AAPT UNITED STATES PHYSICS TEAM
AIP 2015

2015 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

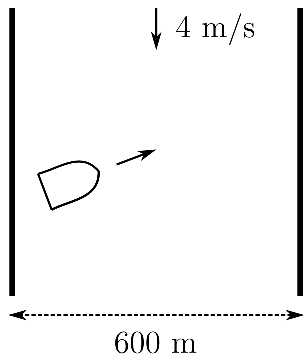
INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2015.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. A 600 meter wide river flows directly south at 4.0 m/s. A small motor boat travels at 5.0 m/s in still water and points in such a direction so that it will travel directly east relative to the land.



The time it takes to cross the river is closest to

(A) 67 s

(B) 120 s

(C) 150 s

(D) 200 s ← **CORRECT**

(E) 600 s

Solution

The speed of the boat relative to the land is 3.0 m/s; a vector diagram will illustrate that.

2. A car travels directly north on a straight highway at a constant speed of 80 km/hr for a distance of 25 km. The car then continues directly north at a constant speed of 50 km/hr for a distance of 75 more kilometers. The average speed of the car for the entire journey is closest to

   (A) 55.2 km/hr ← **CORRECT**

   (B) 57.5 km/hr

   (C) 65 km/hr

   (D) 69.6 km/hr

   (E) 72.5 km/hr

Solution

Average speed is total distance divided by total time. The time required for the first 25 km is 0.31 hours; the time required for the second 75 km is 1.5 hours.

3. The force of friction on an airplane in level flight is given by $F_f = kv^2$, where k is some constant, and v is the speed of the airplane. When the power output from the engines is P_0, the plane is able to fly at a speed v_0. If the power output of the engines is increased by 100% to $2P_0$, the airplane will be able to fly at a new speed given by

   (A) $1.12v_0$

   (B) $1.26v_0$ ← **CORRECT**

   (C) $1.41v_0$

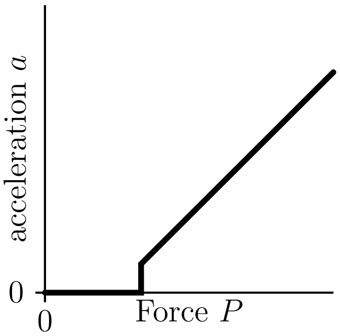
   (D) $2.82v_0$

   (E) $8v_0$

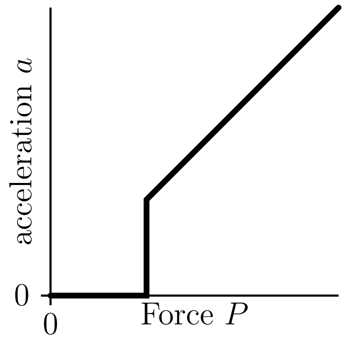
Solution

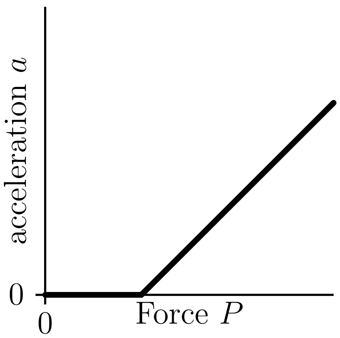
$P = Fv$, so $P = kv^3$. Then $v/v_0 = \sqrt[3]{P/P_0}$.

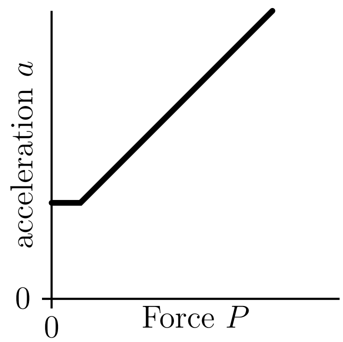
4. A 2.0 kg box is originally at rest on a horizontal surface where the coefficient of static friction between the box and the surface is μ_s and the coefficient of the kinetic friction between the box and the surface is $\mu_k = 0.90\mu_s$. An external horizontal force of magnitude P is then applied to the box. Which of the following is a graph of the acceleration of the box a versus the external force P?

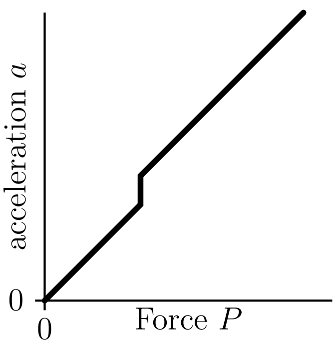
(A) 

← **CORRECT**

(B) 

(C) 

(D) 

(E) 

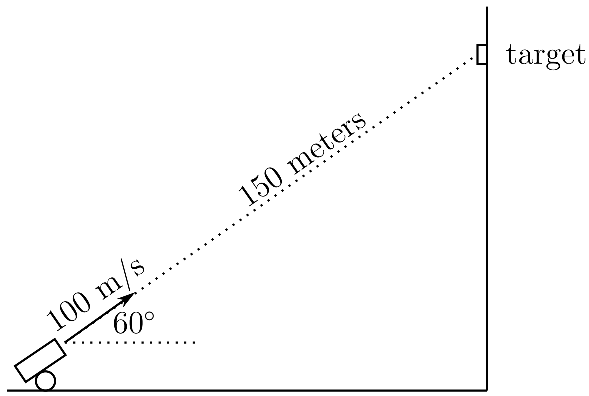
Solution

Box won't move unless $P > \mu_s mg$.

If box moves, the acceleration is given by $a = P/m - \mu_k g$. Note that the graph is discontinuous at $P = \mu_s mg$, and has slope $1/m$.

One might consider what happens if the force is *not* horizontal.

5. A 470 gram lead ball is launched at a 60 degree angle above the horizontal with an initial speed of 100 m/s directly toward a target on a vertical cliff wall that is 150 meters away as shown in the figure.



Ignoring air friction, by what distance does the lead ball miss the target when it hits the cliff wall?

(A) 1.3 m
(B) 2.2 m
(C) 5.0 m
(D) 7.1 m
(E) 11 m ← **CORRECT**

Solution

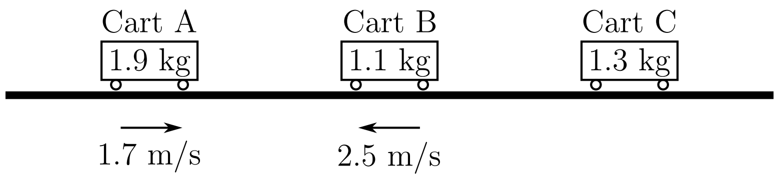
The time it would take to hit the target if there was no gravity is given by

$$150 \text{ m}/100 \text{ m/s} = 1.5 \text{ s}$$

During that time the ball falls vertically a distance of

$$d = \frac{1}{2}gt^2 = 11.25 \text{ m}$$

6. Three trolley carts are free to move on a one dimensional frictionless horizontal track. Cart A has a mass of 1.9 kg and an initial speed of 1.7 m/s to the right; Cart B has a mass of 1.1 kg and an initial speed of 2.5 m/s to the left; cart C has a mass of 1.3 kg and is originally at rest. Collisions between carts A and B are perfectly elastic; collisions between carts B and C are perfectly inelastic.



What is the velocity of the center of mass of the system of the three carts after the last collision?

(A) 0.11 m/s ← **CORRECT**

(B) 0.16 m/s

(C) 1.4 m/s

(D) 2.0 m/s

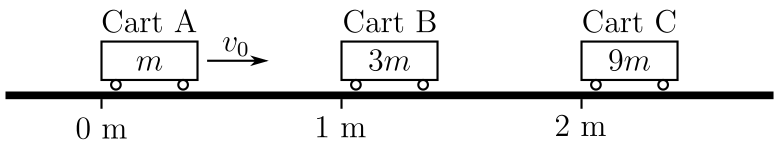
(E) 3.23 m/s

Solution

The center of mass velocity is unchanged by any collisions, so

$$v_{cm} = \frac{(1.9\text{ kg})(1.7\text{ m/s}) + (1.1\text{ kg})(-2.5\text{ m/s}) + (1.3\text{ kg})(0\text{ m/s})}{(1.9\text{ kg}) + (1.1\text{ kg}) + (1.3\text{ kg})} = 0.11\text{ m/s}$$

The following information applies to questions 7 and 8



Carts A, B, and C are on a long horizontal frictionless track. The masses of the carts are m, $3m$, and $9m$. Originally cart B is at rest at the 1.0 meter mark and cart C is at rest on the 2.0 meter mark. Cart A is originally at the zero meter mark moving toward the cart B at a speed of v_0.

7. Assuming that all collisions are completely *inelastic*, what is the final speed of cart C?

   (A) $v_0/13$ ← **CORRECT**

   (B) $v_0/10$

   (C) $v_0/9$

   (D) $v_0/3$

   (E) $2v_0/5$

Solution

After the first collision the two carts are moving at a speed $v_0/(1+3)$. After the second collision the carts are moving at a speed $v_0/(1+3+9)$.

8. Assuming that all collisions are completely *elastic*, what is the final speed of cart C?

   (A) $v_0/8$

   (B) $v_0/4 \leftarrow$ **CORRECT**

   (C) $v_0/2$

   (D) v_0

   (E) $2v_0$

Solution

Solve using conservation of kinetic energy and momentum. The standard result for one cart at rest and the other initially moving is

$$v_{1f} = v_0 \frac{m_1 - m_2}{m_1 + m_2}$$

and

$$v_{2f} = v_0 \frac{2m_1}{m_1 + m_2}$$

As such, the first cart bounces back and the second cart moves forward with speed $v_0/2$. Repeat the process, and the final speed of the third cart is $v_0/4$.

The following information applies to questions 9 and 10

A 0.650 kg ball moving at 5.00 m/s collides with a 0.750 kg ball that is originally at rest. After the collision, the 0.750 kg ball moves off with a speed of 4.00 m/s, and the 0.650 kg ball moves off at a right angle to the final direction of motion of the 0.750 kg ball.

9. What is the final speed of the 0.650 kg ball?

   (A) 1.92 m/s ← **CORRECT**

   (B) 2.32 m/s

   (C) 3.00 m/s

   (D) 4.64 m/s

   (E) 5.77 m/s

Solution

Yes, we purposefully choose the velocities to satisfy the 3-4-5 triangle, albeit incorrectly. Conserve momentum, sketch a right triangle. Then

$$p_{2f}{}^2 + p_{1f}{}^2 = p_{1i}^2,$$

with

$$p_{1i} = (0.65 \text{ kg})(5.0 \text{ m/s}) = 3.25 \text{ kg} \cdot \text{m/s}$$

and

$$p_{2f} = (0.75 \text{ kg})(4.0 \text{ m/s}) = 3.0 \text{ kg} \cdot \text{m/s}$$

therefore,

$$p_{1f} = 1.25 \text{ kg} \cdot \text{m/s}$$

and then

$$v_{1f} = (1.25)/(0.65) \text{ m/s} = 25/13 \text{ m/s}.$$

10. Let the change in total kinetic energy in this collision be defined by $\Delta K = K_f - K_i$, where K_f is the total final kinetic energy, and K_i is the total initial kinetic energy. Which of the following is true?

(A) $\Delta K = (K_i + K_f)/2$

(B) $K_f < \Delta K < K_i$

(C) $0 < \Delta K < K_f$

(D) $\Delta K = 0$

(E) $-K_i < \Delta K < 0$ ← **CORRECT**

Solution

Plug in the numbers from the previous question, or be fancy.

Consider the expression

$$p_{2f}{}^2 + p_{1f}{}^2 = p_{1i}^2,$$

and then

$$\Delta K = \frac{1}{2}\left(\frac{p_{2f}{}^2}{m_2} + \frac{p_{1f}{}^2}{m_1} - \frac{p_{1i}^2}{m_1}\right) = \frac{1}{2}\left(\frac{p_{2f}{}^2}{m_2} - \frac{p_{2f}{}^2}{m_1}\right)$$

so that

$$\Delta K = \frac{1}{2}\frac{p_{2f}{}^2}{m_1}\left(\frac{m_1}{m_2} - 1\right)$$

This is clearly a negative number, and clearly smaller in magnitude than K_i

11. A sphere floats in water with 2/3 of the volume of the sphere submerged. The sphere is removed and placed in oil that has 3/4 the density of water. If it floats in the oil, what fraction of the sphere would be submerged in the oil?

    (A) 1/12
    (B) 1/2
    (C) 8/9 ← **CORRECT**
    (D) 17/12
    (E) The sphere will not float, it will sink in the oil.

Solution

Since the fraction submerged is the ratio of the densities, $\rho_{\text{sphere}} = 2/3\rho_{\text{water}}$. But then in the oil the fraction submerged will be

$$\frac{\rho_{\text{sphere}}}{\rho_{\text{oil}}} = \frac{2/3}{3/4} = \frac{8}{9}$$

The following information applies to questions 12 and 13

A pendulum consists of a small bob of mass m attached to a fixed point by a string of length L. The pendulum bob swings down from rest from an initial angle $\theta_{\text{max}} < 90$ degrees.

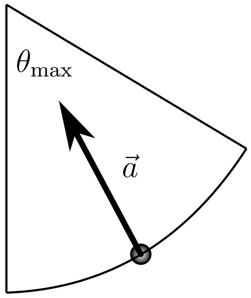
12. Which of the following statements about the pendulum bob's acceleration is true?

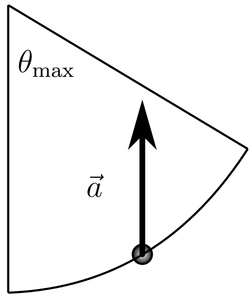
    (A) The magnitude of the acceleration is constant for the motion.
    (B) The magnitude of the acceleration at the lowest point is g, the acceleration of free fall.
    (C) The magnitude of the acceleration is zero at some point of the pendulum's swing.
    (D) The acceleration is always directed toward the center of the circle.
    (E) The acceleration at the bottom of the swing is pointing vertically upward.← **CORRECT**

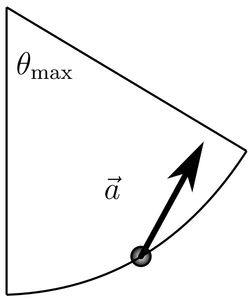
Solution

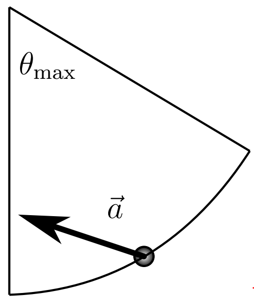
Consider each scenario in turn.

13. Consider the pendulum bob when it is at an angle $\theta = \frac{1}{2}\theta_{\text{max}}$ on the way up (moving toward θ_{max}). What is the direction of the acceleration vector?

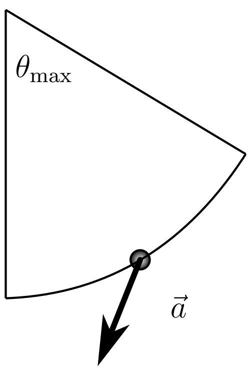
(A) 

(B) 

(C) 

(D) 

← **CORRECT**

(E) 

Solution

There must be acceleration directed toward the center of the circle. There must be acceleration directed tangent to the circle toward the bottom of the path.

The following information applies to questions 14 and 15

A 3.0 meter long massless rod is free to rotate horizontally about its center. Two 5.0 kg point objects are originally located at the ends of the rod; they are free to slide on the frictionless rod and are kept from flying off the rod by an inflexible massless rope that connects the two objects.

Originally the system is rotating at 4.0 radians per second; assume the system is completely frictionless; and ignore any concerns about instability of the system.

14. Calculate the original tension in the rope.

    (A) 60 N

    (B) 106 N

    (C) 120 N ← **CORRECT**

    (D) 240 N

    (E) 480 N

Solution

It is as easy as

$$F = mr\omega^2 = (5.0 \text{ kg})(1.5 \text{ m})(4.0 \text{ rad/s})^2 = 120 \text{ N}$$

15. The rope is slowly tightened by a small massless motor attached to one of the objects. It is done in such a way as to pull the two objects closer to the center of the rotating rod. How much work is done by the motor in pulling the two objects from the ends of the rod until they are each 0.5 meters from the center of rotation?

(A) 120 J

(B) 180 J

(C) 240 J

(D) 1440 J ← **CORRECT**

(E) 1620 J

Solution

Work is change in energy, so

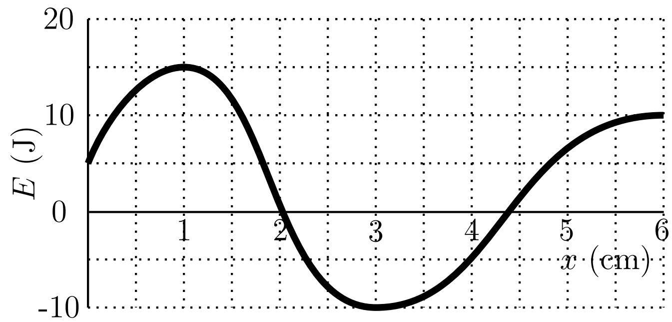
$$W = \frac{1}{2I_f}L^2 - \frac{1}{2I_i}L^2,$$

where L is the *conserved* angular momentum, or $L = I_i\omega_i$, and I refers to the rotational inertia, or mr^2.

Then $I_i = 22.5\ \mathrm{kg \cdot m^2}$, $I_f = 2.5\ \mathrm{kg \cdot m^2}$, and $L = 90\ \mathrm{kg \cdot m^2/s}$.

Finally, $W = 1440$ J.

16. Shown below is a graph of potential energy as a function of position for a 0.50 kg object.



Which of the following statements is NOT true in the range 0 cm $< x <$ 6 cm?

(A) The object could be at equilibrium at either $x = 1$ cm or $x = 3$ cm.

(B) The minimum possible total energy for this object in the range is -10 J.

(C) The magnitude of the force on the object at 4 cm is approximately 1000 N.

(D) If the total energy of the particle is 0 J then the object will have a maximum kinetic energy of 10 J.

(E) The magnitude of the acceleration of the object at $x = 2$ cm is approximately 4 cm/s^2.$\leftarrow$ **CORRECT**

Solution

Consider each scenario in turn.

17. A flywheel can rotate in order to store kinetic energy. The flywheel is a uniform disk made of a material with a density ρ and tensile strength σ (measured in Pascals), a radius r, and a thickness h. The flywheel is rotating at the maximum possible angular velocity so that it does not break. Which of the following expression correctly gives the maximum kinetic energy per kilogram that can be stored in the flywheel? Assume that α is a dimensionless constant.

    (A) $\alpha\sqrt{\rho\sigma/r}$

    (B) $\alpha h\sqrt{\rho\sigma/r}$

    (C) $\alpha\sqrt{(h/r)}(\sigma/\rho)^2$

    (D) $\alpha(h/r)(\sigma/\rho)$

    (E) $\alpha\sigma/\rho \leftarrow$ **CORRECT**

Solution

Dimensional analysis is the way to go. We want to find a combination of exponents a, b, c, d that results in

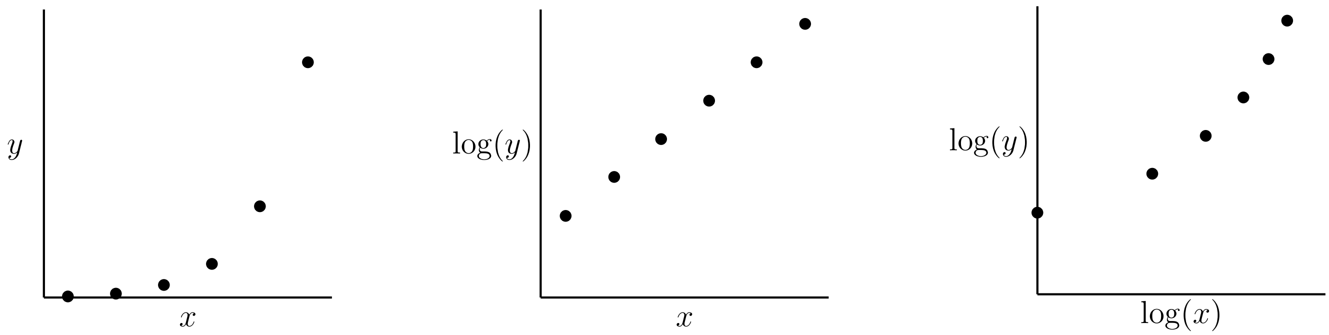
$$\rho^a \sigma^b r^c h^d$$

having dimensions of energy per mass. Then

$$(M/L^3)^a (M/LT^2)^b (L)^c (L)^d = (L^2/T^2)$$

The solution is of the form $b = 1$, $a = -1$, and $c = -d$. It is not possible to resolve d by dimensional analysis, but it can be reasoned out that if you double the thickness of the disk the maximum possible rotation speed before it disintegrates won't change, so you will have twice the energy and twice the mass, so $d = 0$.

18. Shown below are three graphs of the same data.



Which is the correct functional relationship between the data points? Assume a and b are constants.

(A) $y = ax + b$

(B) $y = ax^2 + b$

(C) $y = ax^b$

(D) $y = ae^{bx}$ ← **CORRECT**

(E) $y = a \log x + b$

Solution

Since the log-linear graph is a straight line, it is the correct functional form:

$$\log y = Ax + B$$

or

$$y = ae^{bx}$$

The following information applies to questions 19 and 20

A U-tube manometer consists of a uniform diameter cylindrical tube that is bent into a U shape. It is originally filled with water that has a density ρ_w. The *total* length of the column of water is L. Ignore surface tension and viscosity.

19. The water is displaced slightly so that one side moves up a distance x and the other side lowers a distance x. Find the frequency of oscillation.

(A) $\frac{1}{2\pi}\sqrt{2g/L}$ ← **CORRECT**

(B) $2\pi\sqrt{g/L}$

(C) $\frac{1}{2\pi}\sqrt{2L/g}$

(D) $\frac{1}{2\pi}\sqrt{g/\rho_w}$

(E) $2\pi\sqrt{\rho_w g L}$

Solution

Frequency of oscillation is given by

$$f = \frac{1}{2\pi}\sqrt{k/m}$$

where m is the mass, in this case $LA\rho$, with L the total length of the fluid, and k is an effective spring constant.

If the fluid is displaced a distance x, then the net restoring force on the fluid is given by $2xgA\rho$, so $k = 2gA\rho$. Then

$$f = \frac{1}{2\pi}\sqrt{2g/L}$$

20. Oil with a density half that of water is added to one side of the tube until the total length of oil is equal to the total length of water. Determine the equilibrium height difference between the two sides

(A) L

(B) $L/2$ ← **CORRECT**

(C) $L/3$

(D) $3L/4$

(E) $L/4$

Solution

Require the forces to balance. If x is the height of the water on the side with only water, then

$$x\rho_w = (L - x)\rho_w + L\rho_o$$

or

$$x = L - x + L/2$$

or $x = 3L/4$. The height difference is therefore

$$(2L - x) - x = 2(L - x) = L/2$$

21. An object launched vertically upward from the ground with a speed of 50 m/s bounces off of the ground on the return trip with a coefficient of restitution given by $C_R = 0.9$, meaning that immediately after a bounce the upward speed is 90% of the previous downward speed. The ball continues to bounce like this; what is the total amount of time between when the ball is launched and when it finally comes to a rest? Assume the collision time is zero; the bounce is instantaneous. Treat the problem as ideally classical and ignore any quantum effects that might happen for very small bounces.

(A) 71 s

(B) 100 s ← **CORRECT**

(C) 141 s

(D) 1000 s

(E) ∞ (the ball never comes to a rest)

Solution

For a bounce, $C_R = v_f/v_i$. The time of flight between two bounces is given by $t = 2v/g$, where v is the initial speed as the object rises off of a bounce. We then have an infinite sequence:

$$t_n = \frac{2}{g}v_n = \frac{2}{g}v_0 {C_R}^n$$

Summing the resulting series to get the total time of flight,

$$T = \sum \frac{2}{g}v_0 {C_R}^n = \frac{2}{g}v_0 \sum {C_R}^n = \frac{2}{g}v_0 \frac{1}{1 - C_R}$$

Therefore

$$T = \frac{2}{10 \text{ m/s}^2}(50 \text{ m/s})\frac{1}{1 - 0.9} = 100 \text{ s}$$

22. A solid ball is released from rest down inclines of various inclination angles θ but through a fixed vertical height h. The coefficient of static and kinetic friction are both equal to μ. Which of the following graphs best represents the total kinetic energy of the ball at the bottom of the incline as a function of the angle of the incline?

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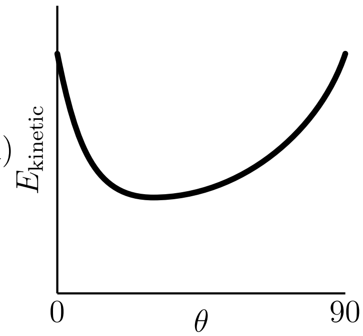

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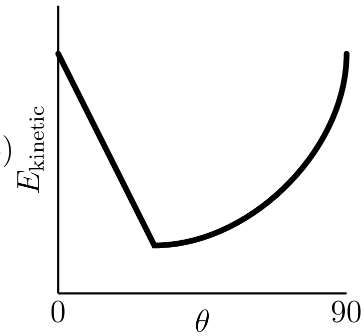

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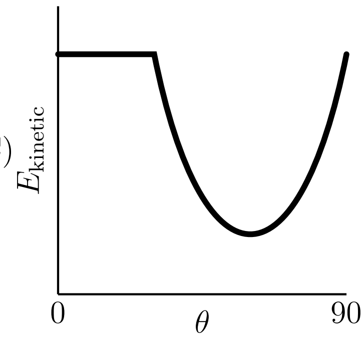
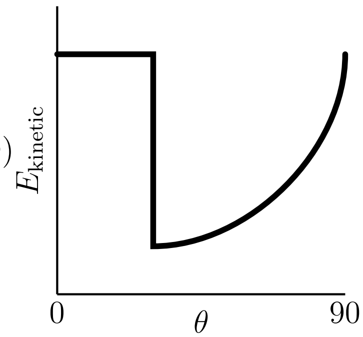


← **CORRECT**

(D)


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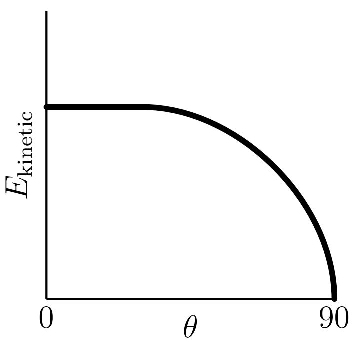


Solution

The ball rolls without slipping so long as θ is less than some critical angle. For θ less than the critical angle no energy is lost to friction. For θ greater than the critical angle the ball will slip, and energy will be lost to friction. If the angle is 90 degrees the ball falls straight down and there is no friction, so no energy lost. There must then be some angle where the most energy is lost to friction.

23. A 2.0 kg object falls from rest a distance of 5.0 meters onto a 6.0 kg object that is supported by a vertical massless spring with spring constant $k = 72$ N/m. The two objects stick together after the collision, which results in the mass/spring system oscillating. What is the maximum magnitude of the displacement of the 6.0 kg object from its original location before it is struck by the falling object?

(A) 0.27 m

(B) 1.1 m ← **CORRECT**

(C) 2.5 m

(D) 2.8 m

(E) 3.1 m

Solution

Conserve energy to find the speed of the 2.0 kg object at the instant before the collision:

$$m_1 g y = (2.0\text{ kg})(10\text{ m/s})(5.0\text{ m}) = 100\text{ J}$$

and this equals the kinetic energy so

$$v = \sqrt{2K_1/m_1} = 10.0\text{ m/s}.$$

The collision is inelastic, so

$$v_f = v_1 \frac{m_1}{m_1 + m_2} = 2.5\text{ m/s}$$

Now conserve energy again

$$K_f = \frac{1}{2} m_t v^2 = \frac{1}{2}(8.0\text{ kg})(2.5\text{ m/s})^2 = 25\text{ J}$$

This compresses the spring *and* changes the gravitational potential energy, so we must solve for x in the equation

$$\frac{1}{2} k x^2 + m_t g x = K_f$$

or

$$36x^2 + 20x - 25 = 0$$

which has solutions $x = -1.15$ m and $x = 0.6$ m.

It was pointed out by a very astute and determined reader that 1.15 m should have been rounded to 1.2 m as opposed to 1.1 m.

Okay, I have received enough complaints about this problem that I will add to the solution.

First, there *is* a typo: the equation

$$\frac{1}{2} k x^2 + m_t g x = K_f$$

should have read

$$\frac{1}{2}kx^2 + m_1 gx = K_f$$

but there isn't a mistake in the physics or the math, as we did indeed use $m = 2.0$ kg in the solution like we should have. So the answer is correct.

But a number of persistent individuals have continued to insist that we use $m = 8.0$ kg in this expression. After all, they ponder, since both masses are moving, and we included both masses in the kinetic energy expression, why aren't we including both masses in the potential energy expression?

This is a subtlety attached to the quadratic nature of springs. If we want to consider an energy expression that has the change in potential of the 6.0 kg mass, then we need to measure the spring from the unstretched length *before* the 6.0 kg mass is suspended from the spring.

The energy expression is then

$$(m_1 + m_2)g(x + h) + \frac{1}{2}k(x + h)^2 + U_0 = K_f,$$

where both h and x are measured positive if up, so

$$m_2 g = -kh.$$

The quantity U_0 is a constant energy term that makes the two approaches numerically equivalent; we want the total potential energy of the system to be zero *before* the second mass m_2 hits it, and we want the second mass to also have no potential energy at $x = 0$.

Expand and combine,

$$-kh(x + h) + m_1 g(x + h) + \frac{1}{2}kx^2 + kxh + \frac{1}{2}kh^2 + U_0 = K_f$$

or

$$\frac{1}{2}kx^2 + m_1 gx + m_2 gh - \frac{1}{2}kh^2 + U_0 = K_f$$

But U_0 is chosen so that the two energy expression approaches are equivalent, so

$$m_2 gh - \frac{1}{2}kh^2 + U_0 = 0$$

and therefore they are: the total potential energy of the system is zero just before the masses collide in either approach.

I suspect there are a few readers who still think that slight of hand was performed in this last step.

Isn't physics wonderful?

Now I hope this is the last edit I need to make on this solution.

24. The speed of a transverse wave on a long cylindrical steel string is given by

$$v = \sqrt{\frac{T}{M/L}},$$

where T is the tension in the string, M is the mass, and L is the length of the string. Ignore any string stiffness, and assume that it does not stretch when tightened.

Consider two steel strings of the same length, the first with radius r_1 and a second thicker string with radius $r_2 = 4r_1$. Each string is tightened to the maximum possible tension without breaking.

What is the ratio f_1/f_2 of the fundamental frequencies of vibration on the two strings?

(A) 1 $\leftarrow$ **CORRECT**

(B) $\sqrt{2}$

(C) 2

(D) $2\sqrt{2}$

(E) 4

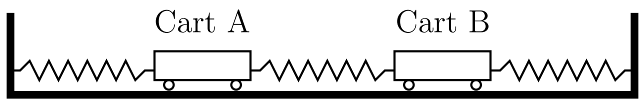
Solution

The string will break if $T > \sigma A$, where σ is the tensile strength and A is the cross sectional area, so

$$v = \sqrt{\frac{T}{M/L}} = \sqrt{\frac{\sigma AL}{M}} = \sqrt{\frac{\sigma}{\rho}},$$

where ρ is the density. This expression is independent of the length of the string, so the wave speed on the two strings is the same. Since $v = f\lambda$ and the strings are the same length, they have the same fundamental frequency.

25. Two identical carts A and B each with mass m are connected via a spring with spring constant k. Two additional springs, identical to the first, connect the carts to two fixed points. The carts are free to oscillate under the effect of the springs in one dimensional frictionless motion.



Under suitable initial conditions, the two carts will oscillate in phase according to

$$x_{\mathrm{A}}(t) = x_0 \sin \omega_1 t = x_{\mathrm{B}}(t)$$

where x_{A} and x_{B} are the locations of carts A and B relative to their respective equilibrium positions.

Under other suitable initial conditions, the two carts will oscillate exactly out of phase according to

$$x_{\mathrm{A}}(t) = x_0 \sin \omega_2 t = -x_{\mathrm{B}}(t)$$

Determine the ratio ω_2/ω_1

(A) $\sqrt{3}$ ← **CORRECT**

(B) 2

(C) $2\sqrt{2}$

(D) 3

(E) 5

Solution

Though you can solve this problem with matrix methods, it can also be solved by carefully digesting the problem.

For the in-phase situation, the separation between the masses is constant, so the force of A on B is constant. In that case, only one spring exerts a varying force, and then the angular frequency must be given by

$$\omega_1 = \sqrt{k/m}$$

For the out of phase situation, the separation between the objects is not constant, but there exists a point on the connecting spring that does not move. As such, either object can be though of moving under the influence of two springs in parallel, one with length L, the there with length $L/2$. That is equivalent to a spring with constant k in parallel with a spring of constant $2k$. The net spring constant is then $k + 2k = 3k$.

As such, the frequency is given by

$$\omega_2 = \sqrt{3k/m} = \sqrt{3}\ \omega_1$$