British Physics Olympiad 2008

Paper 3

Monday 25th February 2008.

Time allowed 3hrs plus 15 minutes reading time. Graph paper is needed for this examination.

All questions should be attempted. The maximum mark for question 1 is 40 marks , questions 2,3 and 4 are marked out of 20.

Speed of light in free space	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Elementary charge	e	1.60×10^{-19}	C
Mass neutron/proton (to 3 sig figs)	m_p, m_n	1.67×10^{-27}	kg
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Radius of the Sun	R_{Sun}	6.96×10^{8}	m
Mass of the Sun	M_{Sun}	1.99×10^{30}	kg
Radius of Jupiter	R_{Jup}	7.10×10^{7}	m
Mass of Jupiter	M_{Jup}	1.90×10^{27}	kg
Radius of Earth's orbit around the Sun	R_{OE}	1.50×10^{11}	m
Radius of Jupiter's orbit around the Sun	R_{OJ}	7.78×10^{11}	m
Period of Jupiter orbiting the Sun	T_J	3.58×10^{8}	s
Period of the Earth orbiting the Sun	T_{Earth}	3.00×10^{7}	s
Radius of the Earth	R_{Earth}	6.37×10^{6}	m
Surface tension of water	τ	7.28×10^{-2}	$\mathrm{Nm^{-1}}$
Gas Constant	R	8.3	$\mathrm{Jmol^{-1}K^{-1}}$
Permittivity of free space	ε_o	8.86×10^{-12}	$\mathrm{Fm^{-1}}$
Boltzmann Constant	k	1.38×10^{-23}	$\mathrm{JK^{-1}}$

Useful formulae:

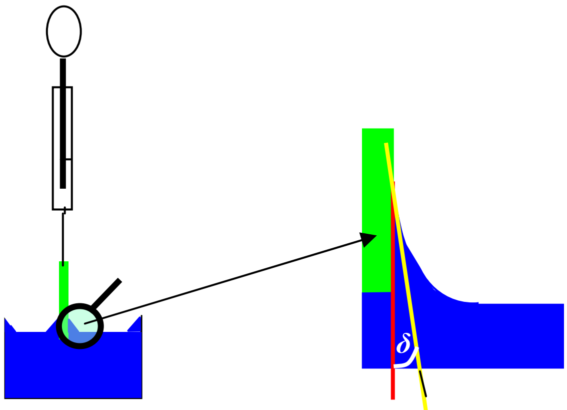
$PV=NRT$; for a perfect gas. P is the pressure, V is the volume, N number of mols and R is the gas constant. $p = \frac{1}{3} nm < c^2 >$ p is the pressure, n the number of molecules per unit volume, m the mass of a molecule and c is the speed of a molecule – not the velocity of light.

Q1

1a *This question is about surface tension. You are unlikely to have studied it in depth, if at all; however sufficient help is given to enable you to solve the problems.*

(i)

Figure 1a1



The surface tension τ is the force per unit length in the surface of a liquid when the liquid is in contact with a solid perpendicular to the line of contact of the surface with the solid. An example is given in Figure 1a1 where a clean glass microscope slide is hung on a balance and lowered into clean water such that the water just touches the base of the slide. The water wets the glass and the surface of the water adjusts so that it climbs and makes a very small angle (δ) with the surface of the glass. This angle is named the angle of contact and is practically zero for clean water and clean glass. There is tension between the molecules of water and it is that tension that pulls down the glass slide. The length of the slide "into the paper" is l giving the total perimeter approximately $2l$. Show that

$$\tau = \frac{W_{st} - W}{2l Cos\delta},$$

where W_{st} is the weight of the slide after immersion and W is the weight of the slide before immersion.

Figure1a2

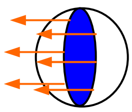


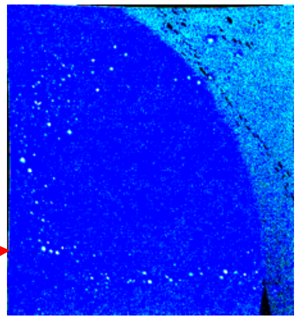
Figure1a2 shows a *small* water droplet, radius R. The dark blue is the circular water surface if the droplet is cut in half. The perimeter of the blue circle is the line along which the tension acts, illustrated by the orange arrows. By considering the stability of one hemisphere, or otherwise, show that:

$$p = \frac{2\tau}{R} \qquad \boxed{\text{Equation 1a2}}$$

where p is the excess pressure in the droplet above the surroundings.

1b

Figure 1b1



Bubble formation caused by sound waves.

The phenomenon of cavitation is caused by low pressure. This a very difficult problem affecting a ship's propellers, pumps etc. Parts of the liquid reach a lower pressure causing the liquid to superheat locally and cause bubbles to form. These grow and then may suffer catastrophic collapse. The temperature in a collapsing bubble may reach values greater than 10^4 K. Some have speculated that this might be used to initiate fusion. Sound waves may initiate the formation of bubbles and light may be emitted during their collapse, Figure 1b1. Equation 1a2 shows that the pressure in a very small bubble will be high so that a higher water temperature than normal boiling point may be needed to initiate the bubble formation. A charged bubble will reduce the temperature needed to start the bubble. This is used to show the tracks of charged particles through hydrogen liquid bubble chambers.

Figure 1b2

Figure 1b3

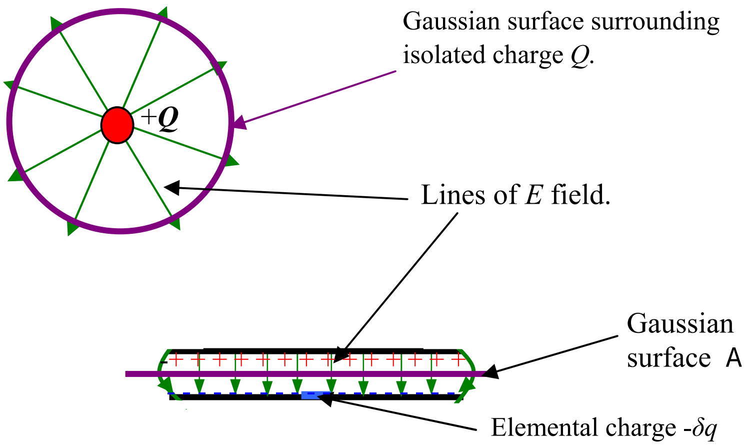


Figure 1b3 shows a cross section of two large metal plates with a charge of $+q$ and $-q$. The area of a plate is A. Ignoring edge effects show that the force of attraction between the plates is given by:

$$F = \frac{q^2}{2\varepsilon_o A}.$$

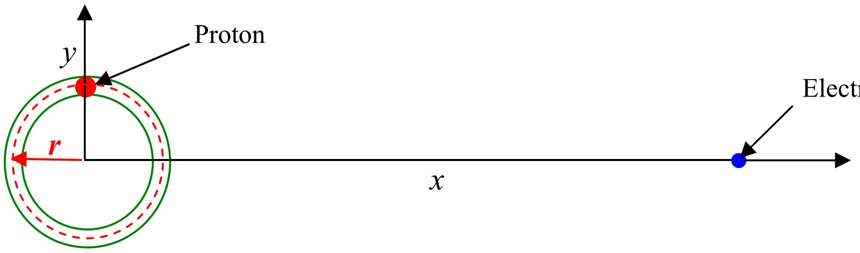
(**Hint**. *The theorem of Gauss states that if a surface A, is found such that the lines of electric field pierce that surface at right angles to the surface then for a simple symmetrical case,*

$$E\, A\, \varepsilon_o = q,$$

where E is the electric field, ε_o is a constant, q is the charge enclosed by the surface and A is the surface area of the surface that surrounds the charge. ***Any surface that encloses a charge in this way is called a Gaussian surface****. Figure 1b2 shows a spherical Gaussian surface around a small isolated charge Q . The purple line in Figure 1b3 shows the Gaussian surface enclosing the charge q. Consider the force on an elemental charge -δq due to the E field it is in.***)**

Hence or otherwise find the outward pressure on an isolated spherical water drop with a charge q, radius r. Find the outward pressure due to the electrostatic effect on a bubble in water with a radius of 20 µm whose charge is caused by 10^9 excess electrons. How does this compare with normal atmospheric pressure?

Figure 1c1



A proton is constrained to move in a circle of radius r at an angular velocity of ω in a non conducting ring, in the x,y plane Figure 1c1. Note that $r\omega << c$ the velocity of light Consider an electron a large distance x away from the centre of the ring, find an expression for the force on the stationary electron. Is the force the same if the electron moves? Explain your answer.
Why will a charged particle moving in a circular track in a very good vacuum lose energy?

Q2

Figure 2.1a

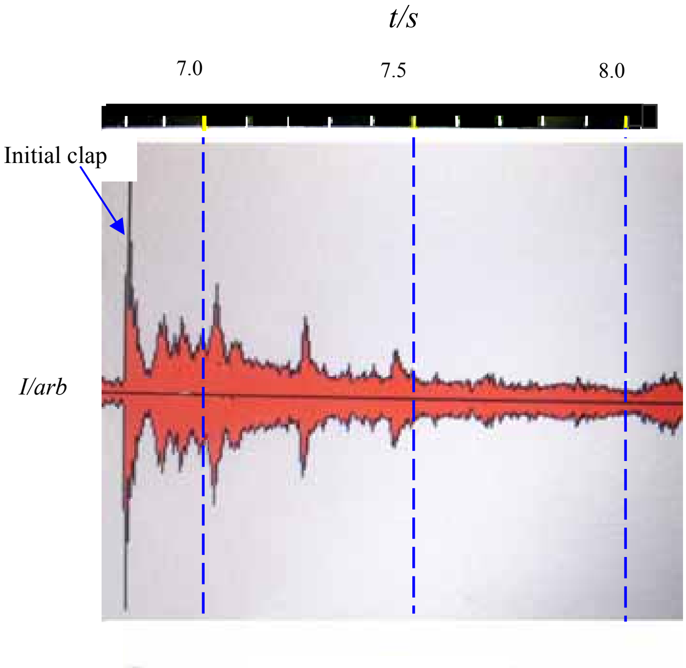
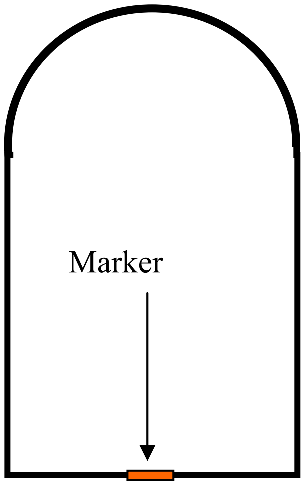


Figure 2.1b



a) Figure 2.1b shows a simplified cross section of the Jamie mosque in Isfahan in Iran (venue of the International Olympiad in 2007). On the floor of the mosque there is a metal marker. If one claps one's hands near the marker it is suggested that multiple echoes should be heard. A digital camera was placed on the marker. Hands were clapped and the sounds recorded by a digital camera in "movie mode". Figure 2.1a shows a recording of the intensity of the sound. A time scale is shown in Figure 2.1a. There was a discussion amongst the physics leaders as to whether the marker is at the centre of the circular dome (if it is circular in cross section) or at the focus of the dome. **N.B** the sketch in Figure 2.1b is _not_ to scale or correct proportion.

(i) Is the marker at the focus or the centre of curvature of the dome you can assume is spherical? How is it possible to decide this from the given data?
{ii) Use the information given to determine the height of the centre of the interior of the dome. Carefully estimate the errors.
(iii) The intensity of sound, *I*, is given by

$$I = I_o Sin(\omega t)\,,$$

ω is the angular frequency, I_o is the max intensity, a constant, and t is the time.

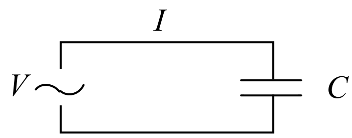
It is suggested that the max intensity of the sound at the first reflection (I_1) is related to that of the second reflection (I_2) by an equation mathematically similar to that of radioactive decay measured at equal time intervals i.e.

$$\frac{I_o}{I_1} = \frac{I_1}{I_2}.$$

By analysing the diagram Figure 2.1a find evidence to support, or reject, this assertion. Explain why your conclusion should be expected on theoretical grounds.

Q3

Figure 3.1



a) The charge on a capacitor, Q, of capacity, C, is related to its pd, V, by the formula $Q = CV$.
In the circuit Figure 3.1 at time t

$$V = V_o \sin \omega t,$$

V_o is the maximum value of V and ω is the angular frequency of the applied ac. Show that the current in the wires is given by

$$I = I_o \cos \omega t.$$

Find an expression for V_o in terms of I_o and ω.

b) The magnitude of the impedance of the capacitor is defined as $\frac{V_o}{I_o}$.

Figure 3.3

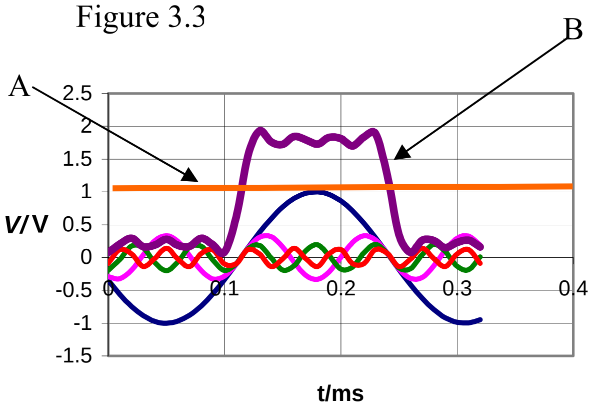


Figure 3.2

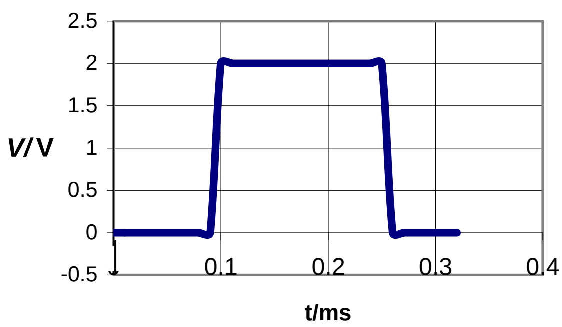


Figure 3.2 shows a periodic square pulse of 2 V with a period of 0.30 ms. This can be approximated by the addition of four sine waves and a dc of 1 V, see A in Figure 3.2. Line B, Figure 3.3, is the plot of

$$V = 1 + (Sin 2\pi f_o t) - \frac{1}{3} Sin(2\pi 6 f_o t) + \frac{1}{5} Sin(2\pi 10 f_o t) - \frac{1}{5} Sin(2\pi 14 f_o t).$$

Where f_o is the lowest frequency in Figure 3.3.

Suggest a circuit, to insert into Figure 3.4, giving suggested component value(s) such that the current in the load would be approximately a sine wave. The load has a resistance of 300 Ω.

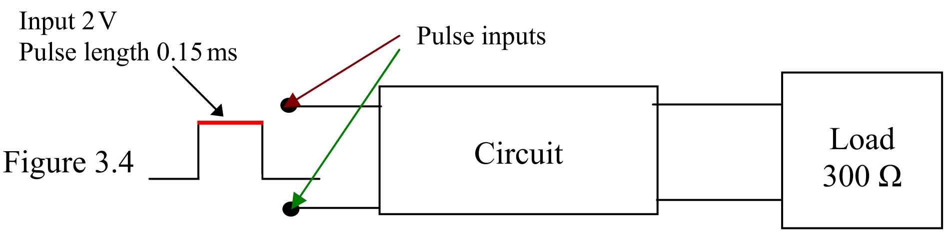


Figure 3.4

Q4

(a) Light from a sodium gas discharge lamp is incident on a gas flame into which common salt crystals are thrown. On a screen behind the flame a dark shadow of the flame is seen. Explain this observation.

(b) In recent years many planets have been spotted orbiting stars. The first ones were spotted by the movement of a star round the common centre of mass of the star-planet system.

(i) Show that when a light source, wavelength λ , is moving away from an observer at a speed v the displacement of the line, $\delta\lambda$, due to orbital motion, will be given by

$$\frac{\delta\lambda}{\lambda} \approx \frac{v}{c}$$

v<<c.

(ii) Find the maximum speed at which the Sun appears to approach the centre of mass of the solar system as viewed from a planet on a distant star. Ignore the effect of all the planets except Jupiter. Then the equation above to determine $\delta\lambda$ for a sodium absorption line of wavelength λ = 569.0 nm.

(iii) Assume the Sun's atmosphere is at a temperature of 6400 K and it is a perfect gas. Find the rms speed of an atom of Fe_{26}^{56} and an atom of H_1^1 in the Sun's atmosphere. How do these speeds compare with the speeds found in (ii)? Comment.

c) Another very recent method of detecting planets round other stars is the reduction in light due to the planet passing across the star. Find the fractional reduction in the intensity of light due to the passage of Jupiter across the face of the Sun as seen by a very distant observer at a distance of the order of about 10^{17} m. Assume that Jupiter is totally opaque.

d)

(i) It is of interest to know whether water is present on these remote planets. The water vapour in the atmosphere of the earth, and possibly the remote planet will absorb certain wavelengths. However in practice we can distinguish between these two effects. How?

(ii) Assume that the atmosphere of the Earth is 20 km thick and is sufficiently optically thick to remove all the light from the relevant water vapour spectral lines. Imagine that a very distant observer records the intensity reduction in these water vapour lines due to the passage of the earth across the face of the Sun. Calculate the relative reduction in the intensity of a spectral line due to transit of the *atmosphere* of the Earth across the line of sight to a distant observer of the Sun who is on a planet orbiting a distant star.