British Physics Olympiad

Paper 3. 2005

Monday 28 February 2005.

Time allowed 3hrs plus 15 minutes reading time.

All questions should be attempted. Question 1 carries 40 marks, the other questions 20 marks each.

Speed of light in free space	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Speed of sound in air at STP	c_s	3.30×10^{2}	$\mathrm{m\,s^{-1}}$
Elementary charge	e	1.60×10^{-19}	C
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Mass of a proton (rest mass)	m_p	1.672623×10^{-27}	kg
Mass of a neutron (rest mass)	m_n	1.674928×10^{-27}	kg
Mass of a electron (rest mass)	m_e	9.109389×10^{-31}	kg
Mass of a deuterium nucleus (rest mass)	m_D	3.368833×10^{-27}	kg
Radius of the Earth (mean)	R_{E}	6.37×10^{6}	m

Mathematics:

$$\int \frac{1}{x^2}dx = -\frac{1}{x} + c\,.$$

$$\int \frac{1}{x}dx = \ln x + c.$$

$$(1+x)^n \approx 1 + nx \;\text{ if }\; x << 1$$

Q1

In this question you are asked to make reasoned estimates and assumptions. These must be clearly stated.

a)

(i) Large print heading in the National Geographic Magazine reads: "Olivine is sharper than steel". Comment and explain. Note olivine is a very hard glass-like volcanic rock.

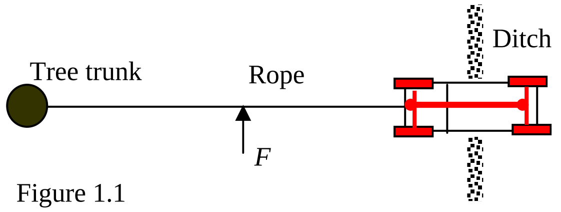


Figure 1.1

(ii) A motorist gets his car stuck in a ditch Figure 1.1. He is advised to attach a rope to a tree trunk, tighten it, and push the rope with a force *F* at right angles to the rope. When the car has moved, he repeats the above procedure. Explain why this procedure is more effective than simply pulling the rope without bothering to use the tree trunk

(iii) A razor, whether electric or manual, needs to be sharp in order to be comfortable when used. Why is this?

(iv) A stationary mass of 0.5 kg is hit by a 2 kg hammer that is moving at a speed of 1.3 ms^{-1}. If the collision is perfectly elastic calculate the subsequent speed of the mass. Since the hardness of the hammer does not enter into the calculation one might as well use a rubber hammer to drive in a nail. Comment.

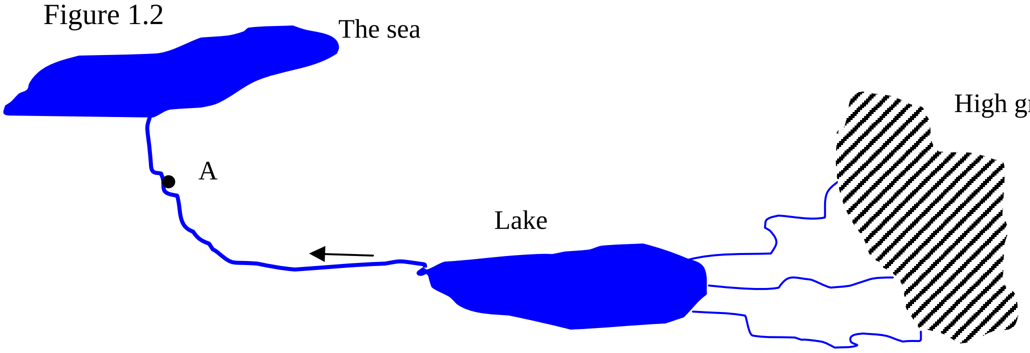
b) Table 1 shows the timetable of a passenger boat on the river Rhine.

↓ Time	Town	↑ Time	Town	Position along the river/km
1700	St Goarhausen	1851	St Goarhausen	642
1830	Bacharach	1801	Bacharach	659

Table 1.

Obtain a numerical estimate for the speed of the Rhine current and the speed of the boats in still water. N.B the position marks follow the river Rhine indicating the distance, *along its course,* from a zero mark.

c) Figure 1.2



It is desired to make an analogue electrical circuit that will replicate the behaviour of the mountain, lake, river system, Figure 1.2. Draw an electrical circuit, analogue, and explain how the circuit may be used, to indicate the rate of rise of river level at A due to sudden rainfall on the high ground.

d) The distance from the centre of the Earth to the poles is 21 km shorter than the radius of the equator. A one second pendulum is taken from the equator (at sea level) to the North Pole. Make a rough estimate of its change in period. You may assume for the purposes of your calculation that the Earth has a constant density.

e) When a perfect monatomic gas expands adiabatically (no heat interchange with its surroundings) the pressure and volume of the gas are related by the formula:

$$P_1V_1^{\gamma} = P_2V_2^{\gamma},$$

where γ in this case is 5/3 and P_1, V_1, P_2, V_2 refer to the initial and final states of the volume and pressure of the gas . A helium balloon at atmospheric pressure is allowed to rise rapidly to a height such that the external pressure is half atmospheric pressure. The envelope of the balloon is at all times loose, of negligible mass, and regarded as a perfect heat insulator.

(i) Calculate the ratio of the initial to final volume of the balloon.

(ii) If the initial temperature of the gas was 300 K, what will be its new temperature?

f) During the 1998 International Olympiad in Iceland competitors and leaders were surprised to see a cone of snow in the sea on the North West horizon. This was the mountain of Snaefell that is about 2000 m high. However only the top 800 m of the mountain could be seen. The mountain is about 120 km from Rekjavik. Use these values to estimate the radius of the Earth.

g) (i) How do tsunamis arise?

(ii) What is the order of magnitude of the wavelength, speed and amplitude of the wave in the deep ocean?

(iii) How do these change when they approach land?

Q2

Figure 2.1

Figure 2.2

In the 2004 International Physics Olympiad in Korea a demonstration showed how a ping pong ball (table tennis ball) could make a hole in three ceramic plates, Figures 2.1 and 2.2. The ceramic plates were placed between two glass tubes. The tubes were then evacuated so that there was a vacuum on each side of the plates. Suddenly the tube was opened on the side of a ball adjacent to the end of the tube, Figure 2.1. The atmospheric pressure on that side shot the ball into the plates making a hole in the plates. The ball made a virtually air tight fit with the glass tubes with little friction.

a) i) Write down the maximum speed with which the ball could hit the plates.
ii) The ball had a mass of 0.010 kg. Calculate its maximum KE.

b) Consider a cubic box, side l, volume V, containing n molecules of mass m all moving in *random* directions at constant speed v. All collisions are to be assumed to be perfectly elastic and the total volume of the molecules is very much less than V.

(i) What is the change in momentum when a molecule, moving in an arbitrary direction, hits a face of the box with speed u normal to the face of the box?
(ii) Calculate the average time it takes this molecule to traverse the box.
(iii) Hence, or otherwise, show that the pressure in the box, p, is related to v, m, n, and V by

$$p = \frac{1}{3V} nmv^2 ,$$

when the molecules are moving in *random* directions with speed v.

c)

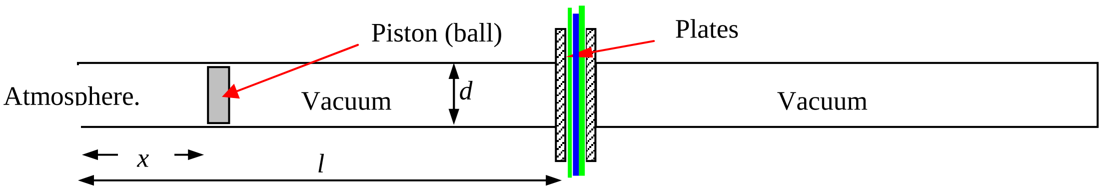


Figure 2.3

Figure 2.3 shows a simplified diagram of the experiment where the ping pong ball, diameter d, has been replaced by a gas tight piston of mass M diameter d. The distance from the end of the tube, length l is x and its speed is u_x at time t.

(i) Assuming that v , the speed of the molecules, remains the same, derive a formula that relates the pressure on the piston to n, m, v, l and d when the piston is moving at speed u_x.
(ii) Find an expression for the acceleration of the piston when it is at a distance x from the end of the tube. Assume that $u_x << c_s$, where c_s is the speed of sound of air at the local temperature.

Q3

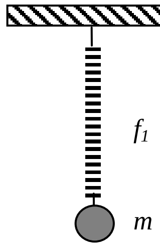


Figure 3.1

a) Figure 3.1 shows a mass *m* hanging from a spring natural length *l*, stiffness *k*, and of negligible mass.

(i) Derive, from first principles, a formula that gives the frequency of longitudinal oscillations, f_1, in terms of the relevant given parameters.

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b)

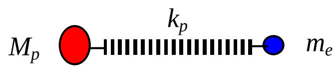


Figure 3.2

Figure 3.2. illustrates a very simple model of a hydrogen atom. The masses in this case are M_p and m_e. The spring stiffness is k_p. Ignore the mass of the springs.

(i) Find the position, relative to the proton, of the point on the spring that does not move when the masses are in motion; the unstreched length of the spring being *l*.

(ii) Find a formula, in terms of the given parameters, that gives the fundamental frequency of the hydrogen atom, f_p.

(iii) Assuming the stiffness, k_p, of the hydrogen atom is the same as that, k_D, of the deuterium atom, determine the value of the ratio $\frac{f_p}{f_D}$, where f_D is the frequency of the deuterium atom.

c) In the earliest circular planetary model of the atom the electron and proton orbited a common centre. The electrostatic forces alone provided the force field. However an accelerating charged body will send out electromagnetic waves and the orbiting charges would consequently lose energy continuously. The model did not, as it stood, predict the existence of discrete energy levels that were known to be a consequence of the discrete system of spectral lines.

In the wave particle model the de Broglie wavelength, λ, is related to the momentum *p* ($p = mv$) by $\lambda = \frac{h}{p}$.
Assume $v < c$ the speed of light in free space.

(i) Use this to find a relationship between the KE of a body and its wavelength.

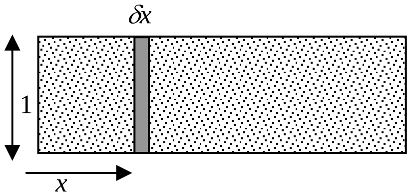
(ii) How, using the de Broglie result, can a discrete energy level model be constructed?

(iii) A neutron has a KE of 1.0 eV. Calculate its de Broglie wavelength.

(iv) Neutron beams are useful for investigating the nature of certain crystals. Why are neutrons used?

Q4

Figure 4.1



a) Figure 4.1 shows a glowing gas in a rectangular glass box. The square cross section, perpendicular to the plane of the diagram, of the ends of the box is $1m^2$. The intensity of the light emitted in the *x* direction due to the glowing atoms in a volume (Figure 4.1) of thickness δx is $k\delta x$, where *k* is a constant. The absorption of the light due to this same volume is $\sigma I_x \delta x$, where σ is a constant and I_x is the incident intensity, in the direction of increasing *x*, normally on the element of thickness δx. Show that this leads to the differential equation

$$\frac{dI_x}{dx} = -\sigma I_x + k.$$

(i) Use this equation to find an explicit relation between *x* and I_x. (*Mathematical integrals are listed on page1. The units of intensity are* Wm^{-2}*).*

(ii) Find an expression for the intensity I_m when *x* is very large and an expression for *x* in terms of $I_{0.5}$ where $I_{0.5} = 0.5 I_m$.

b)

Figure 4.2

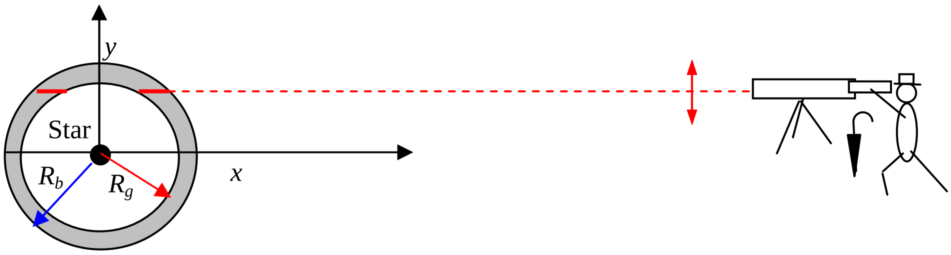


Figure 4.2 shows an observer looking at a luminous cloud around a central star, in the form of a spherical shell, between the radii R_b and R_g. His distance from the star is very much greater than the radius of the cloud. The thickness of the cloud is much less than the radius of the cloud. In the position shown on the diagram the thickness of the cloud in the line of sight is the sum of the lengths of the two full red lines shown in Figure 4.2. The star forms the origin of a Cartesian coordinate system with the *x*-axis parallel to the line of sight and the y-axis at right angles to it.

(i) Find expression/expressions that give the total length of this path through the cloud in terms of *y* and the other relevant parameters.

(ii) Hence find an expression that gives the variation of the intensity of the light, I_x, with *y*.

(iii) Sketch a graph of I_x against *y*.

c)

Figure 4.3

Figure 4.3 shows a picture taken by the Hubble Telescope. A star is seen in the centre of the picture.

(i) Describe qualitatively the main features of Figure 4.3.
(ii) Suggest, with reference to part b, how these features may be explained.