AAPT UNITED STATES PHYSICS TEAM
AIP 2015

2015 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

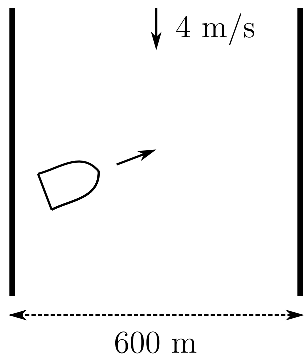
INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2015.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. A 600 meter wide river flows directly south at 4.0 m/s. A small motor boat travels at 5.0 m/s in still water and points in such a direction so that it will travel directly east relative to the land.



The time it takes to cross the river is closest to

(A) 67 s
(B) 120 s
(C) 150 s
(D) 200 s
(E) 600 s

2. A car travels directly north on a straight highway at a constant speed of 80 km/hr for a distance of 25 km. The car then continues directly north at a constant speed of 50 km/hr for a distance of 75 more kilometers. The average speed of the car for the entire journey is closest to

(A) 55.2 km/hr
(B) 57.5 km/hr
(C) 65 km/hr
(D) 69.6 km/hr
(E) 72.5 km/hr

3. The force of friction on an airplane in level flight is given by $F_f = kv^2$, where k is some constant, and v is the speed of the airplane. When the power output from the engines is P_0, the plane is able to fly at a speed v_0. If the power output of the engines is increased by 100% to $2P_0$, the airplane will be able to fly at a new speed given by

   (A) $1.12v_0$
   (B) $1.26v_0$
   (C) $1.41v_0$
   (D) $2.82v_0$
   (E) $8v_0$

4. A 2.0 kg box is originally at rest on a horizontal surface where the coefficient of static friction between the box and the surface is μ_s and the coefficient of the kinetic friction between the box and the surface is $\mu_k = 0.90\mu_s$. An external horizontal force of magnitude P is then applied to the box. Which of the following is a graph of the acceleration of the box a versus the external force P?

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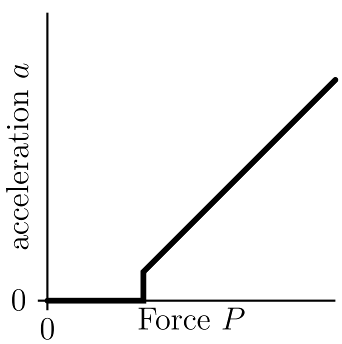

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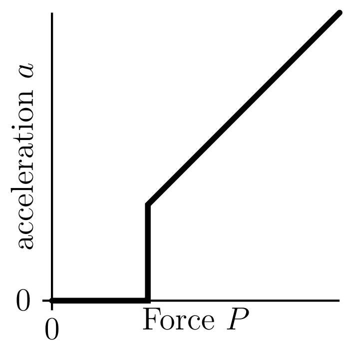

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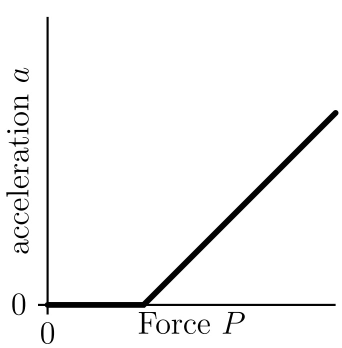

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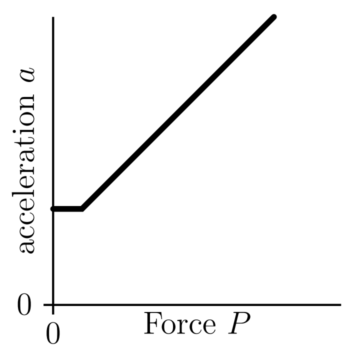

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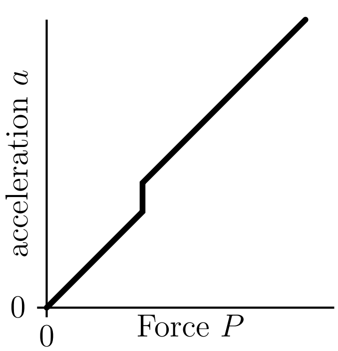
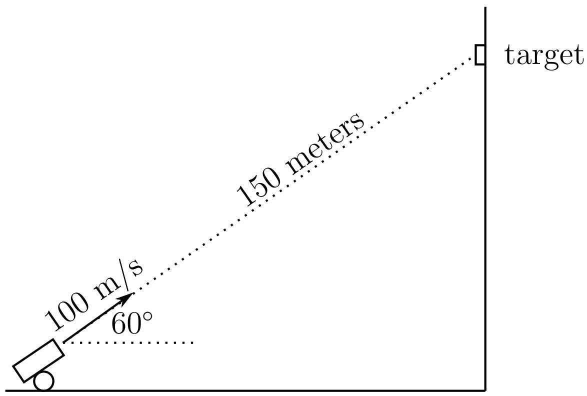


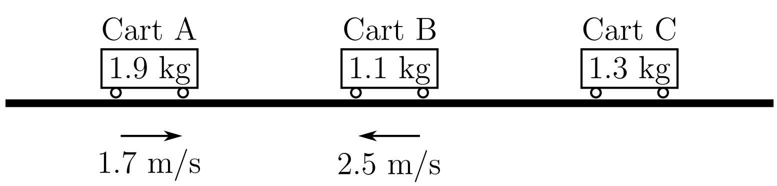
5. A 470 gram lead ball is launched at a 60 degree angle above the horizontal with an initial speed of 100 m/s directly toward a target on a vertical cliff wall that is 150 meters away as shown in the figure.



Ignoring air friction, by what distance does the lead ball miss the target when it hits the cliff wall?

(A) 1.3 m
(B) 2.2 m
(C) 5.0 m
(D) 7.1 m
(E) 11 m

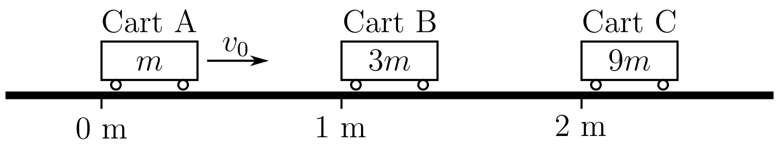
6. Three trolley carts are free to move on a one dimensional frictionless horizontal track. Cart A has a mass of 1.9 kg and an initial speed of 1.7 m/s to the right; Cart B has a mass of 1.1 kg and an initial speed of 2.5 m/s to the left; cart C has a mass of 1.3 kg and is originally at rest. Collisions between carts A and B are perfectly elastic; collisions between carts B and C are perfectly inelastic.



What is the velocity of the center of mass of the system of the three carts after the last collision?

(A) 0.11 m/s
(B) 0.16 m/s
(C) 1.4 m/s
(D) 2.0 m/s
(E) 3.23 m/s

The following information applies to questions 7 and 8



Carts A, B, and C are on a long horizontal frictionless track. The masses of the carts are m, $3m$, and $9m$. Originally cart B is at rest at the 1.0 meter mark and cart C is at rest on the 2.0 meter mark. Cart A is originally at the zero meter mark moving toward the cart B at a speed of v_0.

7. Assuming that all collisions are completely *inelastic*, what is the final speed of cart C?

   (A) $v_0/13$

   (B) $v_0/10$

   (C) $v_0/9$

   (D) $v_0/3$

   (E) $2v_0/5$

8. Assuming that all collisions are completely *elastic*, what is the final speed of cart C?

   (A) $v_0/8$

   (B) $v_0/4$

   (C) $v_0/2$

   (D) v_0

   (E) $2v_0$

The following information applies to questions 9 and 10

A 0.650 kg ball moving at 5.00 m/s collides with a 0.750 kg ball that is originally at rest. After the collision, the 0.750 kg ball moves off with a speed of 4.00 m/s, and the 0.650 kg ball moves off at a right angle to the final direction of motion of the 0.750 kg ball.

9. What is the final speed of the 0.650 kg ball?

   (A) 1.92 m/s

   (B) 2.32 m/s

   (C) 3.00 m/s

   (D) 4.64 m/s

   (E) 5.77 m/s

10. Let the change in total kinetic energy in this collision be defined by $\Delta K = K_f - K_i$, where K_f is the total final kinetic energy, and K_i is the total initial kinetic energy. Which of the following is true?

    (A) $\Delta K = (K_i + K_f)/2$

    (B) $K_f < \Delta K < K_i$

    (C) $0 < \Delta K < K_f$

    (D) $\Delta K = 0$

    (E) $-K_i < \Delta K < 0$

11. A sphere floats in water with 2/3 of the volume of the sphere submerged. The sphere is removed and placed in oil that has 3/4 the density of water. If it floats in the oil, what fraction of the sphere would be submerged in the oil?

    (A) 1/12

    (B) 1/2

    (C) 8/9

    (D) 17/12

    (E) The sphere will not float, it will sink in the oil.

The following information applies to questions 12 and 13

A pendulum consists of a small bob of mass m attached to a fixed point by a string of length L. The pendulum bob swings down from rest from an initial angle $\theta_{\text{max}} < 90$ degrees.

12. Which of the following statements about the pendulum bob's acceleration is true?

    (A) The magnitude of the acceleration is constant for the motion.

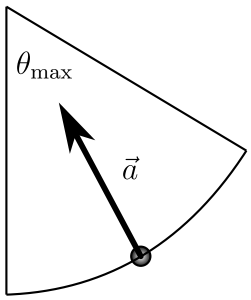
    (B) The magnitude of the acceleration at the lowest point is g, the acceleration of free fall.

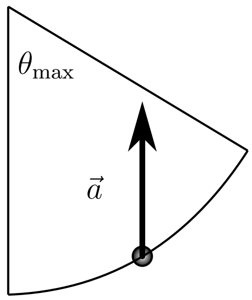
    (C) The magnitude of the acceleration is zero at some point of the pendulum's swing.

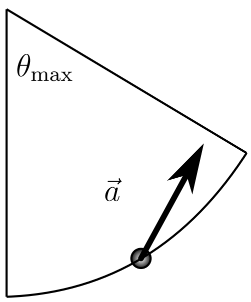
    (D) The acceleration is always directed toward the center of the circle.

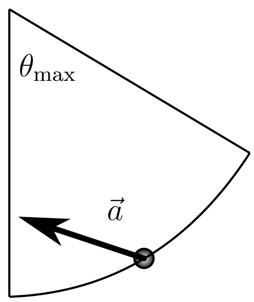
    (E) The acceleration at the bottom of the swing is pointing vertically upward.

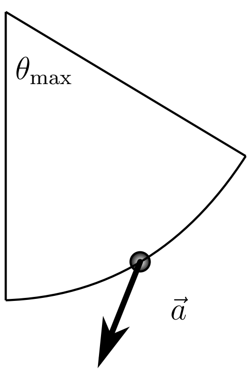
13. Consider the pendulum bob when it is at an angle $\theta = \frac{1}{2}\theta_{\text{max}}$ on the way up (moving toward θ_{max}). What is the direction of the acceleration vector?

(A)


(B)


(C)


(D)


(E)


The following information applies to questions 14 and 15

A 3.0 meter long massless rod is free to rotate horizontally about its center. Two 5.0 kg point objects are originally located at the ends of the rod; they are free to slide on the frictionless rod and are kept from flying off the rod by an inflexible massless rope that connects the two objects.

Originally the system is rotating at 4.0 radians per second; assume the system is completely frictionless; and ignore any concerns about instability of the system.

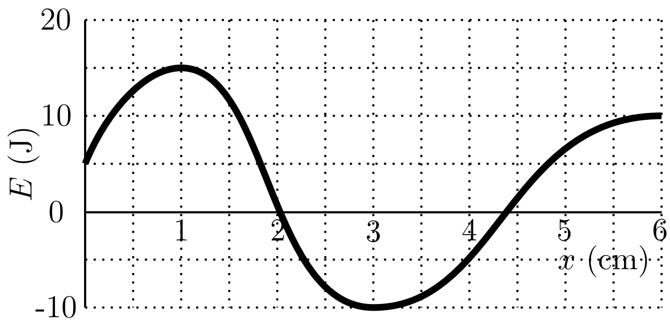
14. Calculate the original tension in the rope.

(A) 60 N
(B) 106 N
(C) 120 N
(D) 240 N
(E) 480 N

15. The rope is slowly tightened by a small massless motor attached to one of the objects. It is done in such a way as to pull the two objects closer to the center of the rotating rod. How much work is done by the motor in pulling the two objects from the ends of the rod until they are each 0.5 meters from the center of rotation?

    (A) 120 J
    (B) 180 J
    (C) 240 J
    (D) 1440 J
    (E) 1620 J

16. Shown below is a graph of potential energy as a function of position for a 0.50 kg object.



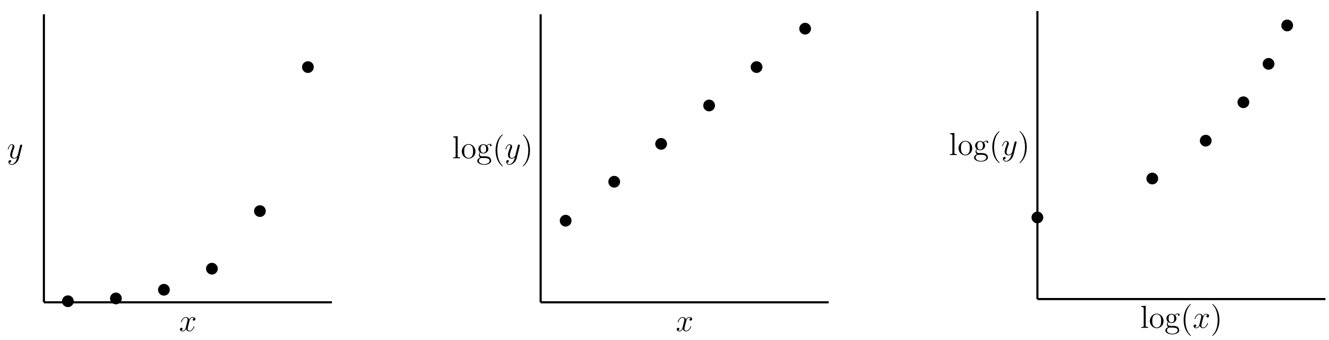
Which of the following statements is NOT true in the range 0 cm $< x <$ 6 cm?

   (A) The object could be at equilibrium at either $x = 1$ cm or $x = 3$ cm.
   (B) The minimum possible total energy for this object in the range is -10 J.
   (C) The magnitude of the force on the object at 4 cm is approximately 1000 N.
   (D) If the total energy of the particle is 0 J then the object will have a maximum kinetic energy of 10 J.
   (E) The magnitude of the acceleration of the object at $x = 2$ cm is approximately 4 cm/s^2.

17. A flywheel can rotate in order to store kinetic energy. The flywheel is a uniform disk made of a material with a density ρ and tensile strength σ (measured in Pascals), a radius r, and a thickness h. The flywheel is rotating at the maximum possible angular velocity so that it does not break. Which of the following expression correctly gives the maximum kinetic energy per kilogram that can be stored in the flywheel? Assume that α is a dimensionless constant.

    (A) $\alpha\sqrt{\rho\sigma/r}$
    (B) $\alpha h\sqrt{\rho\sigma/r}$
    (C) $\alpha\sqrt{(h/r)}(\sigma/\rho)^2$
    (D) $\alpha(h/r)(\sigma/\rho)$
    (E) $\alpha\sigma/\rho$

18. Shown below are three graphs of the same data.



Which is the correct functional relationship between the data points? Assume a and b are constants.

    (A) $y = ax + b$
    (B) $y = ax^2 + b$
    (C) $y = ax^b$
    (D) $y = ae^{bx}$
    (E) $y = a\log x + b$

The following information applies to questions 19 and 20

A U-tube manometer consists of a uniform diameter cylindrical tube that is bent into a U shape. It is originally filled with water that has a density ρ_w. The *total* length of the column of water is L. Ignore surface tension and viscosity.

19. The water is displaced slightly so that one side moves up a distance x and the other side lowers a distance x. Find the frequency of oscillation.

    (A) $\frac{1}{2\pi}\sqrt{2g/L}$
    (B) $2\pi\sqrt{g/L}$
    (C) $\frac{1}{2\pi}\sqrt{2L/g}$
    (D) $\frac{1}{2\pi}\sqrt{g/\rho_w}$
    (E) $2\pi\sqrt{\rho_w g L}$

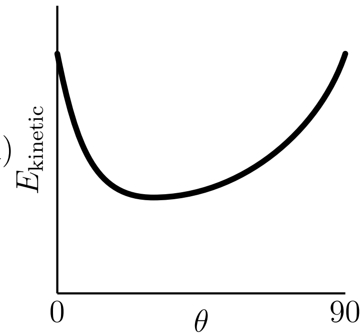
20. Oil with a density half that of water is added to one side of the tube until the total length of oil is equal to the total length of water. Determine the equilibrium height difference between the two sides

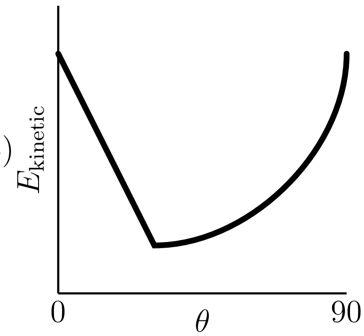
    (A) L
    (B) $L/2$
    (C) $L/3$
    (D) $3L/4$
    (E) $L/4$

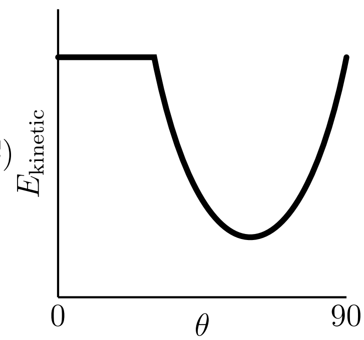
21. An object launched vertically upward from the ground with a speed of 50 m/s bounces off of the ground on the return trip with a coefficient of restitution given by $C_R = 0.9$, meaning that immediately after a bounce the upward speed is 90% of the previous downward speed. The ball continues to bounce like this; what is the total amount of time between when the ball is launched and when it finally comes to a rest? Assume the collision time is zero; the bounce is instantaneous. Treat the problem as ideally classical and ignore any quantum effects that might happen for very small bounces.

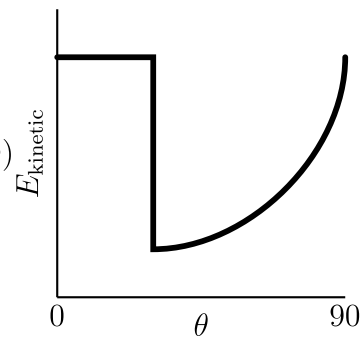
    (A) 71 s
    (B) 100 s
    (C) 141 s
    (D) 1000 s
    (E) ∞ (the ball never comes to a rest)

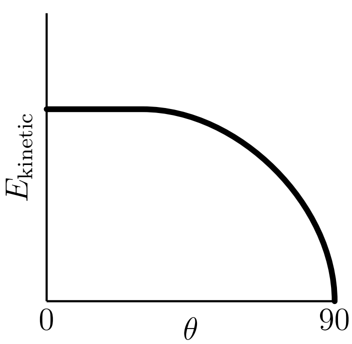
22. A solid ball is released from rest down inclines of various inclination angles θ but through a fixed vertical height h. The coefficient of static and kinetic friction are both equal to μ. Which of the following graphs best represents the total kinetic energy of the ball at the bottom of the incline as a function of the angle of the incline?

(A)


(B)


(C)


(D)


(E)


23. A 2.0 kg object falls from rest a distance of 5.0 meters onto a 6.0 kg object that is supported by a vertical massless spring with spring constant $k = 72$ N/m. The two objects stick together after the collision, which results in the mass/spring system oscillating. What is the maximum magnitude of the displacement of the 6.0 kg object from its original location before it is struck by the falling object?

(A) 0.27 m

(B) 1.1 m

(C) 2.5 m

(D) 2.8 m

(E) 3.1 m

24. The speed of a transverse wave on a long cylindrical steel string is given by

$$v = \sqrt{\frac{T}{M/L}},$$

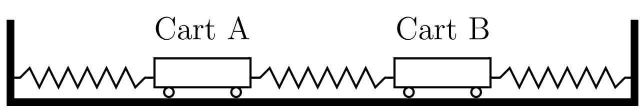
where T is the tension in the string, M is the mass, and L is the length of the string. Ignore any string stiffness, and assume that it does not stretch when tightened.

Consider two steel strings of the same length, the first with radius r_1 and a second thicker string with radius $r_2 = 4r_1$. Each string is tightened to the maximum possible tension without breaking.

What is the ratio f_1/f_2 of the fundamental frequencies of vibration on the two strings?

(A) 1
(B) $\sqrt{2}$
(C) 2
(D) $2\sqrt{2}$
(E) 4

25. Two identical carts A and B each with mass m are connected via a spring with spring constant k. Two additional springs, identical to the first, connect the carts to two fixed points. The carts are free to oscillate under the effect of the springs in one dimensional frictionless motion.



Under suitable initial conditions, the two carts will oscillate in phase according to

$$x_A(t) = x_0 \sin \omega_1 t = x_B(t)$$

where x_A and x_B are the locations of carts A and B relative to their respective equilibrium positions.

Under other suitable initial conditions, the two carts will oscillate exactly out of phase according to

$$x_A(t) = x_0 \sin \omega_2 t = -x_B(t)$$

Determine the ratio ω_2/ω_1

(A) $\sqrt{3}$
(B) 2
(C) $2\sqrt{2}$
(D) 3
(E) 5