# T1: Thermal lens (10 pts)

When an intense laser beam falls onto a semitransparent plate, the transmitted light can self-focus to a point behind the plate due to the inhomogeneous heating of the material. This effect, known as thermal lensing, is observed in materials whose index of refraction increases with temperature, characterized by a positive thermo-optic coefficient $\gamma = \frac{dn}{dT}$.

A semitransparent disk with radius $a = 15.0\,\text{mm}$, thickness $b = 0.2\,\text{mm}$ and optical absorption coefficient $A = 0.1$ is composed of a material having thermal conductivity $k = 0.3\,\text{W}\,\text{m}^{-1}\,\text{K}^{-1}$ and thermo-optic coefficient $\gamma = 2.5 \cdot 10^{-4}\,\text{K}^{-1}$. The outer rim of the disk is thermally connected to a circular metallic holder (not shown in the figure) maintained at a constant temperature $T_\text{h} = 20°\text{C}$. A parallel laser beam of radius $\sigma = 0.5\,\text{mm}$ and power $P_\text{L} = 20\,\text{mW}$ is incident normally onto the center of the disk. The intensity distribution is homogeneous across the cross-section of the beam.
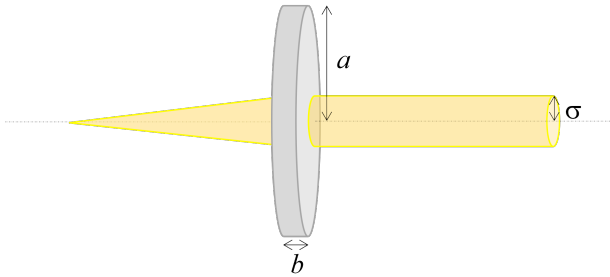

a) (2 pts) Sketch a qualitative graph of the temperature profile $T(r)$, where r denotes the distance from the axis of the beam. Indicate clearly on the graph the illuminated region $0 \leq r \leq \sigma$ and the outer region $\sigma \leq r \leq a$.

b) (4 pts) In the vicinity of the center of the disk, the temperature profile can be represented by a quadratic function $T(r) = T_\text{c} + mr^2$. Calculate the parameters T_c and m.

c) (4 pts) Show that the beam is focused in one point and find the distance f from this point to the disk. If you failed to obtain T_c and m in part b), you may use them as parameters in your final answer.

Do not consider the thermal expansion of the disk. Neglect the thermal radiation and the heat exchange between the disk and the surrounding air. Assume that the index of refraction of air is $n_\text{air} = 1$.

T2: Brick between two planes (10 pts)

A small brick is squeezed between two parallel planes in zero gravity. The planes are perpendicular to the z-axis. The lower plane is moving with constant velocity u_1 along the x-axis, whilst the upper one is moving with constant velocity u_2 along the y-axis.
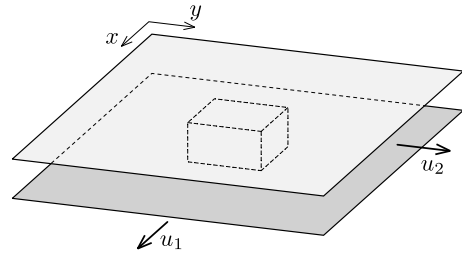

Initially, the brick is at rest. The coefficient of kinetic friction between the brick and each plane is identical.

a) (4 pts) Find the speed of the brick v_∞ after a long time for $u_1 = u_2$.

b) (6 pts) Find the speed of the brick v_∞ after a long time for $u_1 \neq u_2$.

T3: Plate between magnets (10 pts)

Two identical long, cylindrical rod magnets of radius R are close to each other and share the same vertical symmetry axis. The polarity of the two magnets is the same. As a result, the magnetic field in the air gap between the magnets is directed towards the $+z$ direction (see *figure*) and uniform with flux density B. The magnetic field outside the gap is zero. A horizontal large non-magnetic metal plate is placed in the air gap and moved with constant horizontal velocity v in $+y$-direction. The thickness of the plate is δ, the resistivity of the metal is ϱ.
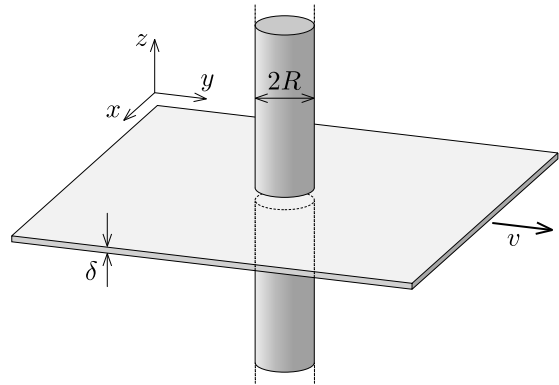
a) (3 pts) Sketch the shape of current streamlines in the metal plate at a given time. Indicate the axes on your sketch.

b) (5 pts) Find and plot the current density inside the plate along a line parallel to the y axis intersecting the symmetry axis of the magnets.

c) (2 pts) Find the horizontal force required to move the plate.