BPhO Round 2 2018

(a) Physicists :

(3)

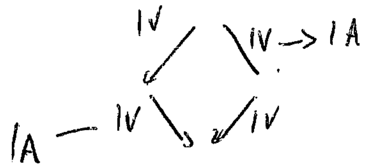
(b) One side plane . One side quite convex.
Converging lens.
Thick . About 5 cm diameter .

(3)

(c) The mist on the windowpane has droplets that are large enough to disrupt the light in that it looks like a fog.
But quite often you can see an interference ring if there is a light behind.
So the droplets have a radius and separation of some wavelengths of light.
They are a single layer thick, close to each other but just separated. If you view the light source you can see a ring of some sort, with maybe an angle of diffraction from the centre of perhaps 6° i.e. 0.1 radian.
So $\theta = \lambda d$ gives a d of 6×10^{-6} m. therefore $d^2 = 36 \times 10^{-12}$ m².
In a large 20 cm diameter patch, the radius is 0.1 m and area is 3×10^{-2} m².
So number of droplets is $N = \frac{3\times10^{-2}}{36\times10^{-12}} = 0.8 \times 10^9$.
So the mass of the layer (density is 10^3 kg/m^{-3}) $m = 0.8 \times 10^9 \times \frac{4}{3}\pi \times (3 \times 10^{-6})^3 \times 3 \times 10^3$ kg $=$ 0.2 g $\approx 10^{-4}$ kg

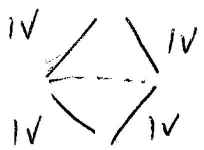
(5)

d) (i) 2 V supply



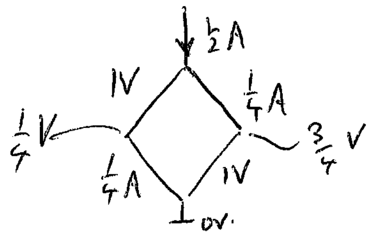
2 A
1A in each arm.

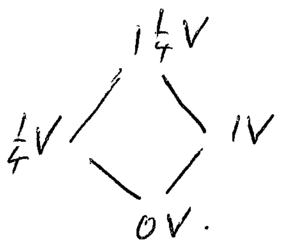
(ii)



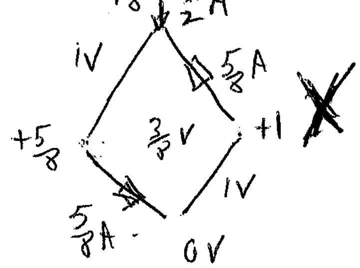
2 A
1A in each arm.

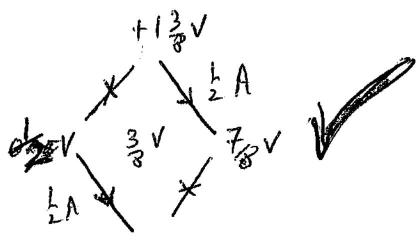
(iii)





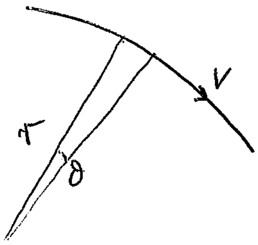
(iv)





(4)

(e)



Normal reaction force $= \frac{mv^2}{r}$ ✓

frictional drag $= \mu \frac{mv^2}{r}$

$WD = dW = -\mu \frac{mv^2}{r} ds$ ✓ minus sign since energy dissipated.

$= -\mu \frac{mv^2}{r} \cdot r\,d\theta$ ✓

$\therefore d(\tfrac{1}{2}mv^2) = -\mu \cancel{m} v^2 d\theta$

$$\int_{V_i}^{V_f} \frac{d(V^2)}{V^2} = -2\mu \int_0^\theta d\theta$$

$$\frac{V_f^2}{V_i^2} = e^{-2\mu\theta}$$

$$\frac{V_f}{V_i} = e^{-\mu\theta}$$ ✓

$V_f = 1.4 e^{-0.25 \times \pi}$

$= 0.64 \text{ ms}^{-1}$ ✓

(5)

20

Gravitational Waves.

a). Correct application of Newton II [1]

Correct result [1]

b). $E = K.E. + P.E.$

$$= \frac{1}{2} m (r\omega)^2 + \frac{1}{2} M (R\omega)^2 - \frac{GMm}{(R+r)}$$ or equivalent [1]

$$= \underbrace{-\frac{1}{2}}_{[1]} \underbrace{\frac{G^{2/3} Mm}{(M+m)^{1/3}} \omega^{2/3}}_{[1]}$$ after using part a)

c). $I = MR^2 + mr^2$ [1] is sufficient now

$$= \frac{mM}{m+M} (r+R)^2$$ [2] either now or in any subsequent part.

d). Correct dimensional analysis

$$P = \alpha \frac{G I^2 \omega^6}{c^5}$$ [1]

$\beta = 1$

[illegible] $= 6$

[illegible] $= -5$

9

h). Any appropriate argument [1]

$m+M \geqslant 4^{3/5} \mathcal{M}$ [1]

i). Combination of Schwarzschild result with Kepler III result of part a). [1]

Identifies $f_c = 300\,\mathrm{Hz}$ [1]

$m+M = 76\,M_\odot$ $\left.\begin{array}{l} M = 53\,M_\odot \\ m = 23\,M_\odot \end{array}\right\}$ [1]

$\pm 15\%$.

j) Correct assumptions for the model. [1]

e.g. large initial separation.

Total energy loss $\simeq \frac{1}{4} \frac{Mm}{(M+m)} c^2$ [1]

$\simeq 4 M_\odot \pm 15\%$ [1]

$\boxed{8}$

$$\frac{1}{2} m r^2 \omega^2 + \frac{1}{2} M R^2 \omega^2$$

$$m r = M R$$

$$\frac{1}{2} m r \omega^2 (r+R) - \frac{G M m}{(r+R)}$$

$$= \frac{1}{2} m r \frac{G(M+m)}{(r+R)^2} - \frac{G M m}{(r+R)}$$

$$= \frac{1}{(r+R)^2}\left[\frac{1}{2} m r G M + \frac{1}{2} M R G m - G M m r - G M m R \right]$$

$$= \frac{G M m}{(\ )^2}\left[\frac{1}{2} \dot{r} + R \right]$$

$$= -\frac{1}{2}\frac{G M m}{(r+R)}$$

$$= -\frac{1}{2}\frac{G M m}{\left[G(M+m)\right]^{\frac{1}{3}}} \omega^{-\frac{2}{3}}$$

$$m = \frac{M^{\frac{6}{5}}}{2^{\frac{1}{5}} M^{\frac{1}{5}}} = \frac{1}{2^{\frac{1}{5}}} M$$

Que 3. Phase-Space Physics

(a) From graph, $p_x = -kx$ (✓)

$$m\frac{dx}{dt} = -kx$$

$$-\frac{m}{k}\int_{x_0}^{x}\frac{dx}{x} = \int_0^t dt$$

$$\ln\frac{x}{x_0} = -\frac{k}{m}t$$

$$x = x_0 e^{-\frac{k}{m}t} \quad (✓)$$

2

The displacement from an origin decreases exponentially with time

(b) (i) $E = k.e. + p.e.$
$= \frac{p^2}{2m} + l(1-\cos\theta)mg$ (✓)

(ii) For small θ, $\cos\theta \simeq 1 - \frac{\theta^2}{2}$

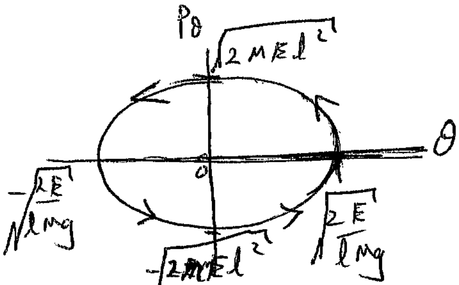
so $E \simeq \frac{p^2}{2m} + \frac{1}{2}l\theta^2 mg$

Since $p_\theta = lp$

$E \simeq \frac{p_\theta^2}{2ml^2} + \frac{1}{2}l\theta^2 mg$ Of the form

$1 = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ - an ellipse

$1 = \frac{p_\theta^2}{2mEl^2} + \frac{lmg}{2E}\cdot\theta^2$

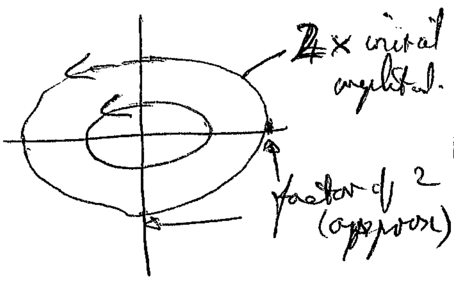


ellipse (✓)
limits (✓)
arrow (✓) on a sketch.

4

(iii) Twice the amplitude, $E \propto A^2 \Rightarrow 4 \times$ initial energy
Area of ellipse $\propto a \times b$ (semi major & semi minor axes) $\{A = \pi ab\}$

$\propto \sqrt{E} \times \sqrt{E}$
$\propto E$
$\propto A^2$ so $4\times$ area



(✓)

For info, area of ellipse is $\frac{1}{2}ET$ (T is the period)
$= \frac{\pi E}{\omega}$.

1

(iv) given $\Delta E \propto -E$

$\Delta E = -kE.\Delta t$ — a decay process like radioactivity.

$E = E_0 e^{-kt}$

Now ~~[illegible]~~ $dE = -2\times10^{-5}\, E \cdot \frac{dt}{T}$ (period, T)

So $k = \frac{2.5\times10^{-5}}{T}$

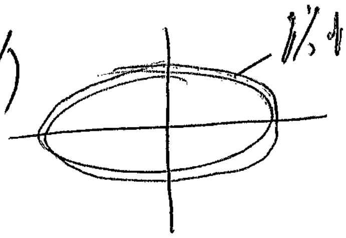
given $t = 400T$

Then $E = E_0 e^{-2.5\times10^{-5}\times400}$

$= E_0 e^{-10^{-2}}$

$= E_0 e^{-0.01}$

$= \underline{\underline{99\% \text{ of } E_0}}$. (✓)

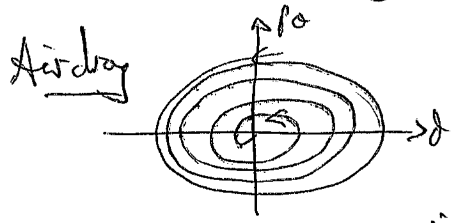


$\boxed{1}$

Area ∝ E.

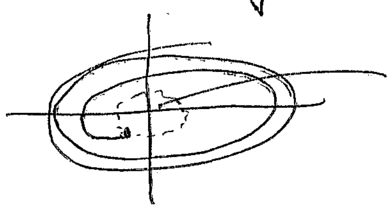
(v) The trajectories cross the axes at right angles, Crossing the $\theta=0$ axis at right angles indicates that the Δp_θ either side is the same, so no energy is dissipated. Similarly at the $p_\theta = 0$ axes, no energy is dissipated [not the same as energy is conserved though]. (✓)

(vi)



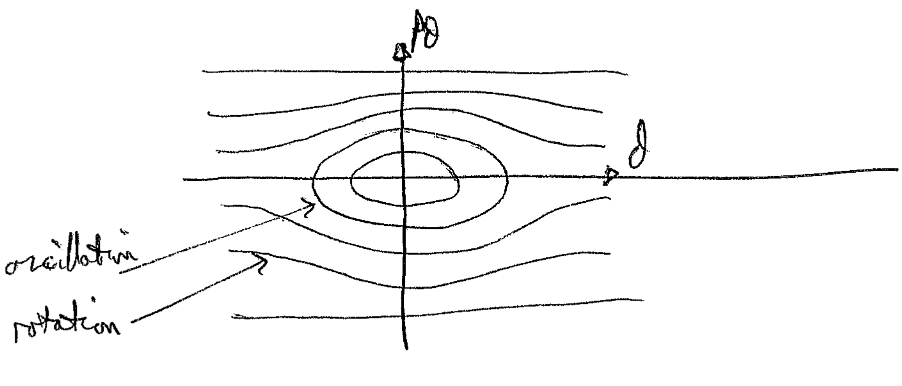
The trajectory will "spiral" in to the origin. (✓) No trajectory ~~crossings~~ crossings occur. The pendulum will eventually swing with an amplitude similar to that caused by collisions with air molecules. Then its periodic motion can not be identified.

(vii) The pendulum will stop by sticking when the amplitude is small.



(✓)

(viii)



(✓)

$\boxed{4}$

(c) (i) Colinear collision: Before $\overset{u_1}{\underset{m_1}{\circ\!\longrightarrow}}$ $\overset{u_2}{\underset{m_2}{\circ\!\longrightarrow}}$

After $\overset{v_1}{\underset{m_1}{\circ\!\longrightarrow}}$ $\overset{v_2}{\underset{m_2}{\circ\!\longrightarrow}}$

$$\left.\begin{array}{lll}\text{Cons. of mom.} & m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 & ①\\ \text{Cons of KE (elastic)} & m_1u_1^2 + m_2u_2^2 = m_1v_1^2 + m_2v_2^2 & ②\end{array}\right\} (\checkmark)$$

from ① $\quad m_1(u_1 - v_1) = m_2(v_2 - u_2)$

from ② $\quad m_1(u_1 - v_1)(u_1 + v_1) = m_2(v_2 - u_2)(v_2 + u_2)$

Assume $u_1 \neq v_1$, $u_2 \neq v_2$ (no collision occurs)

Then, dividing $\quad u_1 + v_1 = v_2 + u_2$

so that $\quad v_2 = u_1 + v_1 - u_2$

Now substituting for v_2 in ①

$$m_1u_1 - m_1v_1 = m_2u_1 + m_2v_1 - m_2u_2 - m_2u_2$$

So $\quad m_1u_1 - m_2u_1 + 2m_2u_2 = (m_1 + m_2)v_1$

Hence $\quad v_1 = \dfrac{(m_1 - m_2)}{(m_1 + m_2)} \cdot u_1 + \dfrac{2m_2}{(m_1 + m_2)} \cdot u_2 \qquad ③ \quad (\checkmark)$

Now substitute for v_1 in ① instead.

$$m_1u_1 - m_1v_1 = m_1u_2 + m_1u_1 = m_2v_2 - m_2u_2$$

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_1 + m_2)v_2$$

Hence, $\quad v_2 = \dfrac{2m_1u_1}{(m_1 + m_2)} - \dfrac{(m_1 - m_2)}{(m_1 + m_2)} \cdot u_2 \qquad ④ \quad (\checkmark)$

(ii)

$v_1 = au_1 + bu_2$

$v_2 = cu_1 + du_2$

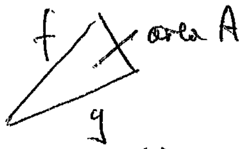
$a = \dfrac{m_1 - m_2}{m_1 + m_2}$ $\qquad c = \dfrac{2m_1}{(m_1 + m_2)}$

$b = \dfrac{2m_2}{m_1 + m_2}$ $\qquad d = -a$

these may be given above.

$\boxed{3}$

Que 3 cont.

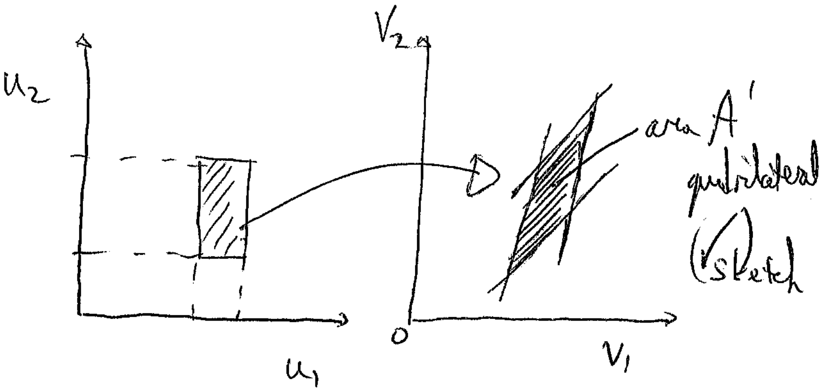


$A = \frac{1}{2}(\underset{\sim}{f} \times \underset{\sim}{g}) \cdot \hat{\underset{\sim}{k}}$

$= \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ f_1 & f_2 & 0 \\ g_1 & g_2 & 0 \end{vmatrix} \cdot \hat{\underset{\sim}{k}}$

$= \frac{1}{2}\hat{\underset{\sim}{k}}(f_1 g_2 - f_2 g_1) \cdot \hat{\underset{\sim}{k}}$

$= \underline{\frac{1}{2}(f_1 g_2 - f_2 g_1)}$



For a fixed value of u_1 (on the graph a vertical line),

$V_1 = \text{const} + b\,u_2$

$V_2 = \text{const} + d\,u_2$

So u_2 transforms linearly onto the V_1-V_2 graph. (✓)

For a different value of u_1, u_2 transforms in the same way, but with a different intercept (so parallel lines.

So the rectangle transforms as a quadrilateral (✓)

The two vectors $\underset{\sim}{V_1}$ and $\underset{\sim}{V_2}$ form a triangle (which is half of the quadrilateral shown)

The area of the quadrilateral is 2× area of triangle above.

$A' = 2 * \frac{1}{2}(a \cdot d - b \cdot c)$

$= -\frac{(m_1 - m_2)^2}{(m_1+m_2)^2} - \frac{4 m_1 m_2}{(m_1 + m_2)^2}$

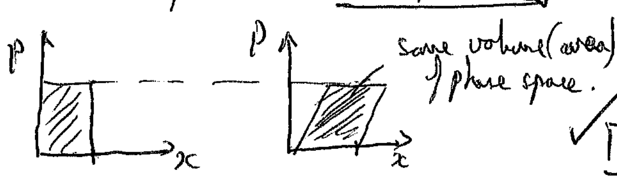
$= \frac{-m_1^2 - m_2^2 + 2m_1 m_2 - 4 m_1 m_2}{(m_1+m_2)^2}$

$= -\frac{(m_1^2 + m_2^2 + 2 m_1 m_2)}{(m_1+m_2)^2}$

$= -1$

modulus of $A' = \underline{1}$ so the transformation in this space is area $\underline{\text{preserving}}$. ✓ $\boxed{4}$

(d) If they do not interact, there is no change of their momenta. But they spread out.



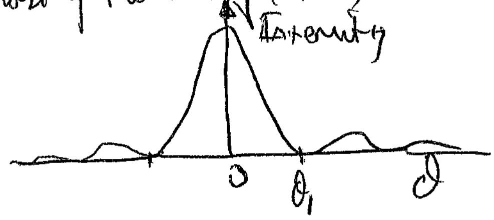
✓ $\boxed{11}$

TOTAL 20 MARKS

Qu 4 Pushing Probes with Lasers

(a) The laser beam from the telescope diffracts similar to a single slit.

Most of the energy (90%) is in the central maxima



The width of the central maximum is $\theta_1 = 1.22 \frac{\lambda}{D}$ (✓)

This is about 90% of the energy in the diffracted beam. (✓)

~~At~~ distance R, $\theta = \frac{d}{R}$

So $\frac{\lambda}{D} = \frac{d}{R}$ (1.22 is not essential). (✓)

$R = \frac{dD}{\lambda}$

$R = \frac{1 \times 10}{600 \times 10^{-9}} \approx 2 \times 10^7$ m (✓)

Factors of 2 (radius ⟺ diameter) are ignored.

$\boxed{4}$

(b) The momentum of a photon is given by $E = pc$

$\therefore \frac{E}{t} = W = \frac{p}{t} \cdot c = Fc$ (✓)

$F = \frac{W}{c}$ $\left(\frac{\text{power}}{c}\right)$

A reflective sail emits light in the other direction: $F \to 2F$ (✓)

Advantage: e.g. reflective sail will not heat up by absorbing energy like a dark surface. (✓)

$\boxed{3}$

Qu 4 cont.

(c) This simplified model has the central mass acting on the sail up to distance R, and then as the sail moves away, the power received by the sail is taken to be zero.

Gravity: if $W \simeq 100\,GW$, then $F = \dfrac{10^{11}}{3\times10^{8}} \simeq 300\,N$ of laser force. This is independent of the mass of the sail, whereas the gravitational attraction of the Earth and Sun (much more significant) is very small on 10^{-3} kg. ($\simeq \frac{1}{100}$ N at the Earth's surface). So we can neglect gravity for this sail. (✓)

∴ Work done by laser is, $WD = F.R$ (with constant F)

KE gained, $\frac{1}{2}mv^2 = F.R$

$\frac{1}{2}mv^2 = ma.R$

$v^2 = 2a.R$ [factor from reflecting sail] (✓)

Substituting for R and a, $v^2 = 2.\dfrac{\cancel{2}W}{mc}.\dfrac{dD}{\lambda}$

so $v = 2\sqrt{\dfrac{W}{mc}.\dfrac{dD}{\lambda}}$ (✓)

as acceleration, a, is ~~[illegible]~~ quite brief (for travelling 4 light years)

time taken, $T = \dfrac{L}{v} = \dfrac{L}{2}\sqrt{\dfrac{mc.\lambda}{W\,dD}}$ (✓)

$$T = \frac{4\times10^{16}}{2}\sqrt{\frac{10^{-3}\times3\times10^{8}\times600\times10^{-9}}{10^{11}\times1\times10}}$$

$= 9\times10^{9}\,s$

$= \underline{300 \text{ years}}$ (✓)

5

Quiz 4 cont.

(d) A single telescope is limited by its diffraction pattern. With many telescopes, a diffraction grating like effect is achieved. The width of the central maximum is still $\theta \sim \frac{\lambda}{D}$, but if D is the diameter of the Earth, then θ is very much smaller.

(✓)

So $D \sim 2 \times 6400$ km

$\sim 1.3 \times 10^{7}$ m

$\underline{\sim 10^{7} \text{ m}}$

(✓)

So $D \to \frac{D}{10^{6}}$

and $T \propto \frac{1}{D^{\frac{1}{2}}}$

Hence $T \to \frac{T}{1000} \sim 0.3$ years

$\underline{\simeq 100 \text{ days}}$

This is a short time for a large distance. $v \simeq 4 \times 10^{9}$ m/s

$\underline{\simeq 10c}$

whereas, previously $v \simeq 4 \times 10^{6}$ m/s

$\underline{\simeq 0.01c}$

So, with the larger diameter source, our non relativistic calculation is not valid.

(✓) Relativity argument

3

TOTAL. 15

Particle Physics Q

(a) (i) $P_{total} = m_1u_1 + m_2u_2$

Want $P'_{total} = m_1(u_1+u') + m_2(u_2+u')$

$$= m_1u_1 + m_2u_2 + (m_1+m_2)u'$$

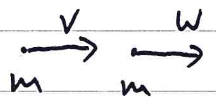
such that $P'_{total} = 0$

$$\Rightarrow \quad u' = -\frac{m_1u_1+m_2u_2}{m_1+m_2}$$ ✓

(ii) $u' = -\frac{mu+0}{2m} = -\frac{u}{2}$

so: 

In ZMF, $P'_{before} = 0 \Rightarrow P'_{after} = 0$ by conservation of momentum

In ZMF after collision: 

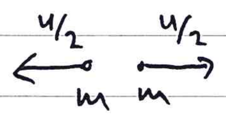
$mv + mw = 0 \Rightarrow \underline{v = -w}$

Since collision elastic KE is conserved $\Rightarrow$ In ZMF:

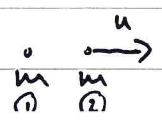
$$2 \times \tfrac{1}{2}m\left(\frac{u}{2}\right)^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m(-v)^2$$

$$\Rightarrow \quad \tfrac{1}{2}u^2 = 2v^2$$

$$\Rightarrow \quad |v| = \frac{|u|}{2}$$

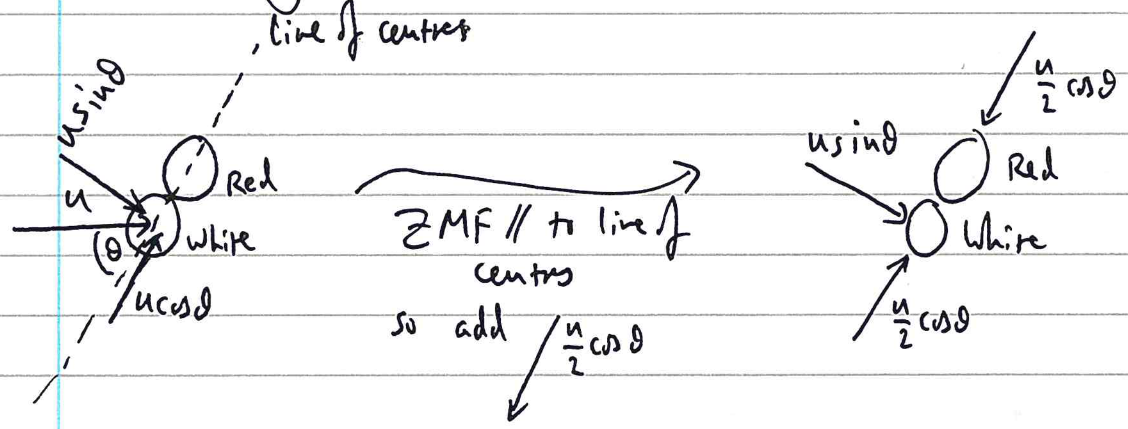
$\therefore$ After collision in ZMF: 

To transform back to lab frame add $\xrightarrow{u/2}$ to each particle

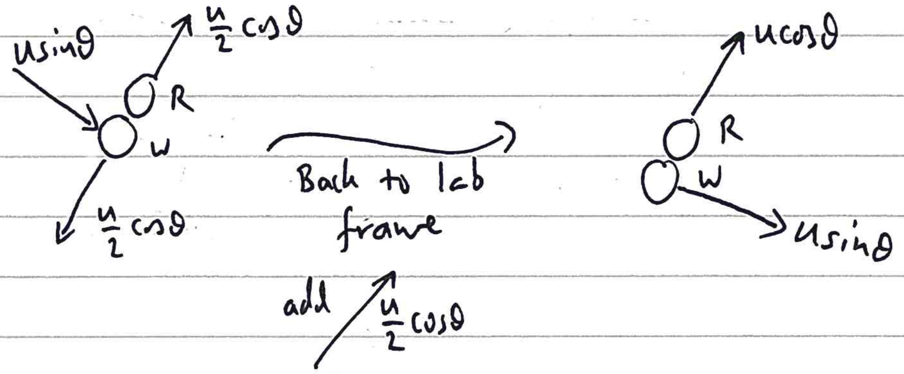
$\Rightarrow$ Final configuration is  //

(pool table)

(iii) Immediately before collision in lab frame:



c.f. (ii), after collision in ZMF:



As white moves off after collision it has no velocity // to line of centres, while red moves only along line of centres. (Collision is assumed frictionless, so no force can be exerted on red in direction perpendicular to line of centres. Impulse only exerted // to line of centres so red must move off in that direction).

(b) (i) $p^2 = \gamma^2 m^2 v^2 = \dfrac{m^2 v^2}{1 - v^2/c^2}$

$$\therefore\ p^2c^2 + m^2c^4 = \frac{m^2v^2c^2}{1-v^2/c^2} + m^2c^4$$

$$= \frac{m^2\cancel{v^2c^2} + m^2c^4 - m^2\cancel{v^2c^2}}{1-v^2/c^2} = \frac{m^2c^4}{1-v^2/c^2} = \gamma^2 m^2 c^4 = E^2 \checkmark$$

For a massive particle at rest, $p=0 \Rightarrow E^2 = m^2c^4$

$$\Rightarrow E = mc^2 \checkmark$$

For a massless particle: $m=0 \Rightarrow E^2 = p^2c^2$

$$\Rightarrow E = pc$$

Using the de Broglie relation $p = \frac{h}{\lambda} \Rightarrow E = \frac{hc}{\lambda} = hf$ ✓

(ii) In the interaction, energy and momentum must both be conserved. Without loss of generality consider a frame in which the e^-e^+ pair have zero momentum (the ZMF):

$$\underset{e^-}{m_e} \xrightarrow{u} \xleftarrow{u} \underset{e^+}{m_e}$$

In principle energy conservation would allow all of the energy (in ZMF: $E_{total} = 2\times E_e = 2\times \gamma_u m_e c^2$) to create a single photon of this energy. However, a single photon has a momentum $p = \frac{h}{\lambda} \neq 0$. Since in the ZMF, $p_{before} = 0 \Rightarrow p_{after} = 0$ it is necessary to have at least one further photon to be emitted to ensure that $p_{total} = 0$ after collision. In the ZMF it is clear (therefore) that the photons must be emitted in opposite directions ("back to back") if there are only two of them:

$$e^- \xrightarrow[u]{} \xleftarrow[u]{} e^+ \quad \xrightarrow[\text{after}]{} \quad \begin{array}{c} \uparrow \text{photon} \\ \downarrow \text{photon} \end{array}$$

(iii) Initially:

$$e^+ \xrightarrow{V} \cdot\, e^- \quad \xrightarrow[\substack{\text{add} \\ \xleftarrow[V']{}}]{\text{ZMF}} \quad \underset{e^+}{\xrightarrow{w}} \quad \underset{e^-}{\xleftarrow{V'}}$$

Total momentum before in lab frame is $p_{e+} + 0 = p_{e+}$

" " " " ZMF is $p'_{e+} + p'_{e-}$

Using xfm for p given:

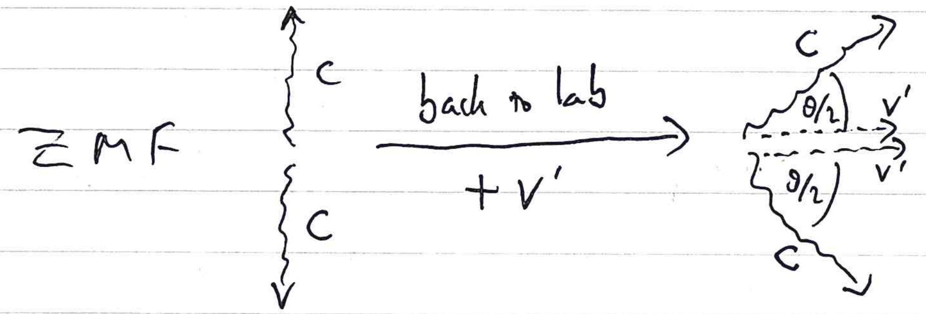
$$p'_{e+} + p'_{e-} = \gamma_{v'}\left(p_{e+} + \frac{v'}{c^2}E_{e+}\right) + \gamma_{v'}\left(0 + \frac{v'}{c^2}E_{e-}\right)$$

$$= \gamma_{v'}\left(p_{e+} + \frac{v'}{c^2}(E_{e+} + E_{e-})\right)$$

The ZMF requires $p_{total} = 0 \Rightarrow p_{e+} + \frac{v'}{c^2}(E_{e+} + E_{e-}) = 0$

$$\therefore V' = -\frac{p_{e+}c^2}{E_{e+} + E_{e-}} = -\frac{\gamma_v m_e V c^2}{\gamma_v m_e c^2 + m_e c^2} = \underline{\underline{-\frac{\gamma_v}{1+\gamma_v}V}}$$

Using Hint, afterwards in ZMF:



$$\therefore \cos\left(\frac{\theta}{2}\right) = \frac{v'}{c} = \frac{\frac{\gamma_v}{1+\gamma_v} v}{c} = \frac{\gamma_v \frac{v}{c}}{1+\gamma_v}$$

Now since $\gamma_v = \frac{1}{\sqrt{\left(1-\frac{v^2}{c^2}\right)}} \Rightarrow \frac{v}{c} = \sqrt{\left(1-\frac{1}{\gamma_v^2}\right)}$

$$\therefore \cos\left(\frac{\theta}{2}\right) = \frac{\gamma_v \sqrt{\left(1-\frac{1}{\gamma_v^2}\right)}}{1+\gamma_v} = \frac{\sqrt{\left(\gamma_v^2-1\right)}}{1+\gamma_v} = \sqrt{\left(\frac{\gamma_v-1}{\gamma_v+1}\right)}$$

~~In limit $v \to 0$, $\gamma_v \to 1 \Rightarrow \cos\left(\frac{\theta}{2}\right)$~~

so $\theta = 2\arccos\sqrt{\left(\frac{\gamma_v-1}{\gamma_v+1}\right)}$

In limit $v \to 0$, $\gamma_v \to 1 \Rightarrow \cos\left(\frac{\theta}{2}\right) \to 0$

$$\Rightarrow \frac{\theta}{2} \to 90^\circ$$

$$\Rightarrow \theta \to 180^\circ$$

(iv) Total energy $E_{e^+} = \gamma m_e c^2 \Rightarrow \gamma = \frac{E_{e^+}}{m_e c^2}$

Energy due to rest mass is $= m_e c^2$

$= 9.11 \times 10^{-31}\,\text{kg} \times (3.0 \times 10^{8}\,\text{ms}^{-1})^2$

$= 8.199 \times 10^{-14}\,\text{J}$

$= \frac{8.199 \times 10^{-14}}{1.60 \times 10^{-19}}\,\text{eV}$

$= 5.12 \times 10^{5}\,\text{eV}$

$= 512\,\text{keV}$

$$\therefore \gamma = \frac{\text{Rest mass energy} + \text{KE}}{\text{rest mass energy}}$$

$$= \frac{512 + 635}{512} \approx 2.24$$

From $\cos\left(\frac{\theta}{2}\right) = \sqrt{\left(\frac{\gamma_v - 1}{\gamma_v + 1}\right)}$, largest γ gives largest $\cos\left(\frac{\theta}{2}\right)$ hence smallest $\frac{\theta}{2}$ hence smallest θ

$$\therefore \cos\left(\frac{\theta_{min}}{2}\right) \approx \sqrt{\left(\frac{2.24 - 1}{2.24 + 1}\right)} \approx 0.62$$

$$\Rightarrow \frac{\theta_{min}}{2} = \arccos(0.62) \approx 52^\circ$$

$$\Rightarrow \underline{\underline{\theta_{min} \approx 104^\circ}}$$

(v) Conservation of Energy: $E_{photon} = E_{e^+} + E_{e^-}$

Conservation of Momentum: $p_{photon} = p_{e^+} + p_{e^-}$

Conservation of energy suggests possible if $E_{photon} \geqslant 2m_e c^2$ (i.e. greater than or equal to the sum of the rest mass energies of the electron and positron).

~~For~~ To consider momentum, transform to frame of reference (ZMF) in which the two photons are emitted 'back to back' so $p'_{e^+} + p'_{e^-} = 0$ in this frame, But conservation of momentum $\Rightarrow p_{photon} = 0$. If photon has no momentum it has no energy and does not exist since $E = pc$ for a massless particle.

$\therefore$ process cannot occur as such.

Note: can also treat explicitly in lab frame to give a contradiction, but this requires more algebra.