BRITISH PHYSICS OLYMPIAD

## 2010 Paper 3

Monday 1st March 2010

Time allowed 3hrs plus 15 minutes reading time. All questions should be attempted.
Questions 1 and 2 both carry 50 marks.
This is a challenging paper.   Do not be discouraged if you cannot attempt all sections.

Speed of light in free space	c	3.00×10^8	$\mathrm{m\,s^{-1}}$
Gravitational constant	G	6.67×10^{-11}	$\mathrm{N\,m^2\,kg^{-2}}$
Acceleration of free fall at Earth's surface	g	9.81	$\mathrm{m\,s^{-2}}$
Earth-Moon distance	R_{EM}	3.82×10^8	m
Radius of the Earth	R_E	6.37×10^6	m
Mass of the Earth	M_{Earth}	5.97×10^{24}	kg
Mass of the Sun	M_{Sun}	1.99×10^{30}	kg
Radius of the Sun	R_{Sun}	6.96×10^8	m
Density of lead	ρ_{pb}	11.0×10^3	$\mathrm{kg\,m^{-3}}$
Density of wood	ρ_{wd}	0.60×10^3	$\mathrm{kg\,m^{-3}}$
Radius of the Earth's orbit around the Sun	R_{ES}	1.50×10^{11}	m
Permeability of free space	μ_o	$4\pi \times 10^{-7}$	$\mathrm{H\,m^{-1}}$
Permittivity of free space	ε_o	8.85×10^{-12}	$\mathrm{F\,m^{-1}}$

Formula :   Moment of inertia of a rectangular body mass M about its centre of mass:
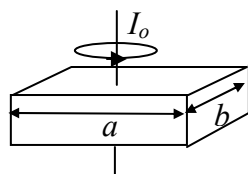
$$I_0 = M\left(\frac{a^2 + b^2}{12}\right)$$

Parallel axis theorem: For a body mass M: $I_d = I_0 + Md^2$. I_o is the Moment of Inertia of the body about its centre of mass. I_d is the M of I of the mass about a parallel axis a distance d from the axis through the centre of mass.

For an EM wave in a vacuum: $\dfrac{1}{\sqrt{\varepsilon_o \mu_o}} = c$.

Q1.a

Figure 1.a.i

Ball

'Net'

100mm

1600mm

Ball
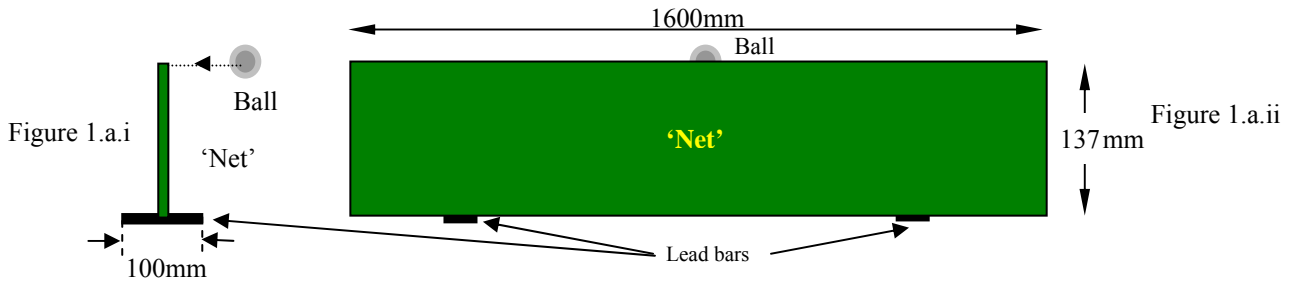
'Net'

137 mm

Figure 1.a.ii

Lead bars

Figure 1.a.i shows the side on view and Figure.2.a.ii. the end on view of a new type of table tennis net. It consists of a thin piece of wood supported by two lead bars 100 mm long. The thickness of the 'net' is 5.0 mm, the width of the bars is 50.0 mm and the thickness of the lead is 3.0 mm. Whilst playing, a ball strikes the top of the net horizontally with its centre in line with the top of the 'net'. Assume the ball is not revolving when it hits the 'net'. The mass of a table tennis ball is 2.7×10^{-3} kg. The ball bounces back with 20% of its incident speed. Calculate the fastest speed it can hit the 'net' without the 'net' toppling over. Assume the 'net' and the bars do *not* slide on the surface of the table.

Q1.b

Figure 1.b.

x

δx
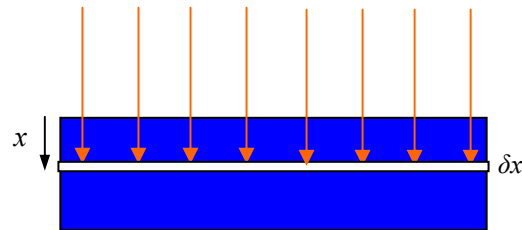
The Sun shines vertically down on to static seawater, Figure 1.b. The fraction of the power, P, absorbed by a thin layer of water thickness δx at depth x, per square metre of surface is $\sigma P \delta x$. Write down an equation that connects the above variables. The thermal conductivity of water, κ, relates the rate of heat flow due to a temperature gradient. Write down an equation for this relation. Obtain a differential equation that relates the temperature of the water θ with the depth of the water x. Assume there are no convection currents. What is the condition for convection currents to start?
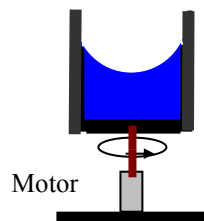
Q1.c

Figure 1.c.

Motor

Figure 1.c. shows a rotating glass cylinder containing a viscous liquid, density ρ. The electric motor rotates the cylinder at an angular velocity of ω. Show that the liquid forms a parabolic surface and find the equation of the parabola. Some astronomical telescopes use rotating dishes filled with mercury to produce a reflecting parabolic surface. Show that a parabolic mirror does produce a focus and derive an expression relating the angular velocity of rotation to the focal length of the mirror. What type of advantages and disadvantages does this type of mirror have?

Q 1.d

Figure 1.d.i.

Figure. 1.d.ii.
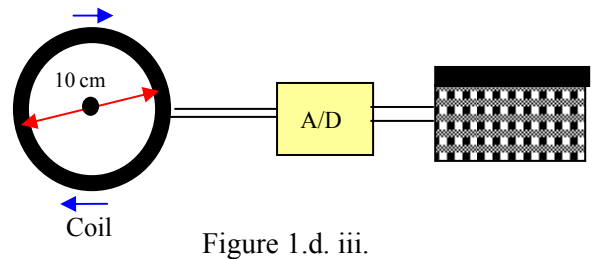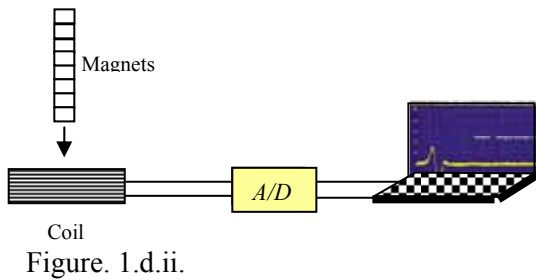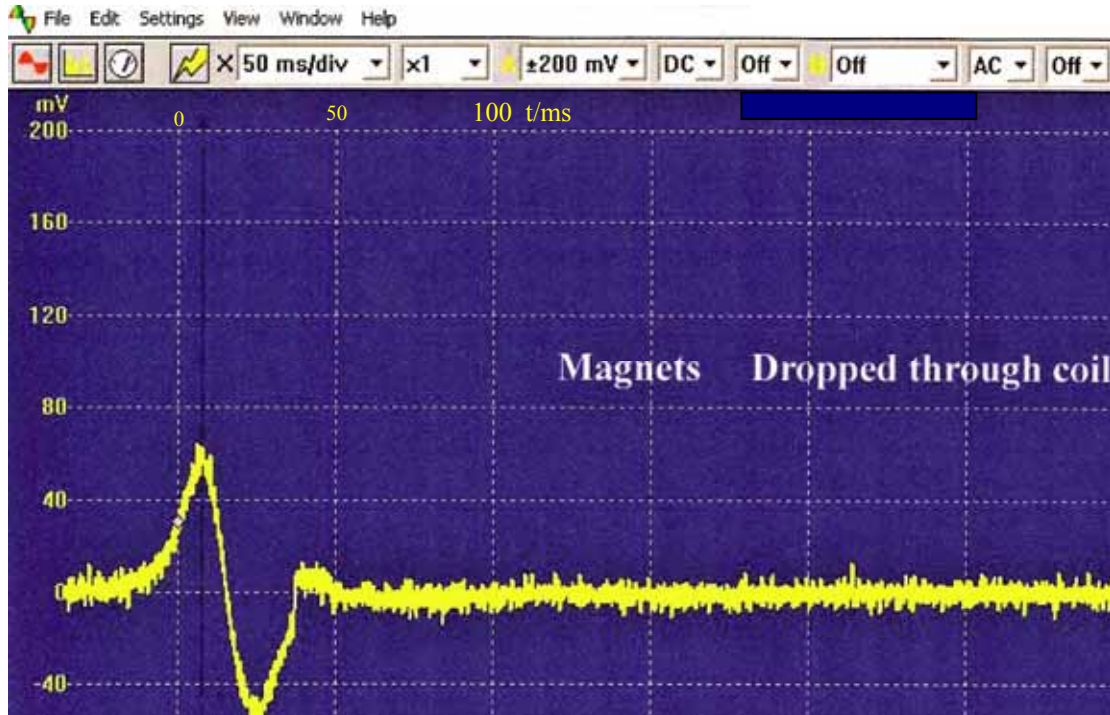
Figure 1.d. iii.

A tube of disc magnets was dropped through a coil.   The computer recorded the EMF generated in the coil Figure 1.d.i. The coil is wound in a clockwise direction looking down, Figure 1.d.ii. and Figure 1.d.iii. The actual display is shown enlarged in Figure1.d.i.   What quantitative and qualitative information can you obtain from the display?   The physical length of the tube of magnets was 8.0 cm. The coil had 100 turns and an average diameter of 10 cm and was 1 cm thick.
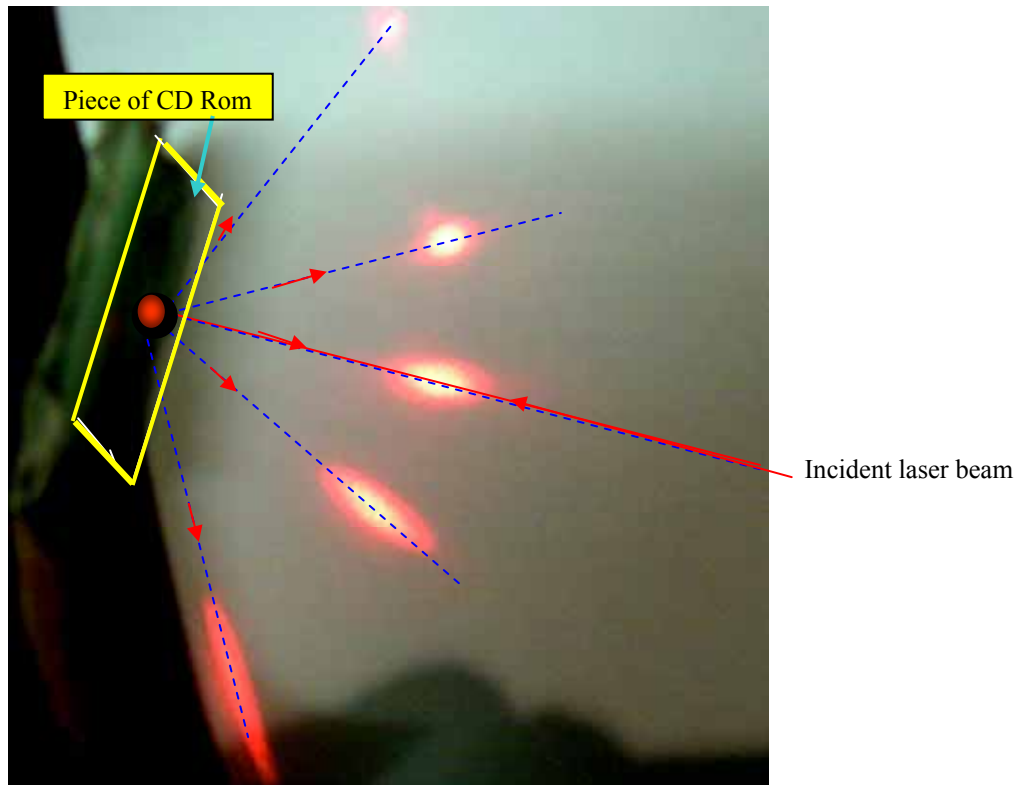
Q 1e

Figure.1.e.

Figure.1.e shows a photo of a laser beam incident at 90^0 on a piece of CD Rom. The laser has a wavelength of 620 nm. The dotted blue lines indicated the directions of the reflected laser light that strikes the paper at the red oblongs shown in the photo. Find the effective reflection grating spacing and hence estimate the maximum number of bits (1 or 0) that might be stored on the CD Rom.

(You can take measurements from the photo).

Q.1.f. Show that the energy stored in a capacitor C storing a charge of Q is $\dfrac{1}{2}\dfrac{Q^2}{C} = \dfrac{1}{2}CV^2$

where V is the potential difference between the plates of the capacitor.

Show that the energy stored in a coil is given by $\dfrac{1}{2}LI^2$ where I is the current in the coil and L is the inductance of the coil. Find the energy stored in the electromagnet of the CMS detector of the Large Hadron Collider. The inductance of the large magnet is $L = 14$ H, the normal current is 19.5 kA, giving a magnetic field intensity at this current of 4.0 T. Calculate the energy stored and the average power output if the circuit was broken and the current reduced to zero in 2 ms.

Q2   This question is about cold dark matter.

Cold dark matter only affects ordinary matter by gravitational interaction.   It will deflect light by gravitational interaction.   Electromagnetic quanta are thought to rarely interact with cold dark matter, therefore "physical collisions" occur very infrequently.   No verified direct detection appears to have yet been recorded and its existence is inferred through indirect observations. These include the deflection of light by invisible bodies and optical effects such as gravitational lensing; double images caused by the gravitational fields and the inferred mass of a group of galaxies rotating about a common centre.   In the last case the total mass of the visible galaxies is far less than the total mass needed to produce stable orbits given the observed orbital speeds and orbital parameters. Part (a) illustrates the bending of light rays by mass.
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(a) According to the General Theory of Relativity the gravitational field of the Sun alters the effective refractive index, n, of outer space so that:

$$n^2 \approx 1 + \frac{4a}{r},$$

where a is a constant which, for the Sun, takes a value of 1.48×10^3 m and r is the distance to the centre of the Sun.
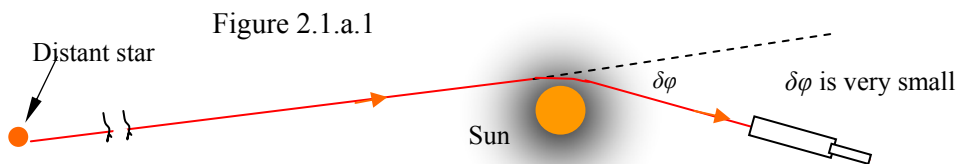
Figure 2.1.a.1

Figure 2.1.a.1 shows the Sun with the surrounding space shaded illustrating the increasing effective refractive index, n, as the distance from the centre of the Sun is decreased.
As the Sun passes across the sky, the skylight of stars closest to the Sun should appear to shift position in the sky due to the effective change in refractive index of the space surrounding the Sun.   However due to the intense light emanating from the Sun the only possible way to measure the position of stars close to the Sun is to observe it during a total solar eclipse. Figure 2.1.a.2. shows a photograph taken during the 1919 solar eclipse.   Measurements of the position of stars from this photo showed an apparent shift in their position from their positions taken during the night. This shift agreed with Einstein's predicted value.

Find the value of the effective refractive index, n, close to the surface of the Sun. Using a simple model, or otherwise, find the angular deflection of the light from a distant star as it passes very close to the Sun.   Draw a clear diagram of your model.

Figure 2.1.a.2.

b)

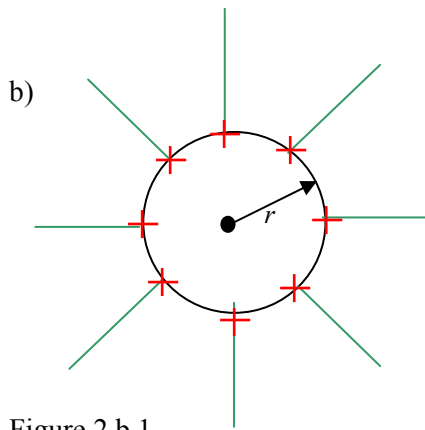Mass (uncharged)
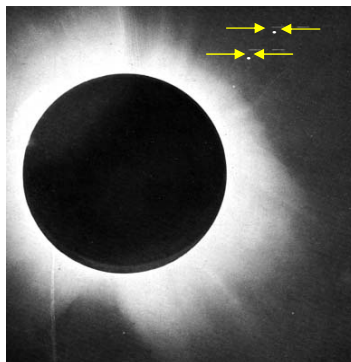
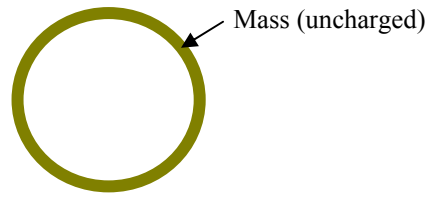Figure 2.b.1   Figure 2.b.2

Figure.2.b.1 shows a hollow metal sphere that is positively charged. The lines of electric field are connected to negative charges that are a long way away.   Show that if the electric field E from a very small charge q is given by $E = \dfrac{q}{4\pi r^2 \varepsilon_0}$, where ε_0 is the dielectric constant of free space, then the electric field inside the sphere is zero.   Figure.2.b.2 shows an uncharged massive spherical shell.   Show that the gravitational field inside the sphere due to the uniform massive shell is also zero.
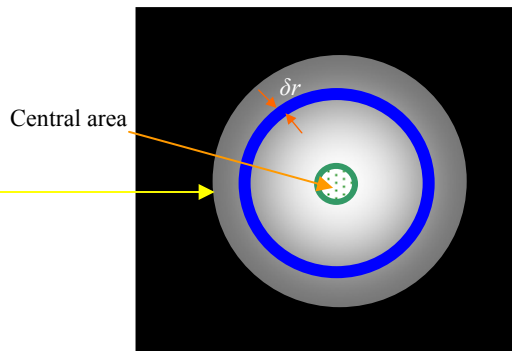
c) Show that, according to Newton's theory of gravitation, the numerical values of the mass of the Sun and radius of the orbit of the Earth, given on page 1 - the data page, are consistent.

d)

Figure 2.d.1.a.   Figure 2.d.1.b.

Model area

Central area

δr
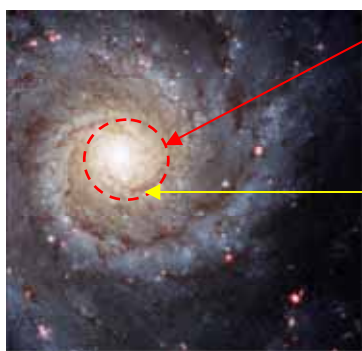
Spiral Galaxy   Model

To perform a mathematical analysis of the distribution of stars and their angular velocity in a galaxy it is expedient to make a simple mathematical model of the galaxy.   The model illustrated in Figure 2.d.1.b.consists of a thin disc of stars inside the dotted circle in Figure 2.d.1.a. The density of the stars is roughly constant in the central area within the green dotted circle Figure.2.d.1.b.

Determine a formula that relates the speeds of the stars with their distance, r, from the centre of the galaxy given that the mass of stars per unit volume, $\rho(r)$ is given by $\rho(r) = k/r$ (except in the central area). k is a constant.   Is your formula consistent with Figure.2.d.e.1?

Central Area
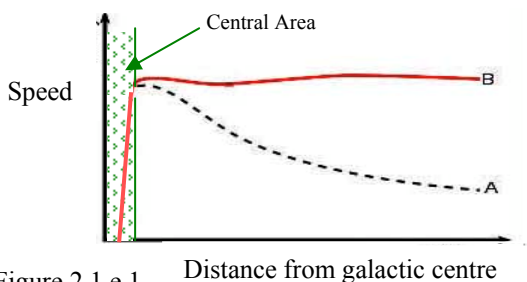
Speed

B

A

Distance from galactic centre

Figure 2.1.e.1.

Rotation curve of a typical spiral galaxy.
Predicted (A) and observed (B).
The difference between the two is attributed to dark matter.

END OF PAPER.
PLEASE make sure the cover sheet is firmly attached to your script.