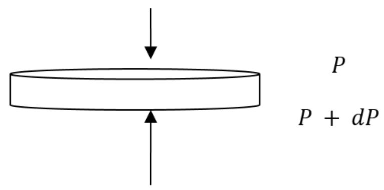


Figure 8: Thin disc of atmosphere with a pressure difference due to the weight of the disc of air.

i. From the hydrostatic equation, $p = \rho g h$ we can can trivially write down an expression for Δp. This contains both p and ρ so you need another equation to eliminate ρ.

ii. From the ideal gas equation, obtain a relation between p and ρ, R, T and M, the molar mass.

iii. Now using the hydrostatic equation, obtain an expression for $\frac{dp}{p}$ in terms of M, g, h, R and T.

iv. The temperature decreases linearly with height, in the form $T = T_0 - \alpha h$. Use this with the previous result to produce an expression for atmospheric pressure as a function of altitude $p(h)$, in terms of sea level pressure p_0, g, the molar mass of air M, the molar gas constant R, the air temperature at sea level T_0, the rate of decrease of temperature with altitude α and altitude h. Humidity should not be considered.

(ii) In a second flight, Charles ascended to $3000\,\mathrm{m}$. Calculate the ambient temperature and pressure at this altitude. Take the molar mass of air to be $0.0290\,\mathrm{kg\,mol^{-1}}$, and the molar gas constant to be $8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$. The rate of decrease of temperature with height is $0.00976\,\mathrm{K\,m^{-1}}$, the reference sea level temperature $T_0 = 288\,\mathrm{K}$ and the reference sea level pressure $p_0 = 1.01 \times 10^5\,\mathrm{Pa}$.

(iii) The balloon expands in volume by $30\,\%$ at a height of $3000\,\mathrm{m}$.

i. Calculate the pressure inside the balloon, assuming no hydrogen has been lost.

ii. Determine the stress in the skin of the balloon.

iii. Estimate the Young's Modulus of the composite skin material.

[25 marks]

Question 4

This question is about orbits and satellites.

There are some useful constants given at the end of the question

In this problem the orbits of the planets will be assumed to be circular.

The Hope probe - this space probe was launched on a journey to Mars on the 20^{th} July 2020 with a mass, m_0 of $1350\,\text{kg}$, including $800\,\text{kg}$ of fuel onboard.

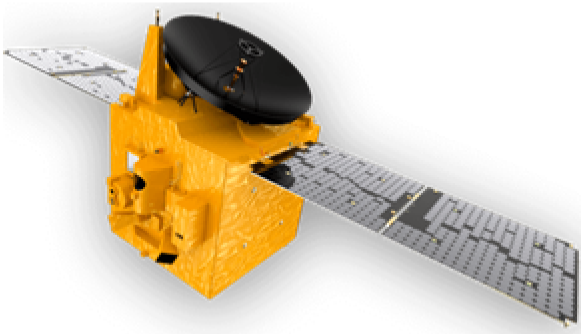


Figure 9: The Hope probe.
Credit: UAE Space Agency

a) (i) At what distance r from the centre of the Earth, with $r > R_\text{E}$, does the gravitational field strength reduce to 1% of its value at the surface? Express this as a fraction of the radius of the Earth.

(ii) The Hope probe was launched from Tanegashima Space Centre in Japan, which lies on $30°$ latitude above the equatorial plane. Calculate the tangential speed of the space centre.

(iii) Estimate the escape speed of a satellite that is launched from the surface of the Earth, with no kinetic energy contribution from Earth's rotation, so that it completely escapes the Earth's gravitational field.

(iv) What is the percentage reduction in energy for a satellite to escape the Earth's gravitational field when

i. launched from Tanegashima Space Centre in Japan,
ii. it is already in a low Earth orbit? (close to the surface of the Earth)

The trajectory of the space probe between Earth and Mars can be represented as part of an elliptical orbit about the Sun, with the Sun at one focus of the ellipse, shown schematically in Figure 10. Thus the space probe cruises without the use of propulsion in an elliptical orbit about the Sun to reach its destination.
For an elliptical orbit, $rv\sin\theta$ = constant, where r is the displacement of the body from one focus, v is the velocity vector, and θ is the angle between them.

(v) By considering the influence of the Sun only, use the idea of total energy to produce expressions for the velocity of the probe at point P (near Earth), v_P and at point A (near Mars), v_A, in terms of the gravitational constant G, the Solar mass M_S, the radius of orbit of the Earth R_{E_o}, and the radius of orbit of Mars R_{M_o}.

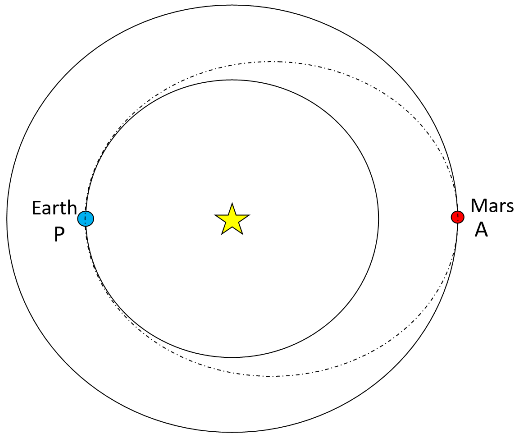


Figure 10: Trajectory of the space probe between Earth and Mars (dashed line). The Sun is represented by a star.

(vi) Determine the difference between v_{A} and the Earth's orbital velocity v_{E}, and the difference between v_{P} and Mars' orbital velocity v_{M}. Thus calculate the total change in tangential velocity needed at point A to allow the probe to cruise to its meeting point with Mars; assume the probe already has sufficient kinetic energy to escape the gravitational potential well of the Earth.

(vii) Calculate the speed of the probe on close approach to Mars.

(viii) On the approach to Mars, the speed of the probe must be slowed to match the orbital speed of Mars to allow it to be captured in an orbit around the planet. This deceleration is achieved by using six thrusters expending half of the fuel over a 27 minute period. Assume the decelerating force and the rate of fuel consumption, k, are constants. For the period of time in which the thrusters are in operation:

i. Sketch a graph of the variation of acceleration against time; append appropriate values to the axes.

ii. Produce an expression for the change in velocity with time $v(t)$ in terms of the total decelerating force F, the initial probe mass, m_0, the rate of fuel consumption k, initial velocity v_0 and time t.

iii. Calculate the force provided by each thruster.

b) The iridium satellite telephone system uses a constellation of 66 satellites orbiting at an altitude of $780\,\mathrm{km}$ above the surface of the Earth.

(i) Calculate the horizon radius of the area covered by any one satellite. (this is the radius of a flat circle within the horizon, i.e. the radius of one of the small circles illustrated in **Fig. 11**).

(ii) In practice, the effective radius is approximately $2350\,\mathrm{km}$. The power distributed by the satellite transmitter to this area is about $200\,\mathrm{W}$. Calculate the intensity of the signal at the Earth's surface.

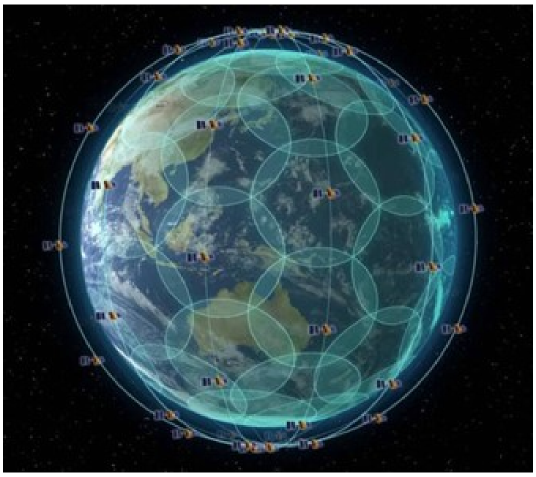


Figure 11: Satellite coverage of earth.

(iii) The movement of the satellites relative to the Earth's surface introduces the Doppler Effect. If the wavelength of the signal from the satellite is λ, the speed of the satellite v, and the speed of light c, produce an expression for the shifted wavelength λ' observed by the receiver as the satellite recedes.

(iv) Using a suitable approximation, provide an expression for the apparent shift in frequency, Δf.

(v) Now consider the satellite moving above the receiver on the Earth's surface. For the effective radius of coverage in *ii)* calculate f_{min} and f_{max} as observed, given the transmission frequency of $1620\,\mathrm{MHz}$, and thus state the bandwidth that must be set aside to manage Doppler shift.

Constant	**Symbol**	**Value**
Mass of Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of Earth	R_E	$6.37 \times 10^{6}\,\mathrm{m}$
Radius of Earth orbit	$R_{\mathrm{E_o}}$	$150 \times 10^{9}\,\mathrm{m}$
Mass of Mars	M_M	$6.42 \times 10^{23}\,\mathrm{kg}$
Radius of Mars	R_M	$6.37 \times 10^{6}\,\mathrm{m}$
Radius of Mars orbit	R_Mo	$227 \times 10^{9}\,\mathrm{m}$

[25 marks]

Question 5

This question is about circuits.

a) A constant potential difference of $2.0\,\mathrm{V}$ is applied across the ends of a metal strip with the following dimensions: length $0.30\,\mathrm{m}$, width $4.0\,\mathrm{mm}$ and thickness $10\,\mu\mathrm{m}$. The metal has a resistivity of $1.5 \times 10^{-7}\,\Omega\,\mathrm{m}$ at $0\,°\mathrm{C}$ and a temperature coefficient of resistance of $\alpha = 3.3 \times 10^{-3}\,°\mathrm{C}^{-1}$. The strip loses heat to its surroundings at a rate proportional to the surface area, and to the excess temperature above the surroundings θ, which are at $0\,°\mathrm{C}$. The constant of proportionality is $22\,\mathrm{W\,m^{-2}\,°C^{-1}}$. Determine:

(i) The resistance of the strip at $0\,°\mathrm{C}$.

(ii) The electrical power transferred when the strip is at $15\,°\mathrm{C}$.

(iii) Write down expressions for the power converted as a function of excess temperature θ, and for the rate of heat loss P as a function of excess temperature θ. Hence estimate the temperature reached after the current has been passing for a long period of time.

b) A tungsten filament lamp consists of a long, $25\,\mathrm{cm}$ filament supported at each end and in the middle. When operated using a $50\,\mathrm{Hz}$ supply there appeared to be four loops in the filament.
Estimate the tension in the filament given the density of tungsten is $19\,\mathrm{g\,cm^{-3}}$?

c) In a conductor of volume V, of uniform cross-section, made of a material of resistivity ρ, the current density is J (the current per unit area normal to the flow). Find an expression in terms of J, ρ and V for the power dissipated as thermal energy.

d) The potential difference across a filament lamp, V, is related to the current I through it by the relationship

$$V = 2I + 8I^2.$$

The lamp is connected to a measurement arm of the bridge circuit shown. Resistors R_1, R_2 and R_3 each have a resistance of $4.0\,\Omega$.
Determine the potential difference V_b such that no current flows through the galvanometer G.

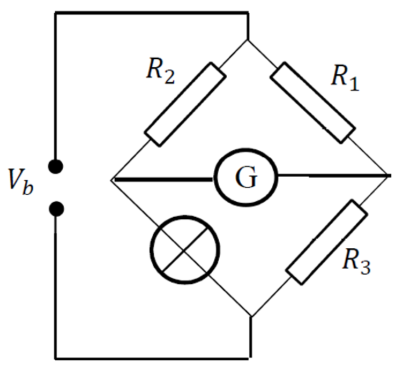


Figure 12: A bridge circuit with a non-linear component.

e) The circuit of **Fig. 13**, includes an element known as a "T-section attenuator". The attenuator is used to adjust the power delivered to the load.
The attenuator is connected to a battery of emf E and internal resistance R, and to a load also of resistance R. The values of x and y are adjusted until the battery delivers maximum power.

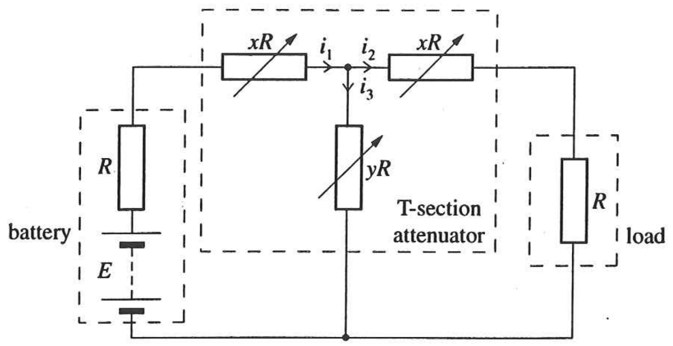


Figure 13: A T-section attenuator circuit

(i) For this maximum power condition, obtain a relation between x and y.

(ii) For the three currents i_1, i_2, i_3 shown in the circuit, obtain a general result for the ratio $\frac{i_1}{i_2}$ in terms of x and y.

(iii) P_{input} is the power delivered at the battery terminals and P_{load} is the power dissipated in the load.
Show that $\frac{P_{\text{input}}}{P_{\text{load}}} = \frac{f(x)^2}{g(x)^2}$ where $f(x)$ and $g(x)$ are simple functions of x.

Hint; in a circuit of emf E and internal resistance r connected to a load resistance R, the condition for the battery to deliver maximum power to the load is when $R = r$.

f) A metal hemisphere of radius $0.1\,\text{m}$ is immersed near the centre of a large conducting tank containing a liquid of resistivity $60\,\Omega\,\text{m}$. The plane surface of the hemisphere is level with the surface of the liquid.

(i) Find an expression for the resistance between two hemispherical shells within the liquid, concentric with the centre of the metal hemisphere, having radii r and $(r + \delta r)$ respectively.

(ii) Hence calculate the resistance between the hemisphere and the tank.

(iii) If the hemisphere is replaced by a sphere in a very large closed container of the liquid, what is the resistance between the sphere and the tank now?

(iv) We now have two small conducting spheres each of radius r separated by a distance a where $a \gg r$, embedded in a resistive material of infinite extent, as in **Figure 14**. What is the resistance between them?
Hint: use the idea of superposition of currents in the material.

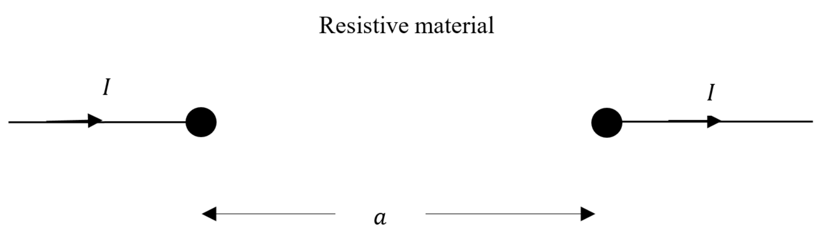


Figure 14: Two small spherical conductors embedded in a resistive material.

[25 marks]

END OF SECTION 2