British Physics Olympiad 2015-16

BPhO Round 2

Monday 25th January 2016

Instructions

Time: 3 hours (approximately 35 minutes on each question) + 15 minutes reading time.

Questions: All five questions should be attempted.

Marks: The questions carry similar marks.

Instructions: To accommodate students sitting the paper at different times, please **do not discuss** any aspect of the paper on the internet until 8 am Saturday 30th January.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets. Graph paper and formula sheets should also be made available. Students should ensure their name and school is clearly written on **all** answer sheets and pages are numbered.

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will not be marked and this is an important aspect of this exam paper.

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Training Dates and the International Physics Olympiad 2016

Following this Round 2 paper, fifteen students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the ***Training Camp*** *to be held in the Physics Department at the University of Oxford from* ***Monday 4th – Thursday 7th April 2016****. Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training.*

From April until late May mentoring by email will cover some topics and problems.

There will be a weekend ***Experimental Training Camp in Oxford*** *over the weekend of* ***Friday 20th – Sunday 22nd May*** *(Friday evening to Sunday afternoon). This is the week before half-term.*

There will be a ***Training Camp at Trinity College, Cambridge*** *(****Sunday 3rd July – Friday 8th July****). The IPhO this year will be held in Zurich, Switzerland, from* ***Sunday 10th July to Monday 18th July.***

Important constants

Speed of light in free space	c	3.00×10^{8} m s^{-1}
Elementary charge	e	1.60×10^{-19} C
Acceleration of free fall at Earth's surface	g	9.81 m s^{-2}
Permittivity of free space	ε_o	8.85×10^{-12} m^{-3} kg^{-1} s^{4} A^{2}
Permeability of free space	μ_o	$4\pi \times 10^{-7}$ H m^{-1}
Mass of a neutron	m_n	1.67×10^{-27} kg
Mass of a proton	m_p	1.67×10^{-27} kg
Boltzmann constant	k	1.38×10^{-23} J K^{-1}
Planck's constant	h	6.63×10^{-34} J s
Radius of the Earth	R_E	6.38×10^{6} m
Radius of the Earth's orbit	r_E	1.49 x 10^{11} m
Mass of the Sun	M_S	2.0×10^{30} kg
Mass of the Earth	M_E	6.0×10^{24} kg
Gravitational constant	G	6.67×10^{-11} m^{3} kg^{-1} s^{-2}
Density of air at ground level	ρ_{air}	1.20 kg m^{-3}
Density of U_{238}	$\rho_{uranium}$	29 000 kg m^{-3}
Density of Lead	ρ_{lead}	11 000 kg m^{-3}
Density of mercury	$P_{mercury}$	13 600 kg m^{-3}
Visible light	λ	400 – 700 nm

N.B. these figures are kept to a small number of significant figures for simplicity

BPhO sponsors

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Question 1

(a) The flow of thermal energy through a conductor due to a temperature difference between two faces is described by an equation of the form,

$$\frac{\mathrm{dQ}}{\mathrm{d}t} = -\lambda A \frac{\Delta T}{\Delta x}$$

in which $\frac{\mathrm{dQ}}{\mathrm{d}t}$ is the rate of heat flow, A is the cross sectional area of the conductor, and $\frac{\Delta T}{\Delta x}$ is the temperature gradient along the line of heat flow. The thermal conduction properties of the medium are given by the coefficient of thermal conductivity, λ.

i) A similar flow through a conductor may be ascribed to conduction of electricity. Write down an equation of the same form for electrical conduction through a material, stating what the corresponding term is in each equation.
Comment on the presence of the minus sign.

ii) In an example in which two cylindrical conductors of heat energy with the same cross sectional areas and similar lengths are placed end to end in series, so that the thermal energy flows through each in turn, what is the equivalent thermal conductivity of the series combination of conductors, λ, in terms of λ_1 and λ_2? (No heat is lost through the sides of the conductors).

(b) A circuit is set up with the current flowing radially through a uniform disc of conducting material of resistivity, ρ, and thickness, t. Current enters a centre metal core of radius r_0 which has a very high conductivity and flows out of the rim of the disc, which has radius r, and has a ribbon of highly conducting metal around it. In terms of the linear dimensions and the resistivity, obtain an expression for the resistance through the disc.

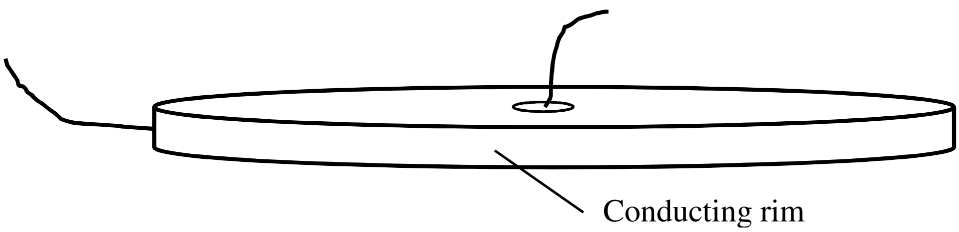

Figure 1.1 Resistive disc with a current flowing radially from centre to edge, from the conducting metal centre core to the conducting metal rim.

(c) Physicists often have to explain effects seen in nature for which there may be a number of contributing explanations. Observation is a key skill of the physicist, being able to see what is in front of him and spot the important relevant features.

In Figures 1.2 and 1.3 below, water was seen flowing down a road from the constant source of a small stream at the side of the road. The smooth flow of the stream breaks up further up the road. The water flows down the uniform slope of the road towards the observer. State what you can observe in the photographs and comment on them from the point of view of a physicist. You are not required to do any calculations or explain all the physics of this complicated situation.

Figure 1.2 Flowing water on a shallow slope showing how the flow becomes uniform again when it spreads out.

Figure 1.3 Flowing water breaking up on a shallow slope.

Question 2

(a) The gravitational force of attraction, F, between two spherical masses M_1 and M_2 is given by

$$F = \frac{GM_1M_2}{R^2}$$

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Measuring G is difficult. Explain why.

(b) One of the earliest methods of measuring G was devised by Henry Cavendish, and a modern version of the sort of setup he used is illustrated below.

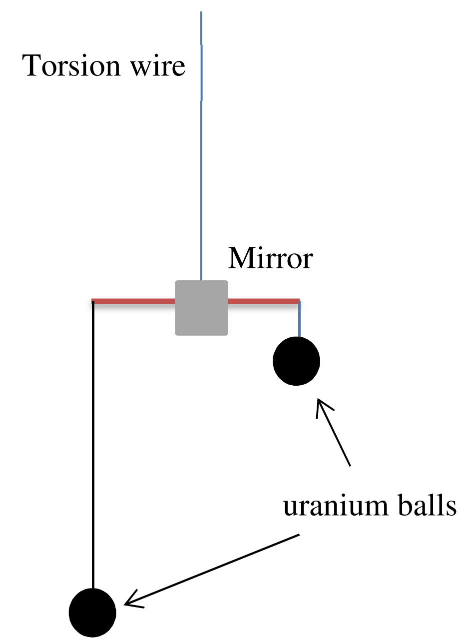

Figure 2.1 Side-on view of apparatus for measuring G. Supporting bar with mirror attached. Two equal, spherical uranium masses hang from the beam (at different heights). The apparatus is inside a draught proof container.

Two identical uranium spheres each of mass 60 g, made of depleted uranium-238, are suspended from a short, light, beam with a mirror attached. This is suspended from a thin torsion wire attached to a rigid support. The two small uranium spheres are hanging at different heights from the beam. Two massive lead balls (not shown) are placed on supports close to each of the uranium spheres, with about 1 cm separation between them. The uranium and lead spheres will not make contact when the beam is released. The lead spheres have a diameter of 7.6 cm and the length of the beam is 8.5 cm.

In Figure 2.2 the layout is now seen from above. This also shows the two large (grey) lead balls viewed from above. The pair of lead balls can be swung from position A as shown Figure 2.2, to position B as in Figure 2.3. The torsion wire supporting the mirror beam will resist twisting with a torque proportional to the angle of the twist, with the constant of proportionality being the *torsion constant*.

Explain how you could get the value of G from this apparatus. Your answer should contain quantitative values as well as comments about the layout of the apparatus and a procedure for determining G. Consider estimating the torsion constant of the wire and the period of oscillation.

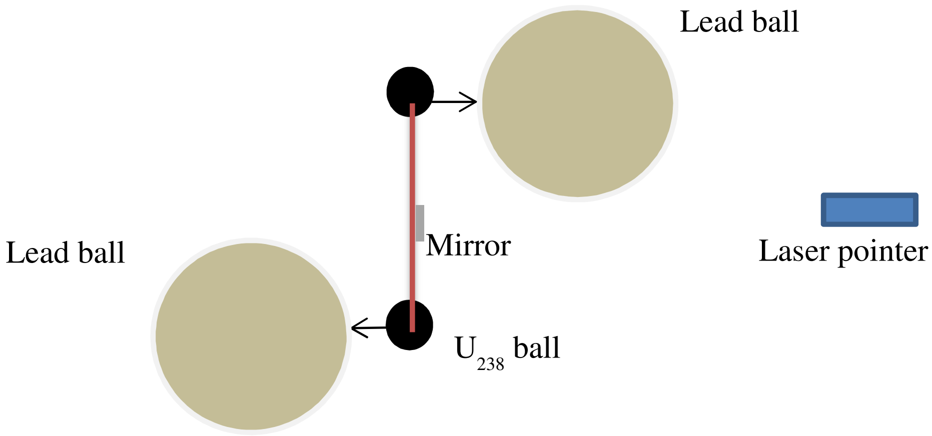

Figure 2.2 View from above. Two lead balls are placed near the two uranium masses suspended from the bar (position A)

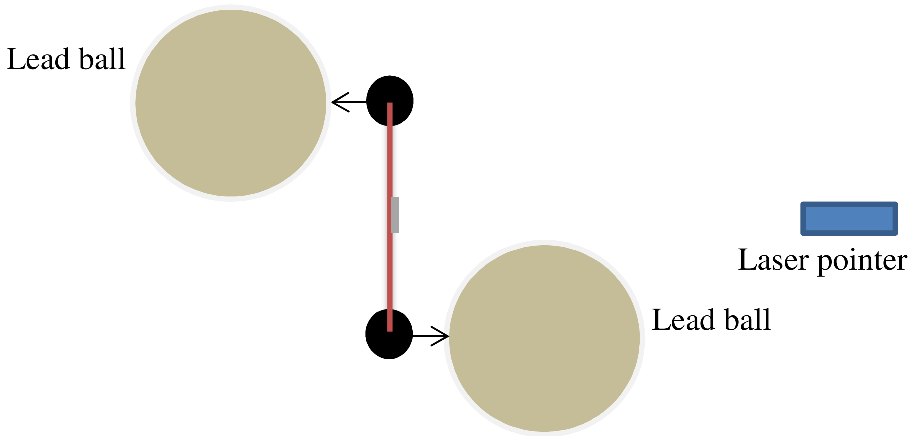

Figure 2.2 The two lead balls swung into position B.

(c) If one of the large balls was moved, then the force on a small ball would change. It might be expected that the force, by similarity to light would propagate at the speed of light and like light photons, would be quantised (gravitons!).

Some years ago a flashing radio wave emitting source was observed by Jocelyn Bell, a PhD student at Cambridge, later Professor Bell and Chair of the BPhO. The team leader, Professor Anthony Hewish received the Nobel Prize in Physics for this discovery of pulsars. Sometime later the mass of the star was determined (the mass is about the same as the Sun but with a diameter of about 25 km), but since the star's surface could not be measured it was difficult to tell what exactly was happening. Binary pulsars have also been found (pulsars that rotate round a common centre of mass) and it is they that are likely to produce detectable changing gravitational fields.

Why would you think that binary pulsars might be a strong source of gravitational waves?

Question 3

(a) Large area optical reflecting telescope mirrors are needed if we are to see further into the universe. Greater light gathering capacity and increased resolving power do, however, mean that the mirror supporting structure must be increasingly expensive. One way to overcome this is to rotate a circular tank of clean liquid mercury at a constant rate to create the correct shape of mirror. Obtain an expression relating the angular frequency of rotation, ω, and the $x - y$ coordinates of the mercury surface.

(b) A telescope mirror in the shape of a parabola can be specified by the equation $y = ax^2$. A ray parallel to the axis of the mirror is shown in Figure 3.1 below, with the normal and the reflected ray. Obtain an expression for $\tan 2\theta$ in terms of a and x.
You may find useful: $\cos 2\theta = \cos^2\theta - \sin^2\theta$ and $\sin 2\theta = 2\sin\theta\cos\theta$

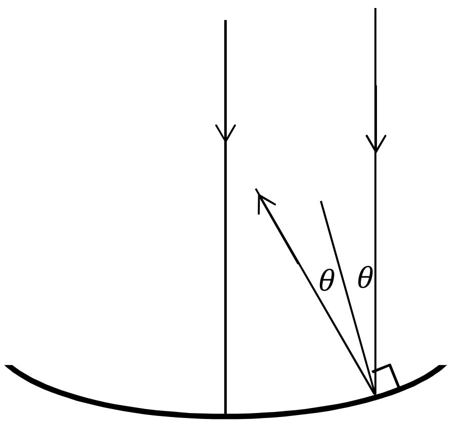

Figure 3.1 Parabolic mirror reflecting a paraxial beam of light. The normal to the mirror is shown.

(c) Show that the parabolic mirror will reflect all paraxial rays through a single focus, and determine the focal length of the mirror, f, in terms of a.

(d) The telescope is required to have a minimum resolution of (i.e. no worse than) 40 milliarcseconds in visible light. To avoid vibrations, the rate of rotation of the mercury mirror is a slow 8.5 revolutions per minute. If the mercury was rotated in a flat bottomed circular container with the same diameter as the mirror, estimate the minimum volume of mercury used to form the mirror.

(e) Mercury evaporates from the surface, as the molecules free themselves from the bulk of the liquid. The rate at which this happens is temperature dependent, but it is about $30\ \mu\text{g hour}^{-1}\ \text{cm}^{-2}$. What fraction of the mercury mirror evaporates away in a week?

(f) The telescope mirror will focus light, but even for the ideal mirror shape, the light cannot be focused to a point. Explain why.
If the intensity of light reaching the telescope from a distant source is $8.0 \times 10^{-14}\ \text{W m}^{-2}$ (a rather faint telescopic 13.7 magnitude star), what is an estimate of the maximum intensity of light which is obtained at the image position?

Question 4

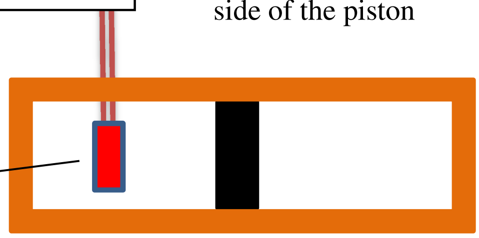

Figure 4.1 Cylinder with insulated walls and a moveable piston.

Figure 4.1 represents a cylinder with thermally insulated walls containing a movable, frictionless, thermally insulated piston. On each side of the piston are n moles of an ideal gas. The initial conditions of pressure, P_0, volume V_0, and temperature T_0, are the same on both sides of the piston.

C_P is the molar heat capacity (i.e. specific heat capacity per mole) of the gas at constant pressure and C_V is the molar heat capacity (i.e. per mole) at constant volume.
The value of γ is 1.5 where γ is the ratio of the molar heat capacities, $\frac{C_P}{C_V}$.
By means of a heating coil in the gas on the left side of the piston, heat is supplied slowly to the gas on this side. It expands and compresses the gas on the right side until its pressure has increased to $\frac{27}{8}P_0$.

Hint: for a gas undergoing an adiabatic change, $P \times V^{\gamma} =$ constant, and also the ideal gas law still applies.

In terms of n, R, C_V, and T_0,

(a) how much work is done on the gas on the right side?

(b) what is the final temperature of the gas on the right?

(c) what is the final temperature of the gas on the left?

(d) how much heat flows into the gas on the left?

Question 5

When illuminated with a laser pointer, the spring of a ballpoint pen produces a diffraction pattern which is similar to the famous Photo 51, the X ray diffraction image of DNA taken in May 1952 at King's College London.
The diffraction pattern of the spring (Figure 5.1) from a ball point pen is shown in Figure 5.2. It is illuminated using a red laser of wavelength of 633 nm. The scale of the diffraction pattern as it appears on the screen is given, but the size of the spring is unknown. The image is a height of 7.8 cm on a screen, which is at a distance of 4.2 m from the spring.

In diffraction, Babinet's principle states that the diffraction pattern of an obstacle is the same as the diffraction pattern of an aperture of the same shape. According to this principle, the diffraction pattern formed by the two straight sections of wire at an angle to each other is equivalent to the diffraction pattern of two single slits oriented at the same angle with respect to each other. The calculations of the angular positions of the maxima and minima will be exactly the same.

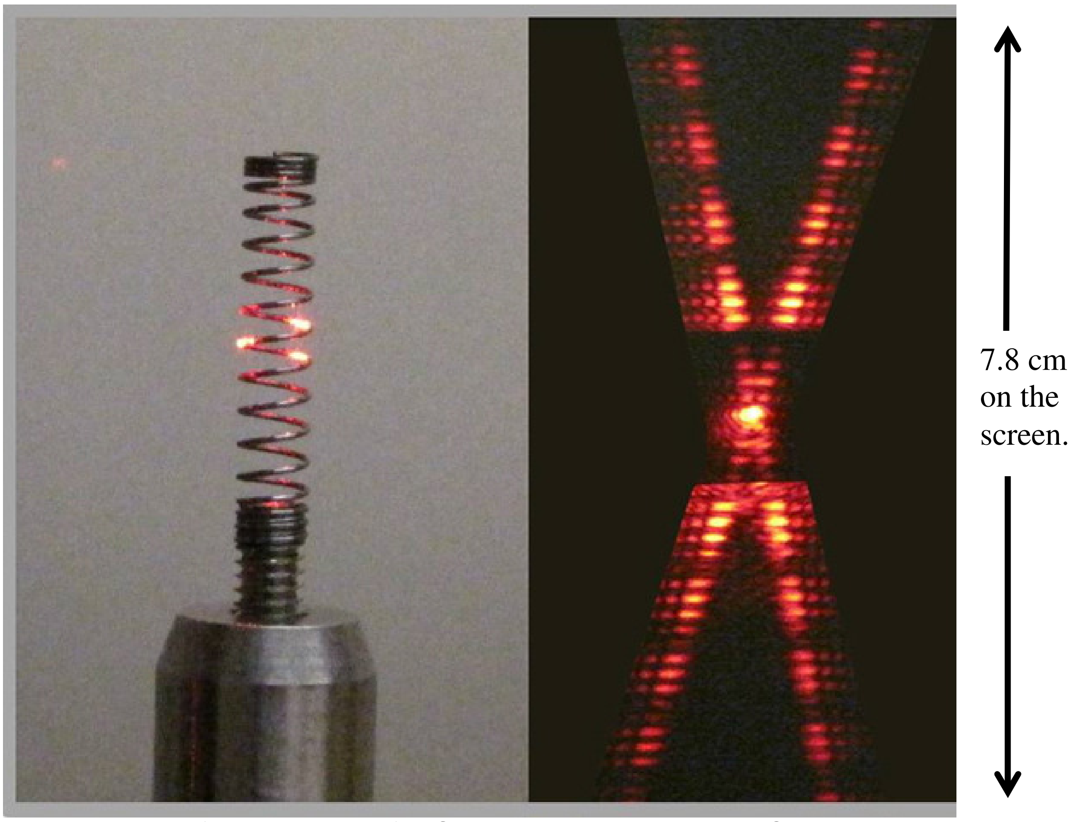

Image courtesy of D. Tierney and of H. Schmitzer, Xavier University in Cincinnati

Figure 5.1 Laser illuminated spring from a ball point pen. No scale shown.

Figure 5.2 Diffraction pattern of the spring. The scale of the diffraction pattern is given.

The scale of the image is shown at the side, but this is not the same scale for the spring.

(a) Using the information given about the scale of the diffraction image, estimate the pitch of the spring, the thickness of the wire of the spring, and the radius of the spring.

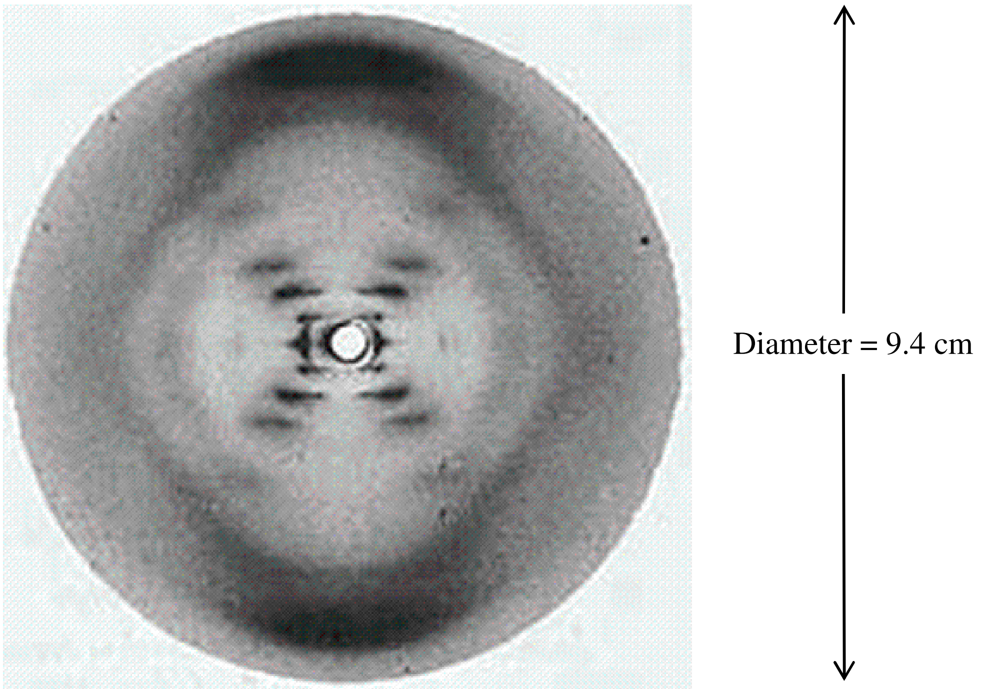

Figure 5.3 X ray diffraction image of DNA taken in this famous Photo 51, crucial in the elucidation of the structure of DNA. The scale of the image is shown at the side.

(b) If Photo 51 was taken with the K_{α} line of copper, which has a wavelength of 0.15 nm, and the distance between the sample and the film was 9.0 cm, estimate the angle, pitch and radius of the DNA molecule. In the image, the zeroth order is not visible in the centre. The first order maximum can just be seen, but the 2^{nd}, 3^{rd} and 5^{th} orders are clear. (The 4^{th} order is missing, an effect of the double helix structure, which you can ignore).

END OF PAPER