British Physics Olympiad 2018-19

Round 2 Competition Paper

Monday 28th January 2019

Instructions

Time: 3 hours (approximately 1 hour for Qu. 1 and then 40 minutes per question).

Questions: All four questions should be attempted.

Marks: Questions 2, 3, 4 carry similar marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets, and graph paper should be provided. Students should ensure their name and school is clearly written on all answer sheets and pages are numbered. **Each question should be started on a new page**. A standard formula booklet with standard physical constants should be supplied.

Instructions: To accommodate students sitting the paper at different times, please do not discuss any aspect of the paper on the internet until 8 am Saturday 2nd February. **This paper must not be taken out of the exam room**.

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam paper.

Training Dates and the International Physics Olympiad

Following this round, the best students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the **Training Camp** *to be held in the Physics Department at the University of Oxford,* **(Saturday 13th April to Wednesday 17th April 2019)**. *Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training. From May there will be mentoring by email to cover some topics and problems. There will be a weekend* **Experimental Training Camp in Oxford 17th – 19th May (Friday evening to Sunday afternoon)**, followed by a **Training Camp in Cambridge beginning on Thursday 27th June.**

The IPhO this year will be held in Tel Aviv, Israel, from 7th to 15th July 2019.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Mass of an electron	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of a neutron	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of a proton	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Radius of a nucleon	r_0	$1.2 \times 10^{-15}\,\mathrm{m}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Molar gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Avogadro constant	N_A	$6.02 \times 10^{23}\,\mathrm{mol^{-1}}$
Specific heat capacity of water	c_w	$4.19 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Mass of the Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth	R_E	$6.38 \times 10^{6}\,\mathrm{m}$

BPhO Sponsors

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Qu 1. General Questions

(a). A powerboat instructor has mass $60\,\mathrm{kg}$. With the bow (front) of her boat just touching the harbour, she removes the outboard motor from the stern (back) and intends to carry it along the length of the boat to shore. Her boat is $5\,\mathrm{m}$ long with a mass of $180\,\mathrm{kg}$. The outboard motor has a mass of $40\,\mathrm{kg}$. As she walks she realises that she has forgotten to affix the boat to shore. Determine how far from the shore she will be when she reaches the bow of the boat in the cases that:

(i) Frictional forces between the boat and water can be ignored;

(ii) the frictional force between the boat and water is proportional to the velocity of the boat.

(b). The BepiColumbo mission to Mercury launched in October 2018 is only the second satellite destined to enter an orbit around Mercury, 6 years after launch in 2024. Given that, at their closest, Mercury and Mars are approximately equidistant from the Earth, explain why a mission to put a satellite in orbit around Mercury requires significantly more energy expenditure than a similar mission to Mars.

(c). A paper sphere is constructed from thin spherical lunes of paper glued together, as illustrated in **Fig. 1**, with a photo of the constructed paper sphere shown in **Fig. 2 (a)**. There is a small hole shown in the top end of the sphere through which air can enter or leave. The paper is very thin, but krinkly and stiff. It supports itself as a sphere when resting on the table, as in **Fig. 2 (c)**.

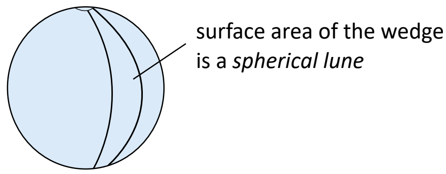

Figure 1

(8)

(a) (b) (c)

Figure 2: Sphere courtesy of NaRiKa Corporation http://www.narika.jp

When a crumpled sphere is bounced gently from one hand to another as in **Fig. 2 (b)**, it gradually takes on a spherical shape, filling with air. Suggest a possible explanation for why it might become filled with more air rather than crumple even more.

(d). A class II 1 mW laser, suitable for use in the classroom, is deemed safe enough in the unfortunate case that it is shone directly in the eye, due to the blink reflex. The diameter of such a laser beam is 6 mm when it reaches the eye, and we can assume that the intensity is uniform across the beam. For an eye focusing the beam on the retina, explain why the beam does not focus to a point, and estimate the factor by which the intensity of the beam is increased on the retina. Make clear any numerical values that you estimate.

(e). Estimate the number of protons that you could pour into an empty teacup before it overflowed. The protons and the cup do not interact electrostatically, but the protons cannot leak through the cup.

(f). The composition of dry air by volume is 78% nitrogen, 21% oxygen and 1% argon, with atomic masses $14.0, 16.0$ and 39.9 respectively. The density of dry air is $1.23\,\mathrm{kg\,m^{-3}}$ at $15\,^{\circ}\mathrm{C}$. Calculate the total kinetic energy of the molecules of $1\,\mathrm{m}^3$ of air under these conditions.

Qu 2. Mechanical Toy

A simple balance toy is made as shown in **Fig. 3**. Two masses, m, are mounted at the end of symmetrical arms of length ℓ each at an angle θ to the body of the toy at a height h above the pivot point. Initially we shall take the body of the toy to be of zero mass.

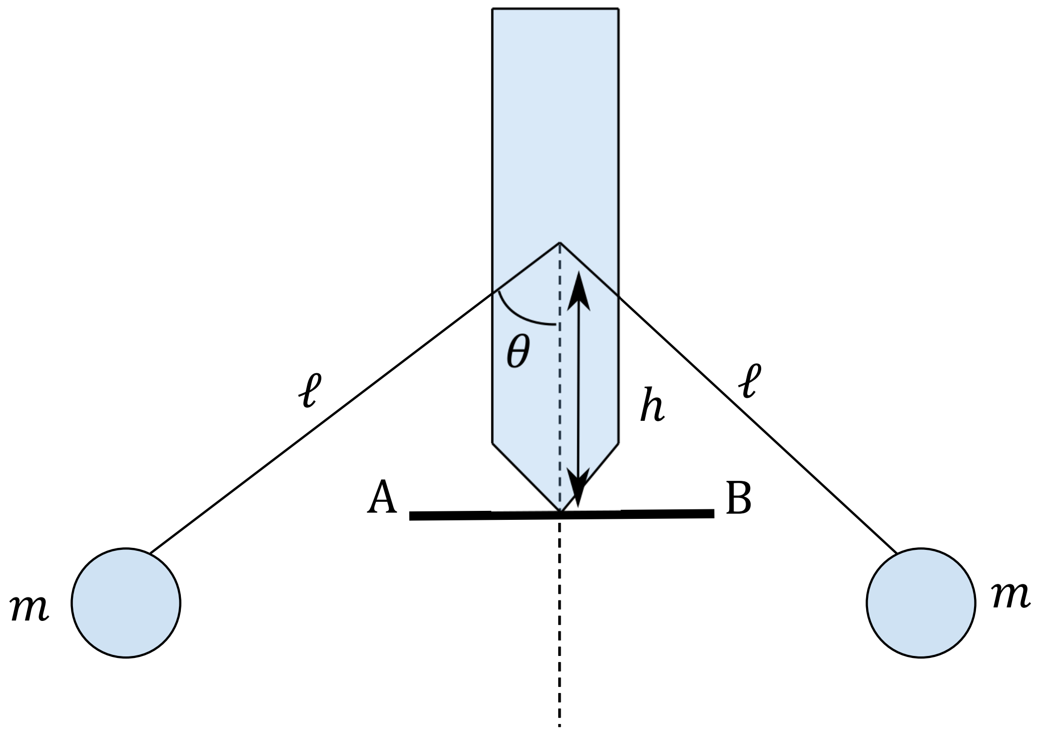

Figure 3

(a). Taking the zero of potential energy as the point where the arms connect to the body of the toy, find an expression for the potential energy, U, of the system if the toy is tilted clockwise through an angle ϕ. A diagram will be helpful.

(b). For the system to be stable, we expect $\frac{\mathrm{d}U}{\mathrm{d}\phi}$ to be zero. Find the angle for which this is true and explain why you reject other solutions.

(c). Further, we require $\frac{\mathrm{d}^2U}{\mathrm{d}\phi^2}$ to be positive.

(i) Explain why the second derivative must be positive.

(ii) Using this condition, explain where the masses must be relative to the toy for stability.

(d). For a small sideways displacement of the point of contact, $\phi \approx x/h$. Use this and the small angle approximation to find the potential and kinetic energy of the toy in terms of x and $\dot{x}$.

(e). Compare your expressions to the equivalent terms for the potential and kinetic energies of a mass on a spring undergoing SHM. Explain why you would expect the toy to undergo SHM for small displacements. Using the expression for the angular frequency of a mass on a spring, $\omega = \sqrt{\frac{k}{m}}$, find an equivalent expression for the toy, by analogy.

(e). Repeat the analysis but allow the body of the toy itself to have a mass M and a centre-of-mass level with the point where the arms contact it. How will the new oscillation frequency compare with the previous one, assuming the system is still stable?

Qu 3. Sisyphus's Coffee

Ben often drinks coffee, but finds that he rarely finishes his cup completely; the coffee is normally too cold by the end for him to drink it all up. This situation puzzles him and, as he is a physicist, he decides to model it. As the coffee cools to a more comfortable temperature, Ben can drink it faster, but when it goes below a critical temperature, T_F, it is too cold and he stops drinking it. He therefore models the rate of drinking as

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{k}{T - T_F} \qquad \text{for} \quad T \geq T_F \tag{1}$$

where $V(t)$ is the volume and $T(t)$ the temperature at time t. k is a positive constant parameterising how fast he is drinking.

(a). Explain how Ben's model accounts for the variation in his rate of drinking and suggest, with reasoning, a value for k.

(b). A picture of Ben's cup is shown below *on the left*. Identify and discuss the physical factors of the coffee and mug you would expect the rate of heat loss to depend on, and the relative contributions they might make. Express the rate of *cooling* as a function of these factors, defining any symbols that you use. *Hint: assume the simplest reasonable dependence on the variables in question. You may also assume his mug is cylindrical.*

Figure 4: *Left:* Ben's cup. *Right:* A cup Ben sees at a conference. Image credits: GAMA Electronics, Thermoserv mug, amazon.com (left); Hotpack, Paper Corn Cup, alibaba.com (right)

(c). Show that Ben's model produces behaviour which accounts for his observations and estimate the amount of coffee he wastes with each mug he drinks, explaining your reasoning carefully. *Hint:* $\frac{\mathrm{d}^2V}{\mathrm{d}t^2} = \frac{\mathrm{d}V}{\mathrm{d}t}\frac{\mathrm{d}}{\mathrm{d}V}\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right)$

(d). Ben travels to a conference where he notices that, in an effort to get participants to drink their coffee up during the break, rather than taking it with them into the lecture theatre, it is being served in cups like the one *on the right* in the image above. The cups are made of polystyrene. Ben does not wish to accept a cup of coffee unless he is certain that he can drink it all up before it goes cold, otherwise he will be stuck with liquid in a cup and nowhere to put it. Explain whether or not you would advise him to accept a coffee. As a physicist, Ben will only be convinced by coherent quantitative reasoning.

Qu 4. Stable Nuclei

A nucleus contains A nucleons and Z protons. Its mass, $M(A, Z)$ can be 'explained' using the semi-empirical mass formula:

$$M(A,Z) \approx (A-Z)m_{\mathrm{n}} + Zm_{\mathrm{p}} - a_1 A + a_2 A^{\frac{2}{3}} + a_3 Z^2 A^{-\frac{1}{3}} + a_4 (A-2Z)^2 A^{-1} + a_5 \delta(A,Z) \quad (2)$$

where

$$\begin{aligned} a_1 &= 15.8\,\mathrm{MeV/c^2} \\ a_2 &= 18.3\,\mathrm{MeV/c^2} \\ a_3 &= 0.714\,\mathrm{MeV/c^2} \\ a_4 &= 23.2\,\mathrm{MeV/c^2} \\ a_5 &= 12\,\mathrm{MeV/c^2} \end{aligned} \quad (3)$$

and

$$\delta(A,Z) \quad : \quad \begin{cases} \delta(\text{even, even}) = -A^{-\frac{1}{2}} \\ \delta(\text{even, even}) = +A^{\frac{1}{2}} \\ \delta(\text{odd, even/odd}) = 0 \end{cases} \quad (4)$$

This model is partly based on theory and partly on experiment, with A and Z taken to vary continuously.

(a). (i) Find an expression for the binding energy of the nucleus, $BE(A, Z)$. The $\delta(A, Z)$ term captures the tendency of nucleons to form pairs in the nucleus so is often referred to as the 'pairing' term. Suggest, with reasoning, the physical origins of the other terms in your expression, giving as much detail as you can. It may help to know that the electrostatic energy of a spherical distribution of charge Q with radius r is $E = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 r}$.

(ii) The term in equation (2) which is accompanied by the constant a_3 is often written alternatively as $a_3 Z(Z-1)A^{-\frac{1}{3}}$. Suggest why this might be.

(b). (i) Represent the binding energy graphically as a function of Z for a given mass number, justifying your reasoning. How many protons does it predict for the most stable nuclei? Show that $Z \approx \frac{A}{2}$ is a decent approximate value.

(ii) Figure 5, on the next page, shows the contributions of the various terms, with the exception of the pairing term, to the binding energy per nucleon as a function of mass number. Considering nuclei with an odd number of nucleons, predict the mass number of the most stable nuclei showing your method clearly.

(c). By analogy with the electrostatic energy of a spherical distribution of charge, $E = \frac{3}{5}\frac{Q^2}{4\pi\varepsilon_0 r}$, include a term for the gravitational energy in your expression for the binding energy per nucleon. Continuing to assume that the number of nucleons is odd, use your new expression to estimate the minimum mass for which a neutron star can be stable.

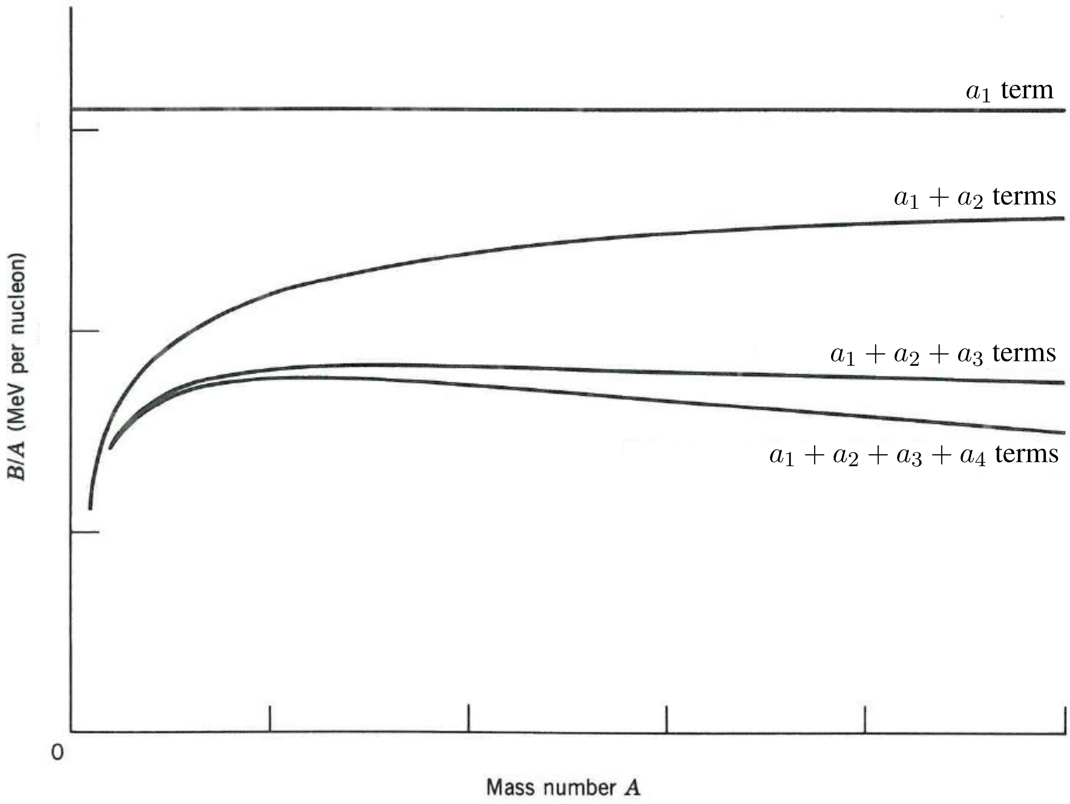

Figure 5: The contributions of the first four terms of the semi-empirical mass formula to the binding energy per nucleon as a function of mass number. Credit: Krane, K. S.: Introductory Nuclear Physics, Wiley (1987) p.69

(d). Returning to **(b)**, make a less basic approximation of the value of Z for which nuclei are stable, and hence a less basic prediction for the mass number of the most stable nuclei.

Hint: Halley's method is a root-finding algorithm which rapidly converges on the root of an equation $f(x) = 0$. If $x = x_n$ is an approximation to a root of $f(x) = 0$, then

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2(f'(x_n))^2 - f(x_n)f''(x_n)} \tag{5}$$

is a better approximation. Here, $f'(x_n)$ refers to the first derivative of $f(x)$ with respect to x, evaluated at $x = x_n$. Likewise $f''(x_n)$ is the second derivative of $f(x)$ with respect to x, evaluated at $x = x_n$

END OF PAPER

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Questions proposed by:
Dr James Bedford (Harrow School)
Dr Benjamin Dive (Austrian Academy of Sciences)