British Physics Olympiad 2023-24

Round 2 Competition Paper

Tuesday 20th February 2024

Instructions

Time: 3 hours (approximately 45 minutes per question).

Questions: All four questions should be attempted.

Marks: The four questions carry similar marks.

Solutions: Answers and calculations are to be written on loose paper or in examination booklets.
Students should ensure their name and school is clearly written on all answer sheets.
A new question should be started on a new page.
Pages must be numbered.

Instructions: Graph paper should be provided.
A standard formula booklet with standard physical constants should be supplied.
To accommodate students sitting the paper at different times, please do not discuss any aspect of the paper on the internet until 8am Monday 26th February.

This paper must not be taken out of the exam room. The paper and any scrap paper and notes must be collected in by the invigilator.

Calculators: Any standard calculator may be used, but calculators must not have symbolic algebra capability. If they are programmable, then they must be cleared or used in "exam mode".

Clarity: Solutions must be written legibly, in black pen (the papers are photocopied), and working down the page. Scribble will definitely not be marked and overall clarity is an important aspect of this competition paper.

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Training Dates and the International Physics Olympiad

Following this round, students eligible to represent the UK at the International Physics Olympiad (IPhO) will be invited to attend the **Training Camp** *to be held in the Physics Department at the University of Oxford,* **(Tuesday 2nd April to Saturday 6th April 2024)**. *Problem solving skills will be developed, practical skills enhanced, as well as some coverage of new material (Thermodynamics, Relativity, etc.). At the Training Camp a practical exam is sat as well as a short Theory Paper. Five students (and a reserve) will be selected for further training. From May there will be mentoring by email to cover some topics and problems. There may be a weekend* **Experimental Training Camp in Oxford during the first weekend of half term in May.(Friday evening - Sunday afternoon)**, followed by a **training camp in Cambridge beginning at the end of June.**

The IPhO this year will be held in Iran in July 2024, and the EuPhO in Georgia, also in July.

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Permittivity of free space	ε_0	$8.85 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Permeability of free space	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Mass of an electron	m_e	$9.11 \times 10^{-31}\,\mathrm{kg}$
Mass of a neutron	m_n	$1.67 \times 10^{-27}\,\mathrm{kg}$
Mass of a proton	m_p	$1.67 \times 10^{-27}\,\mathrm{kg}$
Radius of a nucleon	r_0	$1.2 \times 10^{-15}\,\mathrm{m}$
Planck constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^3\,kg^{-1}\,s^{-2}}$
Boltzmann constant	k	$1.38 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
Molar gas constant	R	$8.31\,\mathrm{J\,mol^{-1}\,K^{-1}}$
Specific heat capacity of water	c_w	$4.19 \times 10^{3}\,\mathrm{J\,kg^{-1}\,K^{-1}}$
Mass of the Sun	M_S	$1.99 \times 10^{30}\,\mathrm{kg}$
Mass of the Earth	M_E	$5.97 \times 10^{24}\,\mathrm{kg}$
Radius of the Earth	R_E	$6.38 \times 10^{6}\,\mathrm{m}$

Qu 1. General Questions

(a). An aircraft is moving around the Earth at 900 km/h in clear daytime conditions. The pilot maintains a course of constant latitude and contrary to the Earth's rotation direction. Determine the latitude at which the pilot observes the Sun to remain in a fixed position in the sky. The equator is at zero latitude and the North pole at 90°.

(b). A bucket of water is tipped out quickly from the top of a tall tower. It arrives at the ground as droplets in the form of rain. Give an explanation for this effect.

(c). Capacitor C_1 has a capacitance $6.0\,\mu\mathrm{F}$ and capacitor C_2 has a capacitance $12.0\,\mu\mathrm{F}$. C_1 is charged in a circuit with an emf of $12.0\,\mathrm{V}$, whilst C_2 is initially charged with an emf of $6.0\,\mathrm{V}$. The two capacitors are then put into a single loop circuit with an open switch, with the same polarities connected together. This is shown in **Fig. 1**. The switch is then closed, and an equilibrium state is reached.

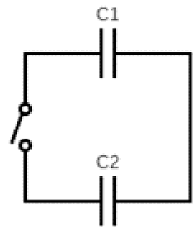

Figure 1

(i) What is the final pd across C_1?

(ii) What fraction of the original energy stored in the capacitors has been dissipated as heat in the wires?

(d). Three particles of equal mass m are free to move under their mutual gravitational interactions. The particles occupy the same fixed circular orbit of radius r, and remain equally spaced throughout the motion. Calculate the orbital period of the system.

(e). The current I through a semiconductor thermistor is measured at a variety of temperatures. The current is given by $I \propto e^{-E_g/k_\mathrm{B}T}$ where T is in kelvin, k_B is the Boltzmann constant, and E_g is the band gap energy in the semiconductor. At $90\,^\circ\mathrm{C}$ the current is $10.90\,\mathrm{mA}$ and at $60\,^\circ\mathrm{C}$ it is $3.97\,\mathrm{mA}$.

(i) Find the value of E_g (in eV)

(ii) Find the current when T is very large

Qu 2. The Physics of Flight

Bernoulli's principle for incompressible and inviscid (zero viscosity) fluids states that

$$\frac{1}{2}\rho v^2 + p = \text{constant} \tag{1}$$

throughout the flow, where ρ is the fluid density, v the fluid speed and p the fluid pressure.

(a) In normal operational flight, air flows over the upper surface of an aircraft wing faster than the bottom surface, as shown in Fig. 2. Use Bernoulli's principle to explain how this generates lift and drag forces on the wing.

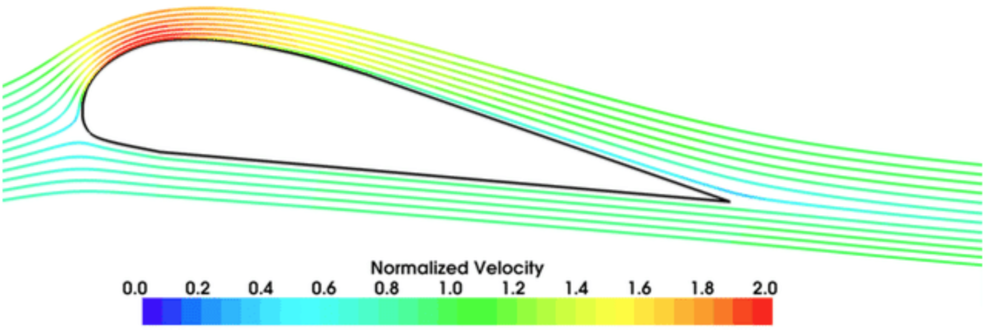

Figure 2: Air flow over an aircraft wing. Source: Becker, Sarah & Bruce, Paul. (2017). Experimental Study of Paraglider Aerodynamics. 10.13140/RG.2.2.33674.16321.

The lift force L and drag force D on an aircraft can be well modelled by

$$L = \frac{1}{2}C_L \rho A v^2 \tag{2}$$

$$D = \frac{1}{2}C_D \rho A v^2 \tag{3}$$

where A is the wing area, ρ is the air density and v is the speed of the aircraft relative to still air. The lift coefficient C_L and drag coefficient C_D both depend upon the wing shape and the *angle of attack* α, which is the angle the wing makes to the undisturbed air-flow direction. This is illustrated in Fig. 3. Typical data for the lift and drag coefficients are shown in Fig. 4.

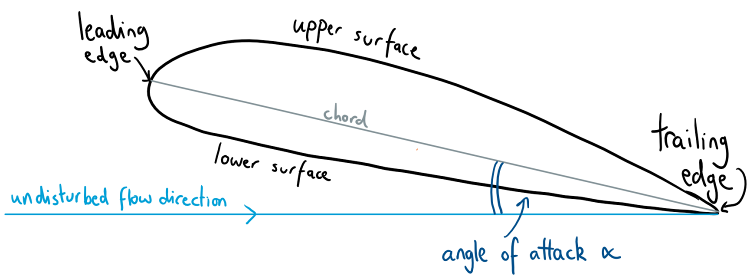

Figure 3: The angle of attack α of a wing is defined as the acute angle subtended by the undisturbed flow direction and the chord of the wing.

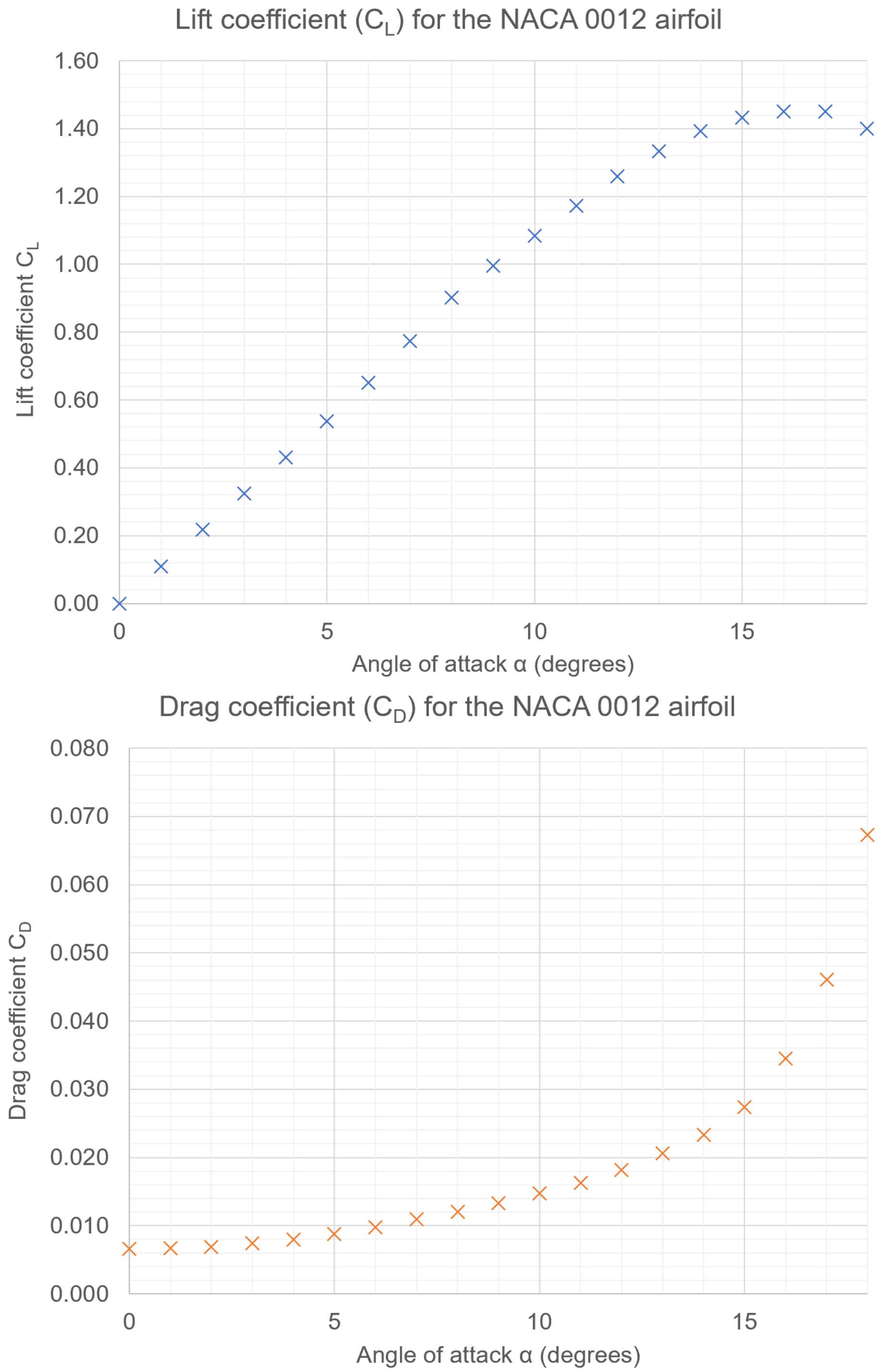

Figure 4: Upper panel: lift coefficient C_L versus angle of attack α for a typical wing shape. Lower panel: drag coefficient C_D versus angle of attack α for the same wing shape. Data source: http://airfoiltools.com/polar/details?polar=xf-n0012-il-1000000-n5

The Airbus A380 is the world's largest passenger aircraft. It has a wing area $A = 845\ \mathrm{m}^2$ and take-off mass $m = 5.60 \times 10^5$ kg. In reality the aircraft mass decreases over time as fuel is burnt but, for simplicity, you should assume this to remain constant.

(b) Shortly after take-off the A380 is moving at $90\,\mathrm{m\,s^{-1}}$ with the wings at an angle of attack of $15°$. Determine the vertical acceleration of the A380. Take the air density at sea-level to be $1.3\,\mathrm{kg\,m^{-3}}$ and use data from **Fig. 4**. Suggest how this vertical acceleration is experienced by a passenger.

(c) At a higher altitude the air density has dropped to $0.80\,\mathrm{kg\,m^{-3}}$. With an angle of attack of $8°$, the A380 flies at constant speed and constant altitude (a condition known to aviators as *steady and level flight*). Determine the engine thrust and speed of the aircraft, and hence the useful power developed by the A380 jet engines.

(d) At its cruising altitude of $11,000\,\mathrm{m}$ the A380 flies at a steady speed of $250\,\mathrm{m\,s^{-1}}$. The air density at this height is $0.36\,\mathrm{kg\,m^{-3}}$. Estimate the necessary angle of attack and the useful power developed by the engines.

(e) An A380 is in steady and level flight. Explain, without calculation, why the drag force can be expected to be large for both very small and very large angles of attack.

(f) Use data from Fig. 4 to estimate the angle of attack that minimises the drag for an A380 in steady and level flight. Suggest the importance to airline companies in knowing this value.

Qu 3. Colourful transparent materials

This question explores the physics of the optical phenomenon demonstrated in Figure 5.

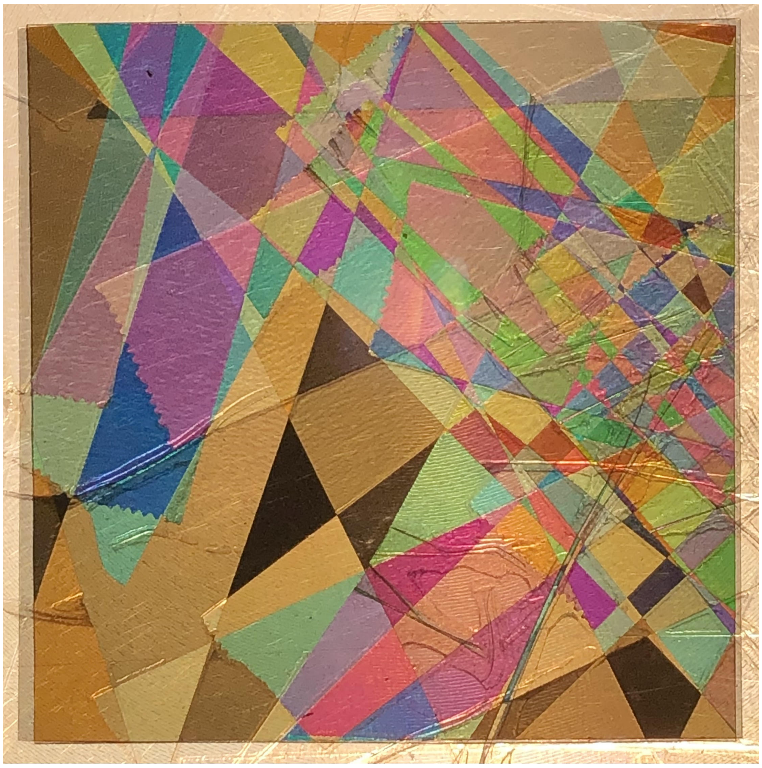

Figure 5: Sandwiched between two crossed polarisation filters, various strips of cellotape (sticky tape) have been set at random orientations, and are in places overlaying one another.

(a) Explain why any source of light appears black when viewed through polarisation filters that are *crossed*, i.e., when the transmission axes of the filters are mutually perpendicular.

(b) Consider a transparent material of thickness t and refractive index n, shown schematically in Figure 6. If light of vacuum wavelength λ strikes the material at normal incidence, argue that the number of wavelengths M that occupy the material is

$$M = \frac{tn}{\lambda}. \tag{4}$$

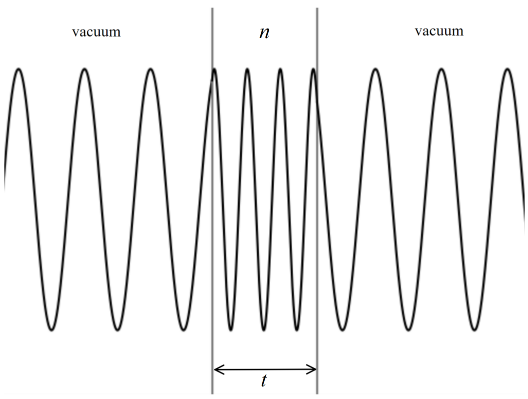

Figure 6: Wavelength compresses when light enters an optically dense material from a vacuum.

Some optical materials display the property of *birefringence*. This means that the speed of propagation of light through the material depends upon the plane of polarisation of that light. Let n_f denote the refractive index for light polarised along the 'fast' axis, and let n_s denote the refractive index for light polarised along the 'slow' axis. These axes are mutually perpendicular and $n_f < n_s$.

(c) Assuming normal incidence, use Eq. 4 to show that the phase difference between the fast and slow waves, as they emerge from a birefringent material of thickness t, is given by

$$\Delta\phi = \frac{2\pi}{\lambda} t \Delta n, \tag{5}$$

where $\Delta n = n_s - n_f$.

The quantity Δn is known as the *birefringence* of the material, and for many materials is well-modelled as a constant value across the visible spectrum. Some examples are given in the Table 1.

Material	**Description**	Δn
Cellotape	Sticky tape	0.0023
Cellophane	Plastic wrap	0.0110
Calcite	Mineral	0.1720

Table 1: Birefringence for various optical media. Source: https://arxiv.org/ftp/arxiv/papers/2206/2206.06983.pdf

The resulting phase difference $\Delta\phi$ between the fast and slow waves can be considered to effectively *rotate* the plane of polarisation of incident light, in a wavelength-dependent fashion. For incident unpolarised light of intensity I_0, uniformly distributed in wavelength, it can be shown that a birefringent material sandwiched between two crossed polarisers will transmit an intensity

$$I = \frac{I_0}{2} \sin^2(2\alpha) \sin^2\left(\frac{\Delta\phi}{2}\right), \tag{6}$$

where $\Delta\phi$ is given by Eq. 5, and α is the angle between the fast axis of the birefringent material and the transmission axis of the first polarising filter. The experimental setup is shown schematically in Figure 7.

(d) State the angles α at which the fast axis of the birefringent material can be set to maximise the transmitted intensity, and explain why there is no transmission when the fast or slow axes are aligned with the transmission axis of the first polarising filter.

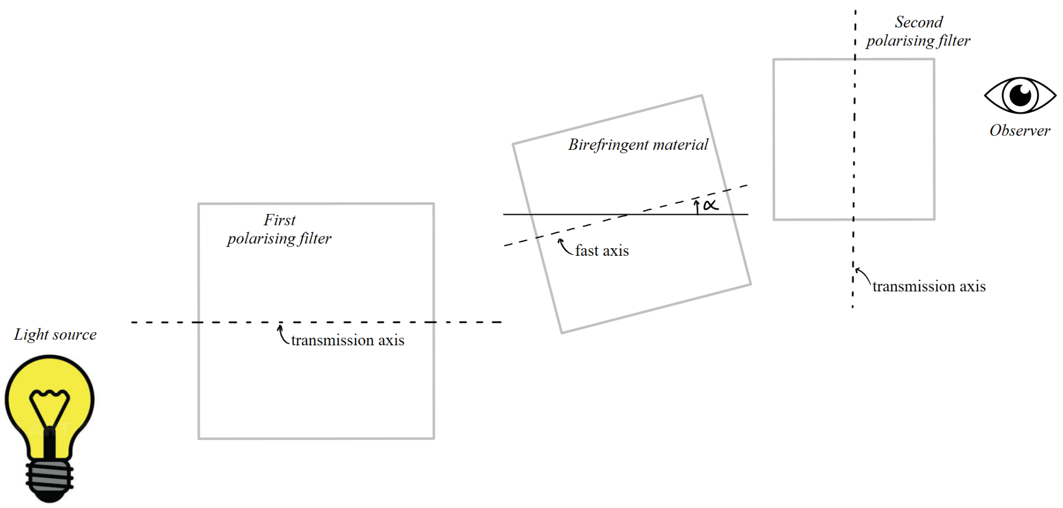

Figure 7: Schematic of experimental setup: a layer of birefringent material is sandwiched between two crossed polarising filters. The light source is viewed at normal incidence through this arrangement.

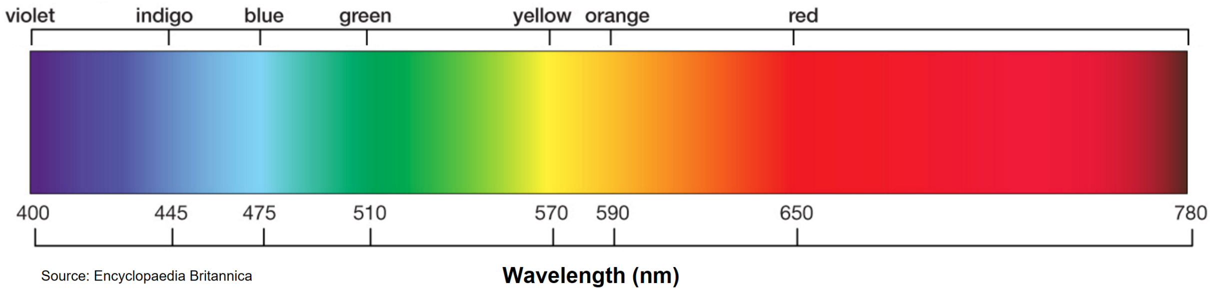

Figure 8: Visible wavelengths roughly span from 400nm to 780nm.

(e) Consider two crossed polarising filters sandwiching a 30 μm layer of cellophane, with its fast axis set at $45°$ to the transmission axes of the polarising filters. Making note of **Figure 8** and **Table 1**, sketch the transmitted intensity across all visible wavelengths. Clearly label your sketch with the letter *E* and suggest the apparent colour of such an arrangement.

(f) Now add to the same axes another sketch, labelled *F*, showing the transmitted intensity for a 90 μm sample of cellophane (again set at $45°$). Suggest the apparent colour of such an arrangement.

(g) Suppose 10 μm of an unknown birefringent material is placed between crossed polarisers. The fast axis is not aligned with the transmission axes of the polarising filters. If the transmitted light is a stark green colour, with almost no blue or red, estimate the birefringence Δn of this material.

(h) Reason why the transmitted light appears white when a very large thickness of birefringent material is used.

(i) With reference to their manufacture, suggest why polymers such as cellotape and cellophane display birefringent properties.

(j) Suggest, experimentally, how you could determine the thickness of a sample of cellophane.

(k) Consider a crystal of calcite placed between two crossed polarisers. Estimate the minimum number of atomic layers of calcite needed to observe colourful birefringent effects. Calcite has a density of 2.71 g cm^{-3} and a molar mass of 100 g mol^{-1}.

Qu 4. Light Bulbs and Fluorescence

This question explores light bulbs and fluorescence.

(a). A fluorescent material absorbs photons and re-emits photons of a longer wavelength. One of its most common applications is in fluorescent lamps, where a UV photon leads to the emission of a visible light photon.

(i) Estimate the efficiency of a fluorescent light bulb. Why will this value necessarily be a maximum?

(ii) The graph below shows the estimated efficiency of incandescent light bulbs as the temperature of the filament varies. The estimation is based on a model where the filament is treated as a non-ideal black body emitting across the visible spectrum.

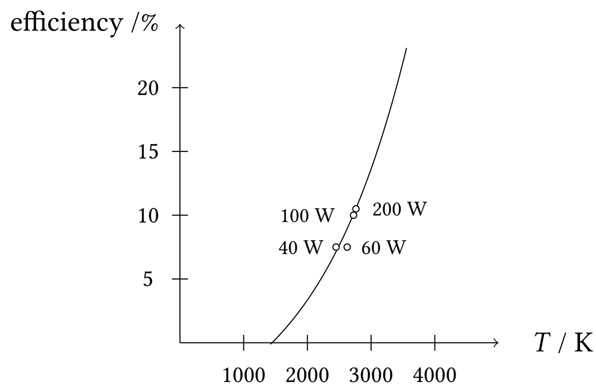

Figure 9: Estimated efficiency vs temperature T of incandescent light bulbs. The open circles show data for 40, 60, 100, and 200 W bulbs.

Compare the efficiency of incandescent bulbs with fluorescent bulbs, commenting on your answer in light of the processes involved.

(b). Electrical conductivity in semiconductors is determined by the susceptibility of electrons to be excited from the valence states ('valence band') to the conduction band ('conduction band'). The difference in energy between these two bands is called the band gap. In an LED, electrons in the conduction band fall to the valence band and emit photons in the process.

(i) Gallium arsenide (GaAs) is a semiconductor with a band gap of $1.42\,\mathrm{eV}$. Fluorescein is a liquid that absorbs light of a wavelength around $494\,\mathrm{nm}$ and fluoresces at around $512\,\mathrm{nm}$. Determine whether a GaAs LED will stimulate fluorescence in fluorescein.

(ii) A laser diode is made from a gallium arsenide-phosphide alloy ($\mathrm{GaAs}_{1-x}\mathrm{P}_x$) with a band gap of $E_g = 2.50\,\mathrm{eV}$. This provides a collimated light source which is used to illuminate a dilute sample of fluorescein that absorbs a proportion ε of the incident photons. The proportion of photons emitted (assumed isotropically) to photons absorbed by a fluorescent material is called its quantum yield, denoted by Φ. A detector of area A connected to a photomultiplier tube is situated at a distance r from the sample, detecting emitted photons and generating a measurable electrical signal. The detector generates electrons via the photoelectric effect which are then amplified by a factor f by the photomultiplier.

If the power of the laser is P_l, find an expression for rate of detection of of photons $\dfrac{\Delta n}{\Delta t}$, explaining your reasoning carefully. You may find that a diagram of the setup is helpful.

(iii) In a particular experiment, the laser has a power of $4\,\mathrm{mW}$, the dilution of fluorescein (whose quantum yield is 0.9) is $1\,\%$, the detector diameter is $1\,\mathrm{cm}$, and the signal is amplified by a factor of 10^6 to give a current of $3\,\mathrm{mA}$. Calculate the distance between the sample and the detector. Why is this likely an overestimate?

(iv) If the uncertainty in the detector area diameter of $\pm 5\,\%$ dominates, estimate the uncertainty in the sample-detector distance.

(c). Fluorescein is an example of a fluorophore, a chemical compound that can re-emit light upon excitation. The rate at which a population of N excited fluorophores decays is proportional (with constant of proportionality k) to the number of excited fluorophores present. The fluorescence lifetime τ is the time taken for a population of N excited fluorophores to decrease to N/e via the loss of energy through fluorescing.

(i) Find an expression for the number of excited fluorophores as a function of time after pulsed excitation, and determine the fluorescence lifetime. What are the units of k and what are the units of τ? Show your working and explain your reasoning.

(ii) A short pulse of laser light is directed towards the sample in described in **(b)**. Following this, the detector current changes from $3\,\mathrm{mA}$ to $0.25\,\mathrm{mA}$ in $10\,\mathrm{ns}$. Calculate the lifetime of fluorescein.

END OF PAPER

Questions set by:
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