British Physics Olympiad 2007

Paper 3

Monday 26th February 2007.

Time allowed 3hrs plus 15 minutes reading time.

All questions should be attempted. The maximum mark for question 1 is 40 marks questions 2,3 and 4 are marked out of 20.

Speed of light in free space	c	3.00×10^{8}	$\mathrm{m\,s^{-1}}$
Elementary charge	e	1.60×10^{-19}	C
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Planck's constant	h	6.62×10^{-34}	Js
Stefan-Boltzmann constant	σ	5.67×10^{-8}	$\mathrm{Wm^{-2}K^{-4}}$
Radius of the Sun	R_{Sun}	6.96×10^{8}	m
Radius of Earth orbit around the Sun	R_o	1.50×10^{11}	m
Radius of the Earth	R_{Earth}	6.37×10^{6}	m
Boltzmann constant	k	1.38×10^{-23}	$\mathrm{J\,K^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}$	$\mathrm{TmA^{-1}}$

Formulae:
These may be helpful.

$$\int \frac{1}{x}dx = \ln x + C$$

$$\sin^2 x + \cos^2 x = 1$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Q1

a)

Figure1.1

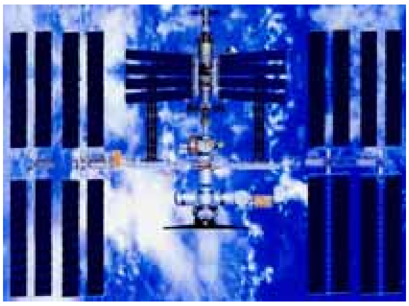


Figure 1.2

Figure 1.1 shows a picture of the International Space Station (ISS). Figure 1.2 shows a photograph taken by an amateur photographer in Normandy of the ISS silhouetted against the Sun. The ISS measures about 200 m by 150 m.

(i) Use the data given about the Sun and the photograph to find a value for the height of the ISS above the surface of the Earth.
(ii) Comment on the accuracy of the answer.
(iii) Doubt has been cast on the validity of the photograph. Why?
(iv) The surface of the Sun appears to get darker towards the edge. Why?

b)

Fig 1.2

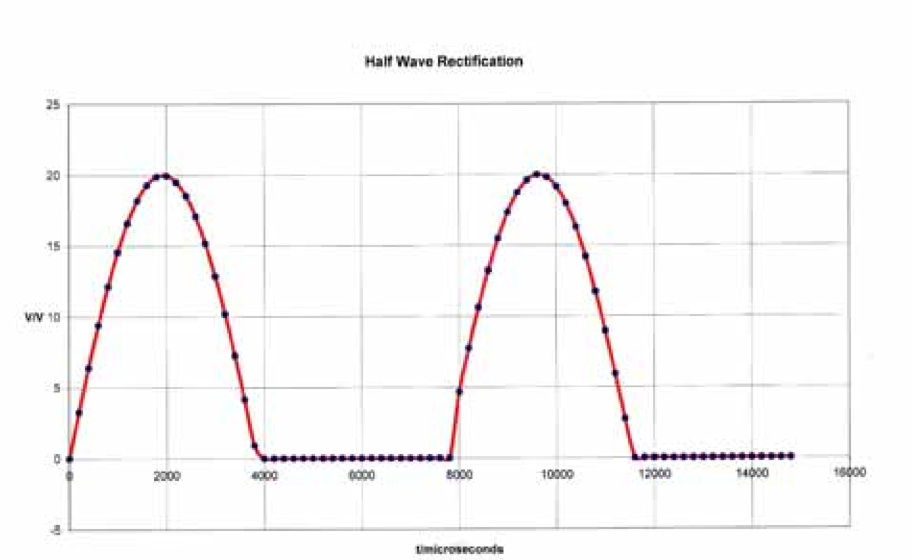
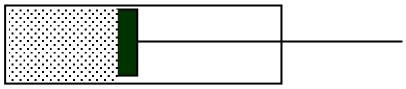


Figure 1.2 shows the pd/time output of a half wave electrical source. Determine:

(i) The average P.D.
(ii) The RMS P.D.
(iii) The heat dissipated in a 200 Ω resistor in 100s.

c)



(i) Fig 1.3 Shows a gas (considered a perfect gas) in an enclosed cylinder. The thermal capacity of the solid components can be considered small and the cylinder and piston are perfect thermal insulators. The piston is pushed in *slowly* and then *slowly* allowed to return to its original position. Sketch a graph of the temperature of the gas against the displacement of the piston from its initial position.

(ii) It is important to have the correct pressure of the air in a tyre on a car. What is the problem if the air in the tyre is at too high a pressure? What is the problem if the air is at too low a pressure?

(iii) A sealed insulated cylinder of air full of a perfect gas is allowed to fall freely under gravity (with the long axis vertical). What happens to (a) the temperature of the air (b) the pressure of the air in the cylinder?

d) Furniture removal companies often charge by volume not weight. To reduce the volume blankets are placed in special sealed flexible bags. A vacuum cleaner is attached to the bag and reduces the pressure of the air inside the bag. What happens to the weight of the bag and contents?

e) There is a small danger of a small asteroid, discovered in 2004, hitting the earth in 2029 or 2036. One of the many proposals to avoid this is to paint the asteroid white hence increasing its albedo from 30% to 80%. This effect is roughly constant over all significant wavelengths. The asteroid has a radius of 160 m and its density is roughly 3000 kgm^{-3}. Calculate its change in velocity due to this paint in the course of one day's illumination from the sun.

The momentum of a photon is hf/c, the energy of a photon is hf.
Albedo is the percentage of photons reflected off an illuminated surface.
The effective temperature of the surface of the Sun is 6400 K.
Assume that the asteroid when the painting occurs is $2R_o$, *from the Sun.*
R_o *is the radius of the orbit of the Earth round the Sun – assumed circular.*

f)

Figure 1.4

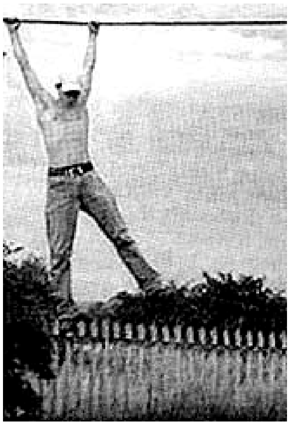


Figure 1.5

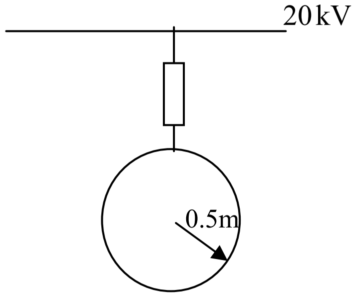


Figure 1.4 shows a photograph of a man hanging on a 20 kV, 50 Hz power cable that supplies an electric train. A newspaper journalist wrote "he survived in spite of 20 kV going through him". Comment. A very simple model of the situation shows a sphere hanging from a wire with a resistance in series. Estimate the maximum current charging the sphere that can be regarded as remote from the ground. Does this model suggest that he will survive? Explain using numerical data where possible.

Data and formula.

(i) Electrical capacity C of an isolated sphere is given by $C = 4\pi\varepsilon_o r$ where r is the radius and $\varepsilon_o = 8.9 \times 10^{-12}\,\mathrm{Fm^{-1}}$.

(ii) The current needed to electrocute a human being is probably a few mA. The maximum "safe" p.d. in dry conditions is considered to be 80 V.

Q2

Figure 2.1

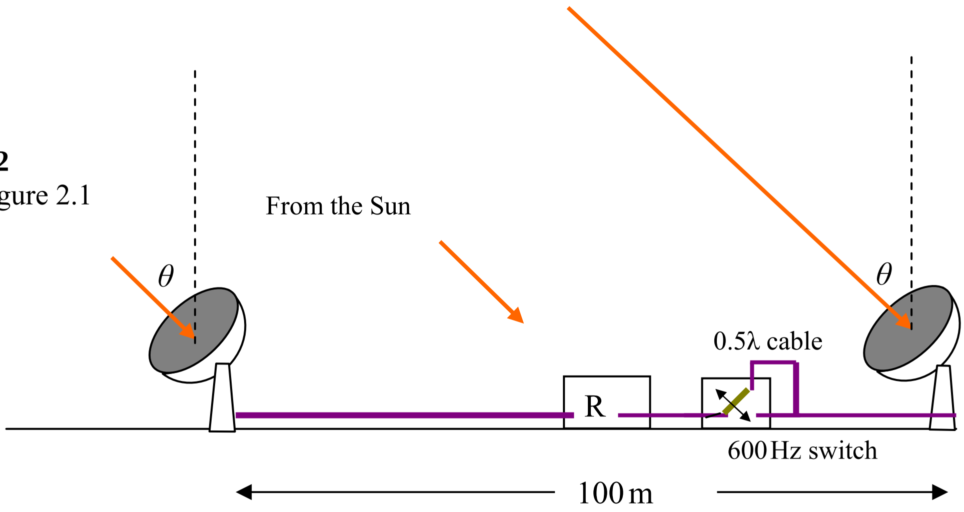


Figure 2.1 illustrates two radio telescope aerials on whose dishes fall part of a radio beam from the quiet Sun. The signal from the dishes are added together in the receiver. The beam makes an angle of θ degrees with the local vertical. The line joining the telescopes is on an E-W line on the equator. The receiver, R, accepts waves of a frequency of 150 MHz ± 1.00 MHz. Consider the date to be March 22nd.

(i) Normal broadcast receivers have a bandwidth of ± 25 kHz. Why has the radio telescope receiver a much wider bandwidth?

(ii) Initially the dishes are simply connected to the receiver by equal length cables and the receiver sums the two signals. Show that the signal amplitude has a very slow component that varies given by,

$$I_\theta = I_o \cos \omega t \,,$$

at time t, around midday. Find the value of ω.

(iii) Sketch a graph showing how the maximum intensity of the signal varies with time between 1000 hrs and 1400 hrs on the 21st of September.

(iv) Sketch on the same graph how the maximum intensity of the signal varies with time between 1000 hrs and 1400 hrs on the 21st of June.

(v) In a modified arrangement a length of cable equivalent to half a wavelength is inserted and removed at a frequency of 600 Hz. After the receiver there is a further amplifier that is so tuned that it amplifies the 600 Hz signal. How will the amplitude of the 600Hz signal vary with θ?

(vi) Why is the modified scheme better?

Note

Bandwidth is loosely defined as the difference between the highest frequency received by a receiver and the lowest frequency received when it is set to receive a certain frequency f_0. *If* $2\Delta f$ *is the bandwidth then the highest frequency received is* $f_0 + \Delta f$.

Q3

Hint: *In this question you may find it a help to consider a small length of rope length δl and the forces on this element of rope.*

a)
Figure 3.1

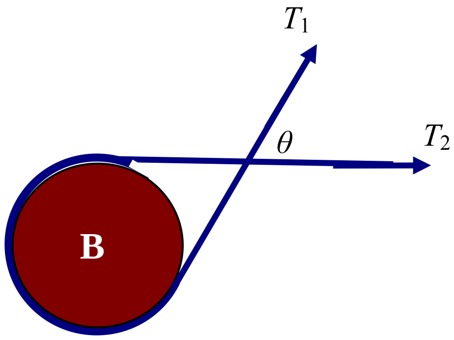


Figure 3.1 shows a bollard B with a rope wrapped round it. T_1 is the tension caused by the rope being attached to a boat. T_2 is the tension due this rope being held by a sailor on the quayside. The angle between the ropes is θ. and the rope is just slipping pulling the sailor along.
Find an expression relating θ, T_2 , T_1 and the coefficient of friction, μ ,between the rope and the bollard.

b)
Figure 3.2

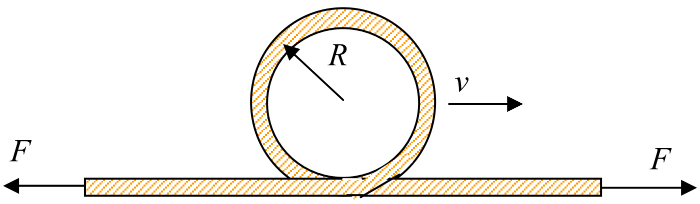
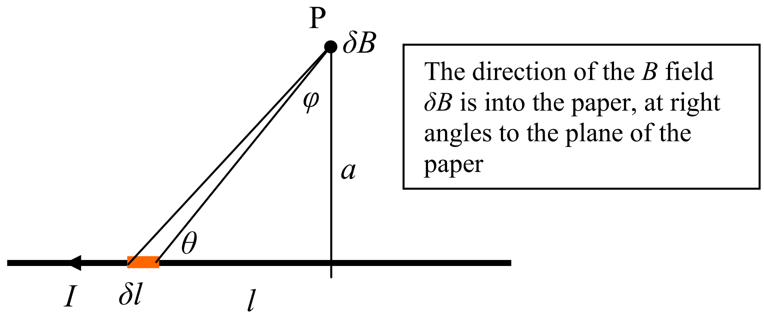


Figure 3.2 shows a long, heavy, flexible rope with mass per unit length ρ that is kept tight by a force F. A sudden movement causes a circular loop radius R to form. This loop moves along the rope in a horizontal plane at a transverse velocity of v.

(i) Calculate the speed v of the loop in terms of the given variables.
(ii) Determine the energy, momentum and angular momentum of the loop.

Q4

Figure 4.1



a) The Biot Savart law relates the magnetic field δB at P to the current I in an element of wire length δl Figure 4.1.

$$\delta B = \left(\frac{\mu_0}{4\pi}\right)\frac{I Sin\theta\delta l}{r^2}$$

where μ_0 is the permeability of free space.

b) Show that for an infinitely long wire this becomes

$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{2I}{a}.$$

c)

Figure 4.2

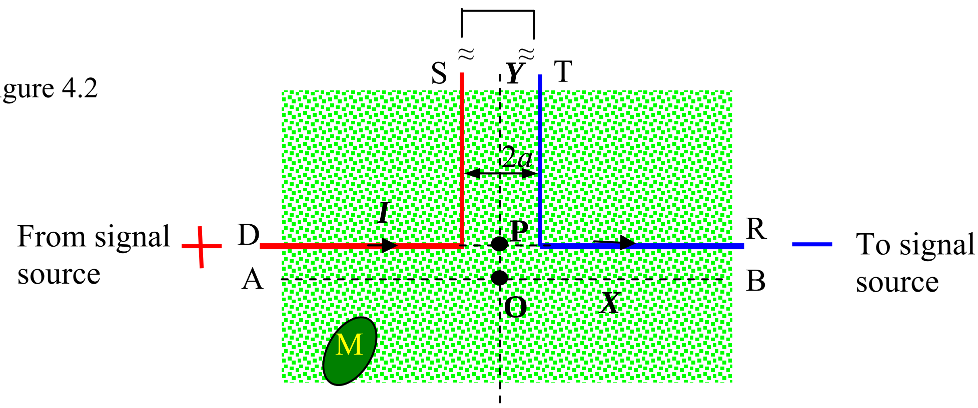


Figure 2 shows the plan view of part of a grass lawn. The mower M is controlled by the magnetic field from the wires, DS and TR, as shown in Figure 4.2. Only a small part of the lawn is shown. The wires are connected in series. The parts of the wires that are not shown in Figure 4.2 are very long. The distance PO is a. The blue and red wires are $2a$ apart where they run parallel.

(i) Use the result in (b) or otherwise to find the value of the magnetic field, B_P, at P.
(ii) Find an expression for the magnetic field for the magnetic field B_O, at O.
(iii) Sketch the value of the magnetic field against distance along the Y axis.

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