

## 2011 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

### INSTRUCTIONS

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

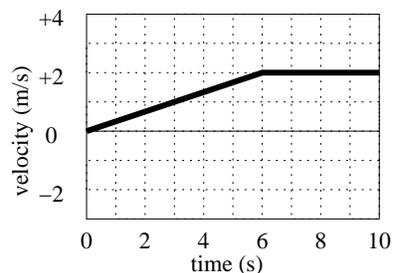
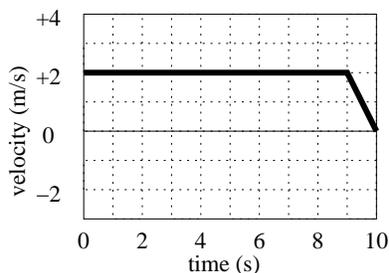
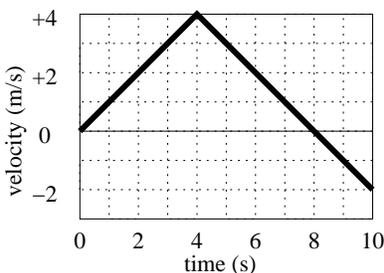
- Use  $g = 10 \text{ N/kg}$  throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers will result in a deduction of  $\frac{1}{4}$  point. There is no penalty for leaving an answer blank.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2011.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

1. A cyclist travels at a constant speed of 22.0 km/hr except for a 20 minute stop. The cyclist's average speed was 17.5 km/hr. How far did the cyclist travel?

- (A) 28.5 km  
 (B) 30.3 km  
 (C) 31.2 km  
 (D) 36.5 km  
 (E) 38.9 km

Questions 2 to 4 refer to the three graphs below which show velocity of three objects as a function of time. Each object is moving only in one dimension.



2. Rank the *magnitudes* of the average acceleration during the ten second interval.

- (A) I > II > III  
 (B) II > I > III  
 (C) III > II > I  
 (D) I > II = III  
 (E) I = II = III

3. Rank the *magnitudes* of the maximum velocity achieved during the ten second interval.

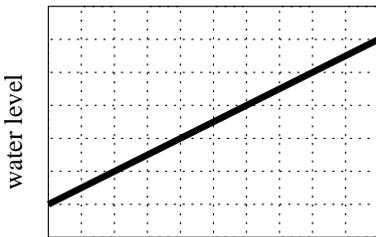
- (A) I > II > III  
 (B) II > I > III  
 (C) III > II > I  
 (D) I > II = III  
 (E) I = II = III

4. Rank the *magnitudes* of the *distance* traveled during the ten second interval.

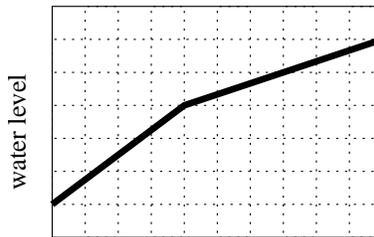
- (A) I > II > III  
 (B) II > I > III  
 (C) III > II > I  
 (D) I = II > III  
 (E) I = II = III

5. A crude approximation is that the Earth travels in a circular orbit about the Sun at constant speed, at a distance of 150,000,000 km from the Sun. Which of the following is the closest for the acceleration of the Earth in this orbit?
- (A) exactly 0 m/s<sup>2</sup>
  - (B) 0.006 m/s<sup>2</sup>
  - (C) 0.6 m/s<sup>2</sup>
  - (D) 6 m/s<sup>2</sup>
  - (E) 10 m/s<sup>2</sup>
6. A child is sliding out of control with velocity  $v_c$  across a frozen lake. He runs head-on into another child, initially at rest, with 3 times the mass of the first child, who holds on so that the two now slide together. What is the velocity of the couple after the collision?
- (A)  $2v_c$
  - (B)  $v_c$
  - (C)  $v_c/2$
  - (D)  $v_c/3$
  - (E)  $v_c/4$
7. An ice skater can rotate about a vertical axis with an angular velocity  $\omega_0$  by holding her arms straight out. She can then pull in her arms close to her body so that her angular velocity changes to  $2\omega_0$ , without the application of any external torque. What is the ratio of her final rotational kinetic energy to her initial rotational kinetic energy?
- (A)  $\sqrt{2}$
  - (B) 2
  - (C)  $2\sqrt{2}$
  - (D) 4
  - (E) 8
8. When a block of wood with a weight of 30 N is completely submerged under water the buoyant force on the block of wood from the water is 50 N. When the block is released it floats at the surface. What fraction of the block will then be visible above the surface of the water when the block is floating?
- (A) 1/15
  - (B) 1/5
  - (C) 1/3
  - (D) 2/5
  - (E) 3/5
9. A spring has an equilibrium length of 2.0 meters and a spring constant of 10 newtons/meter. Alice is pulling on one end of the spring with a force of 3.0 newtons. Bob is pulling on the opposite end of the spring with a force of 3.0 newtons, in the opposite direction. What is the resulting length of the spring?
- (A) 1.7 m
  - (B) 2.0 m
  - (C) 2.3 m
  - (D) 2.6 m
  - (E) 8.0 m

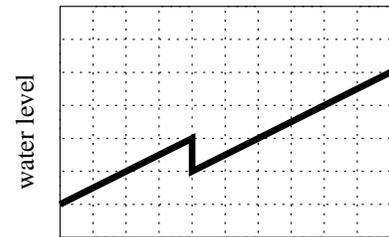
10. Which of the following changes will result in an *increase* in the period of a simple pendulum?
- (A) Decrease the length of the pendulum
  - (B) Increase the mass of the pendulum
  - (C) Increase the amplitude of the pendulum swing
  - (D) Operate the pendulum in an elevator that is accelerating upward
  - (E) Operate the pendulum in an elevator that is moving downward at constant speed.
11. A large metal cylindrical cup floats in a rectangular tub half-filled with water. The tap is placed over the cup and turned on, releasing water at a constant rate. Eventually the cup sinks to the bottom and is completely submerged. Which of the following five graphs could represent the water level in the sink as a function of time?



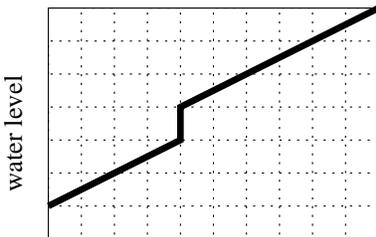
(A)



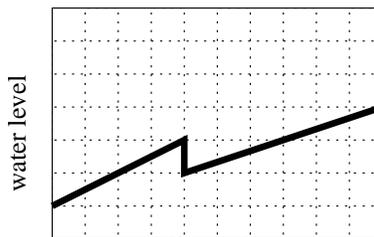
(B)



(C)

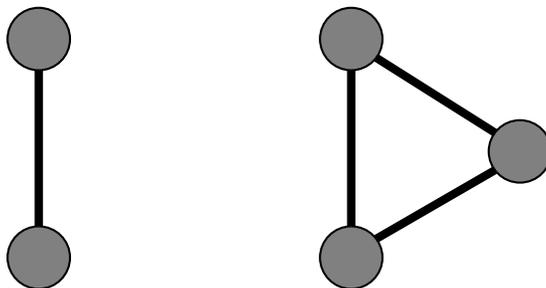


(D)

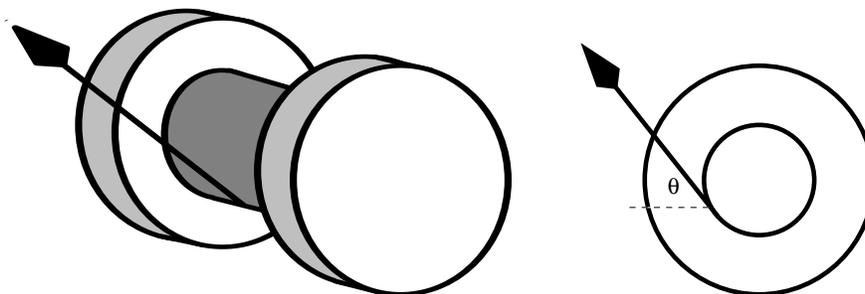


(E)

12. You are given a large collection of identical heavy balls and lightweight rods. When two balls are placed at the ends of one rod and interact through their mutual gravitational attraction (as is shown on the left), the compressive force in the rod is  $F$ . Next, three balls and three rods are placed at the vertices and edges of an equilateral triangle (as is shown on the right). What is the compressive force in each rod in the latter case?

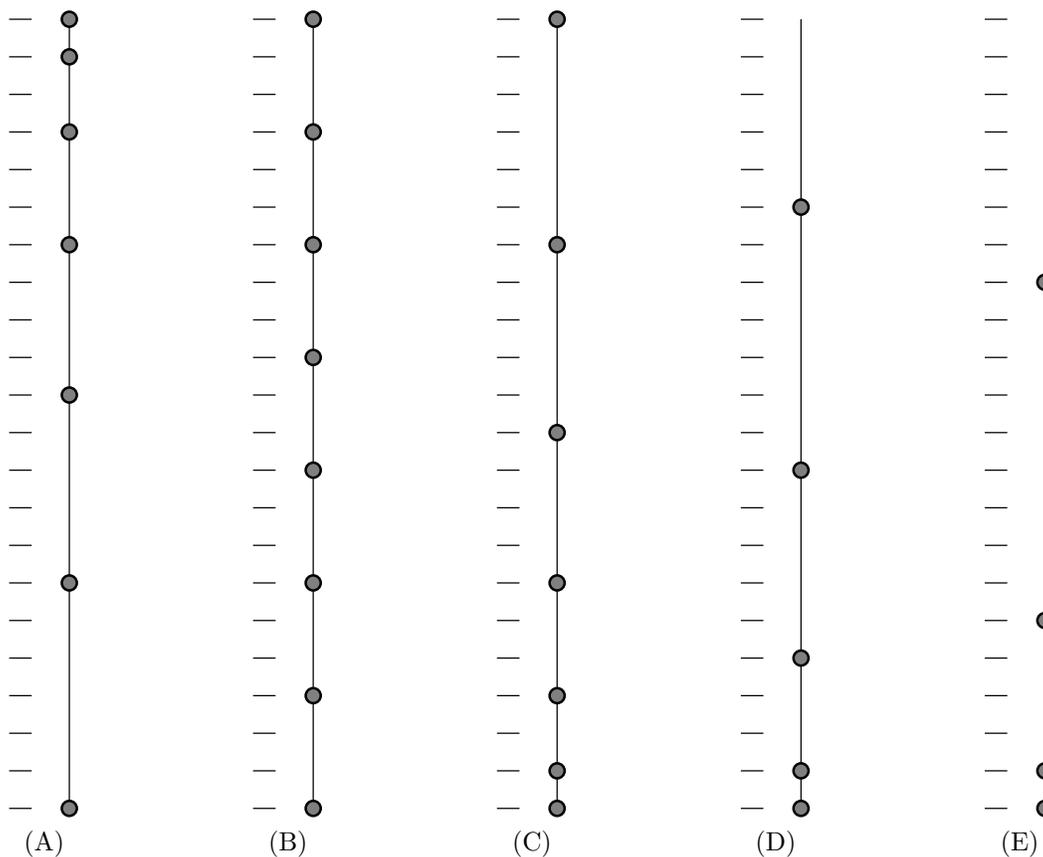


- (A)  $\frac{1}{\sqrt{3}}F$   
 (B)  $\frac{\sqrt{3}}{2}F$   
 (C)  $F$   
 (D)  $\sqrt{3}F$   
 (E)  $2F$
13. The apparatus in the diagram consists of a solid cylinder of radius 1 cm attached at the center to two disks of radius 2 cm. It is placed on a surface where it can roll, but will not slip. A thread is wound around the central cylinder. When the thread is pulled at the angle  $\theta = 90^\circ$  to the horizontal (directly up), the apparatus rolls to the right. Which below is the largest value of  $\theta$  for which it will not roll to the right when pulling on the thread?



- (A)  $\theta = 15^\circ$   
 (B)  $\theta = 30^\circ$   
 (C)  $\theta = 45^\circ$   
 (D)  $\theta = 60^\circ$   
 (E) None, the apparatus will always roll to the right

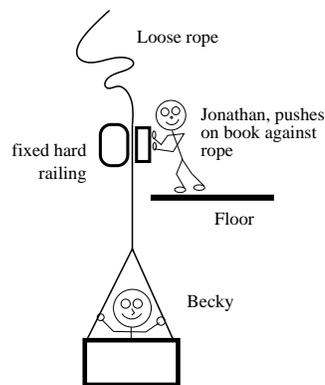
14. You have 5 different strings with weights tied at various point, all hanging from the ceiling, and reaching down to the floor. The string is released at the top, allowing the weights to fall. Which one will create a regular, uniform beating sound as the weights hit the floor?



15. A vertical mass-spring oscillator is displaced 2.0 cm from equilibrium. The 100 g mass passes through the equilibrium point with a speed of 0.75 m/s. What is the spring constant of the spring?
- (A) 90 N/m  
 (B) 100 N/m  
 (C) 110 N/m  
 (D) 140 N/m  
 (E) 160 N/m

Questions 16 and 17 refer to the information and diagram below.

Jonathan is using a rope to lift a box with Becky in it; the box is hanging off the side of a bridge, Jonathan is on top. A rope is hooked up from the box and passes a fixed railing; Jonathan holds the box up by pressing the rope against the railing with a *massless, frictionless* physics textbook. The static friction coefficient between the rope and railing is  $\mu_s$ ; the kinetic friction coefficient between the rope and railing is  $\mu_k < \mu_s$ ; the mass of the box and Becky combined is  $M$ ; and the initial height of the bottom of the box above the ground is  $h$ . Assume a massless rope.



16. What magnitude force does Jonathan need to exert on the physics book to keep the rope from slipping?
- (A)  $Mg$   
 (B)  $\mu_k Mg$   
 (C)  $\mu_k Mg / \mu_s$   
 (D)  $(\mu_s + \mu_k) Mg$   
 (E)  $Mg / \mu_s$
17. Jonathan applies a normal force that is just enough to keep the rope from slipping. Becky makes a small jump, barely leaving contact with the floor of the box. Upon landing on the box, the force of the impact causes the rope to start slipping from Jonathan's hand. At what speed does the box smash into the ground? Assume Jonathan's normal force does not change.
- (A)  $\sqrt{2gH}(\mu_k / \mu_s)$   
 (B)  $\sqrt{2gH}(1 - \mu_k / \mu_s)$   
 (C)  $\sqrt{2gH} \sqrt{\mu_k / \mu_s}$   
 (D)  $\sqrt{2gH} \sqrt{(1 - \mu_k / \mu_s)}$   
 (E)  $\sqrt{2gH}(\mu_s - \mu_k)$
18. A block of mass  $m = 3.0$  kg slides down one ramp, and then up a second ramp. The coefficient of kinetic friction between the block and each ramp is  $\mu_k = 0.40$ . The block begins at a height  $h_1 = 1.0$  m above the horizontal. Both ramps are at a  $30^\circ$  incline above the horizontal. To what height above the horizontal does the block rise on the second ramp?
- (A) 0.18 m  
 (B) 0.52 m  
 (C) 0.59 m  
 (D) 0.69 m  
 (E) 0.71 m

**Questions 19 and 20 refer to the following information**

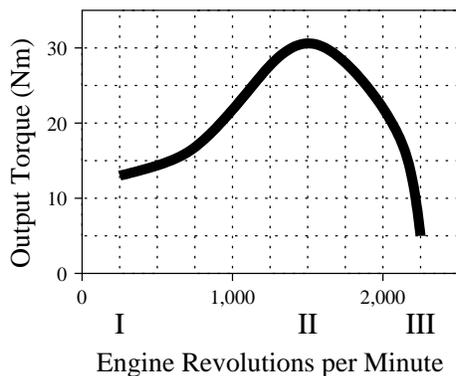
A particle of mass 2.00 kg moves under a force given by

$$\vec{F} = -(8.00 \text{ N/m})(x\hat{i} + y\hat{j})$$

where  $\hat{i}$  and  $\hat{j}$  are unit vectors in the  $x$  and  $y$  directions. The particle is placed at the origin with an initial velocity  $\vec{v} = (3.00 \text{ m/s})\hat{i} + (4.00 \text{ m/s})\hat{j}$ .

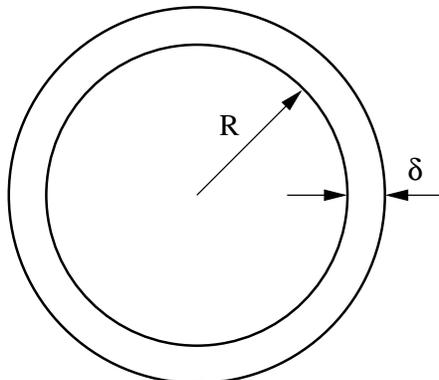
19. After how much time will the particle first return to the origin?
- (A) 0.785 s  
(B) 1.26 s  
(C) 1.57 s  
(D) 2.00 s  
(E) 3.14 s
20. What is the maximum distance between the particle and the origin?
- (A) 2.00 m  
(B) 2.50 m  
(C) 3.50 m  
(D) 5.00 m  
(E) 7.00 m
21. An engineer is given a fixed volume  $V_m$  of metal with which to construct a spherical pressure vessel. Interestingly, assuming the vessel has thin walls and is always pressurized to near its bursting point, the amount of gas the vessel can contain,  $n$  (measured in moles), does not depend on the radius  $r$  of the vessel; instead it depends only on  $V_m$  (measured in  $\text{m}^3$ ), the temperature  $T$  (measured in K), the ideal gas constant  $R$  (measured in  $\text{J}/(\text{K} \cdot \text{mol})$ ), and the tensile strength of the metal  $\sigma$  (measured in  $\text{N}/\text{m}^2$ ). Which of the following gives  $n$  in terms of these parameters?
- (A)  $n = \frac{2}{3} \frac{V_m \sigma}{RT}$   
(B)  $n = \frac{2}{3} \frac{\sqrt[3]{V_m} \sigma}{RT}$   
(C)  $n = \frac{2}{3} \frac{\sqrt[3]{V_m} \sigma^2}{RT}$   
(D)  $n = \frac{2}{3} \frac{\sqrt[3]{V_m^2} \sigma}{RT}$   
(E)  $n = \frac{2}{3} \sqrt[3]{\frac{V_m \sigma^2}{RT}}$

22. This graph depicts the torque output of a hypothetical gasoline engine as a function of rotation frequency. The engine is incapable of running outside of the graphed range.



- At what engine RPM (revolutions per minute) does the engine produce maximum power?
- (A) I
  - (B) At some point between I and II
  - (C) II
  - (D) At some point between II and III
  - (E) III
23. A particle is launched from the surface of a uniform, stationary spherical planet at an angle to the vertical. The particle travels in the absence of air resistance and eventually falls back onto the planet. Spaceman Fred describes the path of the particle as a parabola using the laws of projectile motion. Spacewoman Kate recalls from Kepler's laws that every bound orbit around a point mass is an ellipse (or circle), and that the gravitation due to a uniform sphere is identical to that of a point mass. Which of the following best explains the discrepancy?
- (A) Because the experiment takes place very close to the surface of the sphere, it is no longer valid to replace the sphere with a point mass.
  - (B) Because the particle strikes the ground, it is not in orbit of the planet and therefore can follow a non-elliptical path.
  - (C) Kate disregarded the fact that motions around a point mass may also be parabolas or hyperbolas.
  - (D) Kepler's laws only hold in the limit of large orbits.
  - (E) The path is an ellipse, but is very close to a parabola due to the short length of the flight relative to the distance from the center of the planet.

24. A turntable is supported on a Teflon ring of inner radius  $R$  and outer radius  $R + \delta$  ( $\delta \ll R$ ), as shown in the diagram. To rotate the turntable at a constant rate, power must be supplied to overcome friction. The manufacturer of the turntable wishes to reduce the power required without changing the rotation rate, the weight of the turntable, or the coefficient of friction of the Teflon surface. Engineers propose two solutions: increasing the width of the bearing (increasing  $\delta$ ), or increasing the radius (increasing  $R$ ). What are the effects of these proposed changes?



- (A) Increasing  $\delta$  has no significant effect on the required power; increasing  $R$  increases the required power.  
(B) Increasing  $\delta$  has no significant effect on the required power; increasing  $R$  decreases the required power.  
(C) Increasing  $\delta$  increases the required power; increasing  $R$  has no significant effect on the required power.  
(D) Increasing  $\delta$  decreases the required power; increasing  $R$  has no significant effect on the required power.  
(E) Neither change has a significant effect on the required power.
25. A hollow cylinder with a very thin wall (like a toilet paper tube) and a block are placed at rest at the top of a plane with inclination  $\theta$  above the horizontal. The cylinder rolls down the plane without slipping and the block slides down the plane; it is found that both objects reach the bottom of the plane simultaneously. What is the coefficient of kinetic friction between the block and the plane?
- (A) 0  
(B)  $\frac{1}{3} \tan \theta$   
(C)  $\frac{1}{2} \tan \theta$   
(D)  $\frac{2}{3} \tan \theta$   
(E)  $\tan \theta$