British Physics Olympiad 2021-22

Round 2 Competition Paper

Monday 31st January 2022

Instructions

Time: 3 hours (Q1 approximately 20-30 minutes, Q2, Q3 approx 45 minutes each, Q4 1 hour).

Questions: All four questions should be attempted.

Marks: The questions carry different marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets, and graph paper should be provided. Students should ensure their name and school is clearly written on all answer sheets and **pages are numbered**. A standard formula booklet with standard physical constants should be supplied.

Instructions: To accommodate students sitting the paper at different times, please do not discuss any aspect of the paper on the internet until 8 am Saturday 5th February.

This paper must not be taken out of the exam room.

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will not be marked and overall clarity is an important aspect of this exam paper.

Training Dates and the International Physics Olympiad

Following this round, the best students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the **Training Camp** *to be held in the Physics Department at the University of Oxford* **(Saturday 9th April to Wednesday 13th April 2022)**. *Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training. From May there will be mentoring by email to cover some topics and problems. There will be a weekend* **Experimental Training Camp in Oxford in May (Friday evening to Sunday afternoon)**, followed by a **training camp in Cambridge at the beginning of July.**

The IPhO this year will be held in Belarus in July 2022.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Mass of an electron	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of a neutron	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of a proton	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Radius of a nucleon	r_0	$1.2 \times 10^{-15}\,\mathrm{m}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Molar gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Specific heat capacity of water	c_w	$4.19 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Mass of the Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth	R_E	$6.38 \times 10^{6}\,\mathrm{m}$

Qu 1. General Questions

You are expected to write clear physics, to suggest a model, and to suggest ideas for these effects in Q1. You may find it helpful to bullet point your answers.

(a) In 1998 the McLaren F1 became the world's fastest production car, achieving a top speed of 240 miles per hour. If the maximum useful power developed by the engine is $461\,\mathrm{kW}$, estimate the drag coefficient of the McLaren F1. The drag force is proportional to v^2, the cross-sectional area of the car, and the density of air. The drag coefficient is then the constant of proportionality.

Figure 1: McLaren F1 sports car. Credit: Craig James (en.wikipedia.org/wiki/McLaren_F1)

(b) A resistance R is placed in series with an alternating voltage source, of negligible internal resistance, which has an amplitude V_0 and frequency f. Determine an expression for the average power dissipated by the resistor. If the source can supply a total energy E, calculate how long the source lasts before being drained.

(c) A bag of dried sage and onion stuffing is unpacked from its box. It consists of dried organic particles of a range of sizes. On examination it is seen that the larger particles in the mixture are on top. Suggest, based on your knowledge of physics, why this might be so.

Figure 2: A bag of dried sage and onion stuffing. Credit: James Bedford

(d) A car is left outside overnight in the cold when there is a frost (below zero). Explain why the glass front windscreen of the car can form a thicker layer of ice, compared to that on the side windows of the car.

Qu 2. Stellar Interiors

This question explores nuclear fusion and the interior structure of stars.

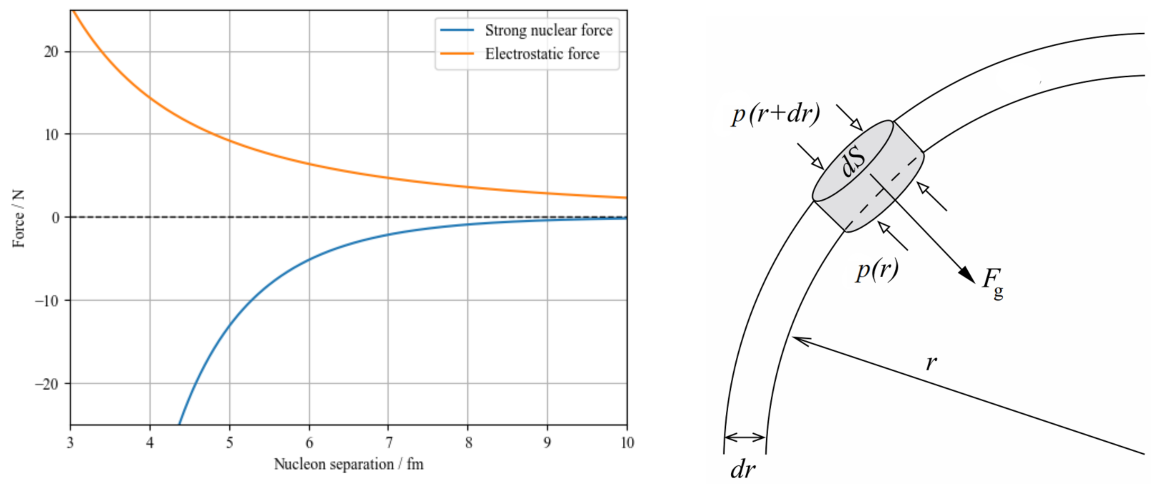

Figure 3: Left: Forces between two protons. Right: Fluid parcel within a spherical shell. Credit: Rupert Allison

(a) As shown in Figure 3, two protons separated by 7 fm experience a mutual electrostatic repulsion of about 5 N. Verify this fact by calculation.

(b) Make a rough copy of the graph of Figure 3 and add your own sketch to it to show the *resultant* force between two protons, as a function of their separation. Explain why very high temperatures are required for fusion to occur. Hence estimate the minimum temperature for proton-proton fusion to occur.

(c) Suggest why your answer to part **(b)** is likely to be an overestimate.

(d) The gravitational field strength within a star is given by $g(r) = -Gm(r)/r^2$, where $m(r) = \int_0^r 4\pi r'^2 \rho(r')dr'$ is the mass enclosed at radius r and $\rho(r)$ is the gas density, i.e. the force of attraction at radius r is only due to the mass of the sphere that lies within radius r. By considering the forces on the fluid parcel indicated in the right-hand panel of Figure 3, show that the pressure gradient within a star in *hydrostatic equilibrium* is given by

$$\frac{dp}{dr} = \rho(r)g(r) \tag{1}$$

and thereby argue that the pressure decreases with increasing radius throughout the star.

(e) The Sun has mass $M = 2 \times 10^{30}$ kg and radius $R = 7 \times 10^5$ km. By taking crude averages (across the whole star) for each of the quantities in Eq. 1, estimate the pressure and temperature of the Sun's core. Is it plausible that the Sun is powered by nuclear fusion? Do protons move relativistically in the Sun's core?

(f) Modelling the Sun with the following density profile, determine more careful estimates for the pressure and temperature at the Sun's core:

$$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right) \tag{2}$$

where ρ_0 is the density at the Sun's core.

(g) Almost all stars we observe have a mass M in the range $0.1M_\odot < M < 100M_\odot$, where $M_\odot$ is the mass of our Sun. Suggest an explanation for this.

Qu 3. The Runner's Pace Function

The *pace function* of a runner specifies how much time she takes to run a certain horizontal distance, according to the slope of the ground on which she travels.

A particular runner has the following pace function:

$$p(m) = 3.500 + 0.210m + 0.035m^2 \tag{3}$$

where p is measured in min km^{-1}, and m is the gradient of the slope expressed as a percentage. For example: if $m = +1.5$, the ground rises 1.5m vertically for every 100m of horizontal distance travelled; if $m = -4$, the ground falls 4m vertically for every 100m of horizontal distance travelled.

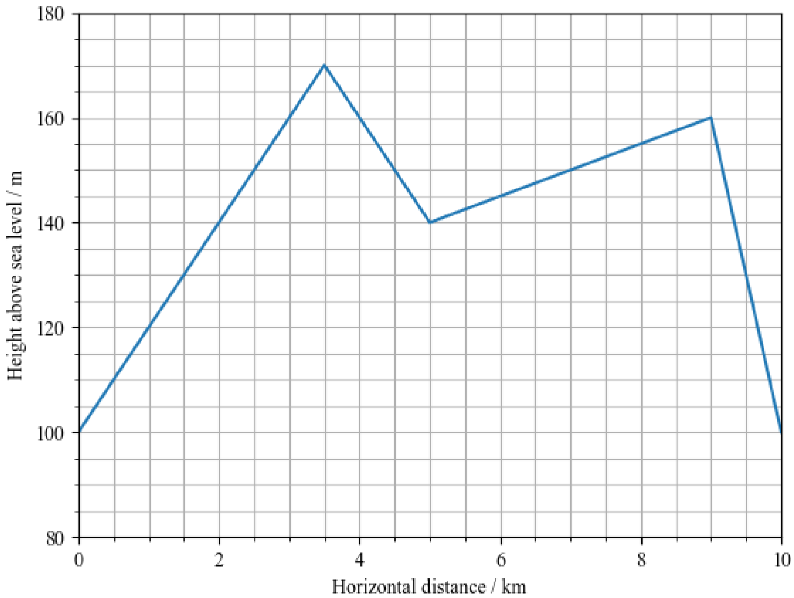

Figure 4: Race elevation profile for Q3(**e**). Credit: Rupert Allison

(a) Suggest why p increases with increasing m for all $m > 0$.

(b) Determine the speed of the runner on flat ground in ms^{-1}.

(c) Determine the gradient on which she runs the fastest, and the corresponding pace in min km^{-1}.

(d) Sketch the pace function for $-15 \leq m \leq 15$, including numerical detail. Would you expect this pace function to be valid for all m?

(e) Figure 4 shows the *elevation profile* of a particular route, which indicates the height above sea level of any particular point along the route. Determine the time the runner takes to run the route and her average pace in min km^{-1}.

(f) Determine the gradient that allows the runner to gain vertical height most rapidly. Mark this *critical gradient* and the corresponding pace on your sketch in part **(d)**.

(g) Another runner has a pace function which is linear, rather than quadratic, in m. For routes that start and finish at the same place, explain the impact that the route elevation profile has in determining their total run time. Does a critical gradient exist in this case?

Qu 4. Cloud Physics

This question explores the formation of clouds in the lower atmosphere.

(a) Consider a sealed flask, connected to a pressure gauge, containing dry air (no water vapour) at atmospheric pressure. A small volume of liquid water is injected into the flask.

(i) With reference to evaporation and condensation, explain why the pressure inside the flask is observed to increase over time, then eventually level-off.

(ii) The flask is now cooled externally. With the aid of a sketch graph, explain qualitatively the subsequent readings on the pressure gauge.

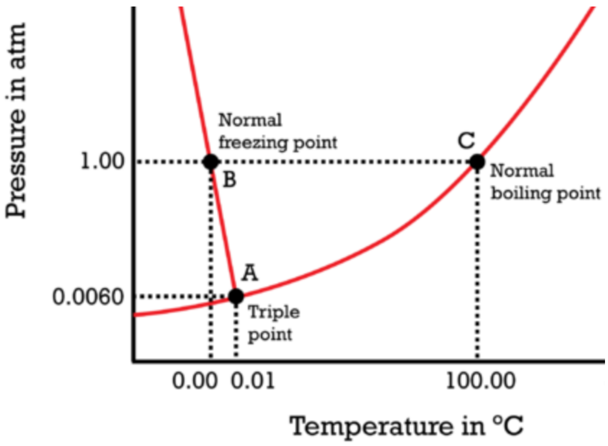

Figure 5: Phase diagram for water. Credit: Adapted from courses.lumenlearning.com/cheminter/chapter/phase-diagram-for-water

Figure 5 shows the so-called *phase diagram* of water.

(b) Sketch out the phase diagram on your answer sheet and mark the regions showing the three possible phases – solid, liquid and vapour – with the letters the S, L and V respectively. Now add arrows to indicate the names of the variety of different phase transitions possible, e.g., melting from S to L.

The coexistence curve $A - C$ shows the possible combinations of pressure p and temperature T for which water exists *in equilibrium* as a mixture of liquid and vapour phases.

The Clausius-Clapeyron equation describes the gradient of this co-existence curve:

$$\frac{dp}{dT} = \frac{LMp}{RT^2} \tag{4}$$

where $L = 2.50 \times 10^6$ Jkg^{-1} is the specific latent heat of vaporisation of water, M is the molar mass of water and R is the molar gas constant. At any given temperature T, the equilibrium pressure exerted by water in its vapour phase is called the *saturation pressure*, $p_{\text{sat}}(T)$.

(c) Using the triple point A, indicated on Figure 5, to provide the relevant initial conditions, show that:

$$\ln\left(\frac{p_{\text{sat}}}{p_0}\right) = \frac{LM}{R}\left(\frac{1}{T_0} - \frac{1}{T}\right) \tag{5}$$

where p_0 and T_0 are the pressure and absolute temperature at the triple point, respectively.

(d) Calculate the saturation pressure of water at 20°C.

Our atmosphere can be effectively modelled as an air/water-vapour mixture of ideal gases. The contribution to the total atmospheric pressure due to the water vapour is called the *partial pressure* of water, denoted p_{w}.

The *relative humidity* ϕ of a parcel of air is defined as the ratio of the partial pressure of the vapour to the saturation pressure, with all quantities evaluated at the temperature of the parcel:

$$\phi(T) = \frac{p_{\mathrm{w}}(T)}{p_{\mathrm{sat}}(T)} \tag{6}$$

(e) With reference to Eq. 6, explain what happens to the relative humidity of an air parcel that rises from the Earth's surface and hence explain how convective clouds form in the lower atmosphere.

For the rest of the question, take the air at sea level to be at 20°C and 35% relative humidity.

(f) Calculate the partial pressure of water vapour in air at sea level. Hence determine the number of water molecules in $1\,\mathrm{m}^3$ of air under these conditions.

(g) Determine the mass fraction of water in the air, making assumptions about the chemical composition of the atmosphere.

(h) Use the following atmospheric data to estimate the altitude at which cloud formation will occur.

Altitude / m	**Atmospheric pressure / atm**	**Temperature / °C**
0	1.00	20
750	0.92	15
1500	0.84	6

(i) Using your understanding developed in the previous parts of the question, and without calculation, suggest the probable atmospheric conditions in Britain when air approaches from each of the three directions, A, B and C, indicated in Figure 6.

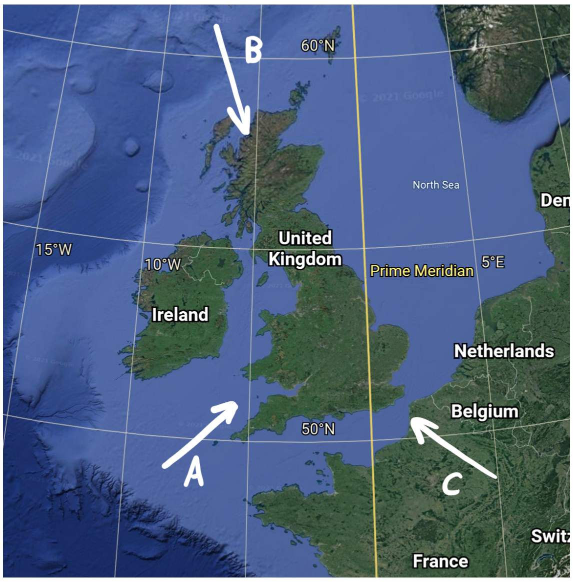

Figure 6: Wind directions for Q4(**i**). Credit: Adapted from Google Earth.

END OF PAPER

Questions proposed by:
Rupert Allison, Harrow School
James Bedford, Harrow School

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