AAPT UNITED STATES PHYSICS TEAM
AIP 2016

2016 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2016.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

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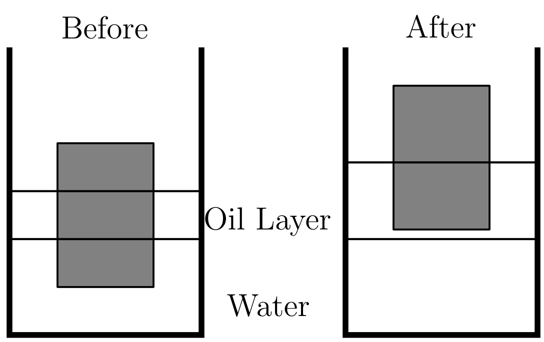
1. A car drives anticlockwise (counterclockwise) around a flat, circular curve at constant speed, so that the left, front wheel traces out a circular path of radius $R = 9.60$ m. If the width of the car is 1.74 m, what is the ratio of the angular velocity about its axle of the left, front wheel to that of the right, front wheel, of the car as it moves through the curve? Assume the wheels roll without slipping.

   (A) 0.331
   (B) 0.630
   (C) 0.708
   (D) 0.815
   (E) 0.847

2. A 3.0 cm thick layer of oil with density $\rho_o = 800\ \text{kg/m}^3$ is floating above water that has density $\rho_w = 1000\ \text{kg/m}^3$. A solid cylinder is floating so that 1/3 is in the water, 1/3 is in the oil, and 1/3 is in the air. Additional oil is added until the cylinder is floating only in oil. What fraction of the cylinder is in the oil?

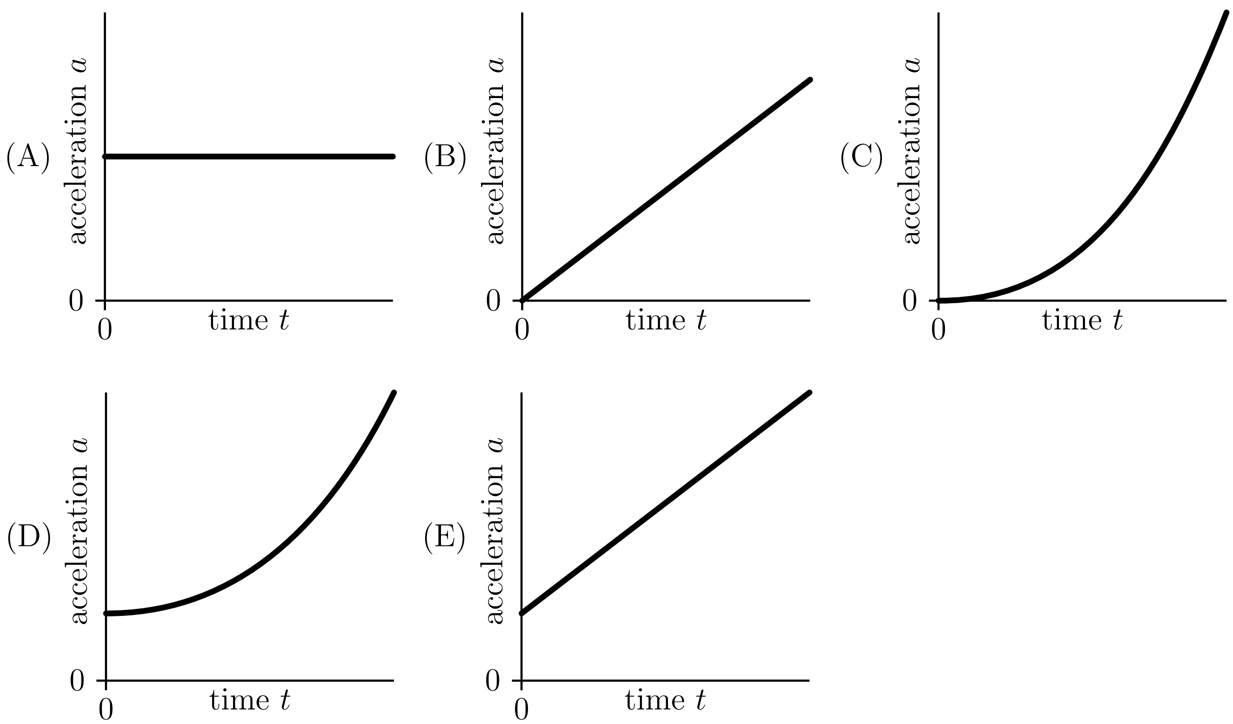


   (A) 3/5
   (B) 3/4
   (C) 2/3
   (D) 8/9
   (E) 4/5

3. An introductory physics student, elated by a first semester grade, celebrates by dropping a textbook from a balcony into a deep layer of soft snow which is 3.00 m below. Upon hitting the snow the book sinks a further 1.00 m into it before coming to a stop. The mass of the book is 5.0 kg. Assuming a constant retarding force, what is the force from the snow on the book?

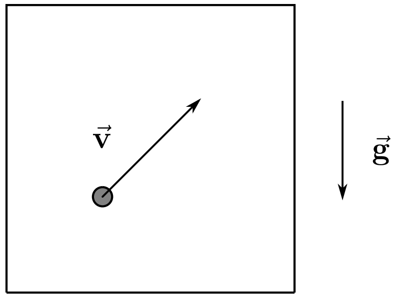
   (A) 85 N
   (B) 100 N
   (C) 120 N
   (D) 150 N
   (E) 200 N

4. A small bead slides from rest along a wire that is shaped like a vertical uniform helix (spring). Which graph below shows the magnitude of the acceleration a as a function of time?



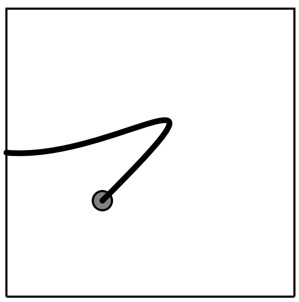
The following information applies to questions 5 and 6.

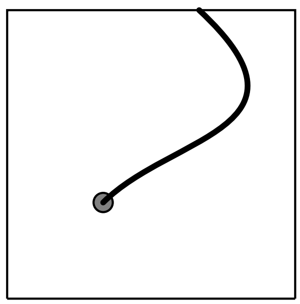
Consider a particle in a box where the force of gravity is down as shown in the figure.

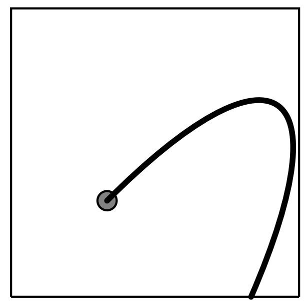


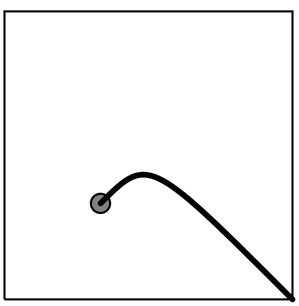
The particle has an initial velocity as shown, and the box has a constant acceleration to the right.

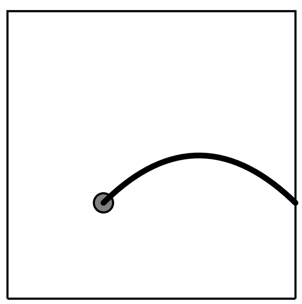
5. In the frame of the box, which of the following is a possible path followed by the particle?

(A) 

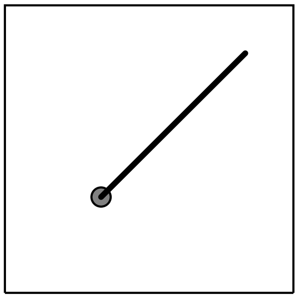
(B) 

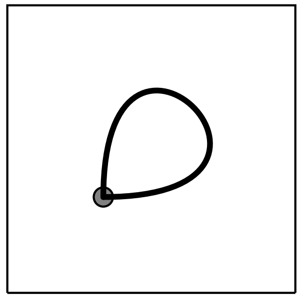
(C) 

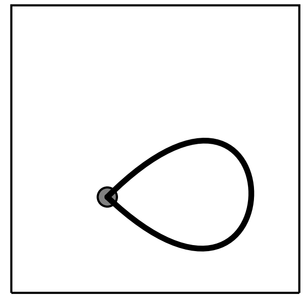
(D) 

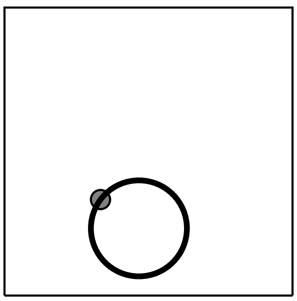
(E) 

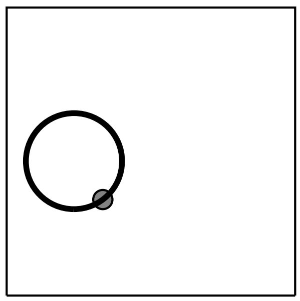
6. If the magnitude of the acceleration of the box is chosen correctly, the launched particle will follow a path that returns to the point that it was launched. In the frame of the box, which path is followed by the particle?

(A) 

(B) 

(C) 

(D) 

(E) 

7. A mass on a frictionless table is attached to the midpoint of an originally unstretched spring fixed at the ends. If the mass is displaced a distance A parallel to the table surface but perpendicular to the spring, it exhibits oscillations. The period T of the oscillations

   (A) does not depend on A.
   (B) increases as A increases, approaching a fixed value.
   (C) decreases as A increases, approaching a fixed value.
   (D) is approximately constant for small values of A, then increases without bound.
   (E) is approximately constant for small values of A, then decreases without bound.

8. Kepler's Laws state that

   I. the orbits of planets are elliptical with one focus at the sun,

   II. a line connecting the sun and a planet sweeps out equal areas in equal times, and

   III. the square the period of a planet's orbit is proportional to the cube of its semimajor axis.

   Which of these laws would remain true if the force of gravity were proportional to $1/r^3$ rather than $1/r^2$?

   (A) Only I.

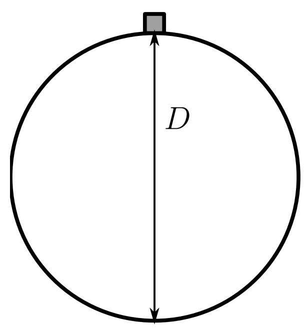
   (B) Only II.

   (C) Only III.

   (D) Both II and III.

   (E) None of the above.

9. A small bead is placed on the top of a frictionless glass sphere of diameter D as shown. The bead is given a slight push and starts sliding down along the sphere. Find the speed v of the bead at the point at which the which the bead leaves the sphere.



   (A) $v = \sqrt{gD}$

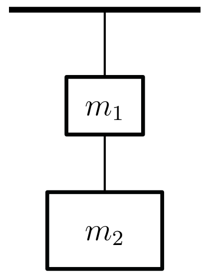
   (B) $v = \sqrt{4gD/5}$

   (C) $v = \sqrt{2gD/3}$

   (D) $v = \sqrt{gD/2}$

   (E) $v = \sqrt{gD/3}$

10. Two blocks are suspended by two massless elastic strings to the ceiling as shown in the figure. The masses of the upper and lower block are $m_1 = 2$ kg and $m_2 = 4$ kg respectively. If the upper string is suddenly cut just above the top block what are the accelerations of the two blocks at the moment when the top block begins to fall?



(A) upper: 10 m/s^2; lower: 0
(B) upper: 10 m/s^2; lower: 10 m/s^2
(C) upper: 20 m/s^2; lower: 10 m/s^2
(D) upper: 30 m/s^2; lower: 0
(E) upper: 30 m/s^2; lower: 10 m/s^2

11. The power output from a certain experimental car design to be shaped like a cube is proportional to the mass m of the car. The force of air friction on the car is proportional to Av^2, where v is the speed of the car and A the cross sectional area. On a level surface the car has a maximum speed v_{max}. Assuming that all versions of this design have the same density, then which of the following is true?

(A) $v_{\text{max}} \propto m^{1/9}$
(B) $v_{\text{max}} \propto m^{1/7}$
(C) $v_{\text{max}} \propto m^{1/3}$
(D) $v_{\text{max}} \propto m^{2/3}$
(E) $v_{\text{max}} \propto m^{3/4}$

12. A block floats partially submerged in a container of liquid. When the entire container is accelerated upward, which of the following happens? Assume that both the liquid and the block are incompressible.

(A) The block descends down lower into the liquid.
(B) The block ascends up higher in the liquid.
(C) The block does not ascend nor descend in the liquid.
(D) The answer depends on the direction of motion of the container.
(E) The answer depends on the rate of change of the acceleration

13. An object of mass m_1 initially moving at speed v_0 collides with an originally stationary object of mass $m_2 = \alpha m_1$, where $\alpha < 1$. The collision could be completely elastic, completely inelastic, or partially inelastic. After the collision the two objects move at speeds v_1 and v_2. Assume that the collision is one dimensional, and that object one cannot pass through object two.

After the collision, the speed ratio $r_2 = v_2/v_0$ of object 2 is bounded by

(A) $(1-\alpha)/(1+\alpha) \le r_2 \le 1$

(B) $(1-\alpha)/(1+\alpha) \le r_2 \le 1/(1+\alpha)$

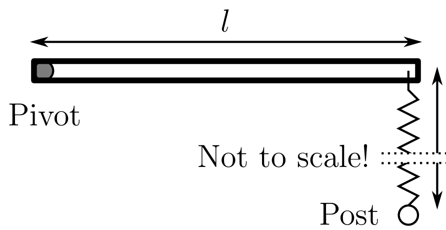
(C) $\alpha/(1+\alpha) \le r_2 \le 1$

(D) $0 \le r_2 \le 2\alpha/(1+\alpha)$

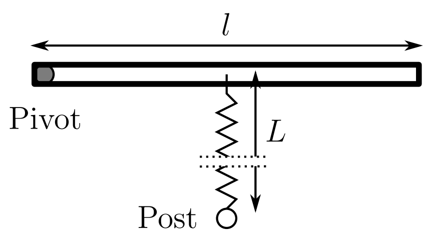
(E) $1/(1+\alpha) \le r_2 \le 2/(1+\alpha)$

The following information applies to questions 14 and 15.

A uniform rod of length l lies on a frictionless horizontal surface. One end of the rod is attached to a pivot. An un-stretched spring of length $L \gg l$ lies on the surface perpendicular to the rod; one end of the spring is attached to the movable end of the rod, and the other end is attached to a fixed post. When the rod is rotated slightly about the pivot, it oscillates at frequency f.



14. The spring attachment is moved to the midpoint of the rod, and the post is moved so the spring remains unstretched and perpendicular to the rod. The system is again set into small oscillations. What is the new frequency of oscillation?



(A) $f/2$

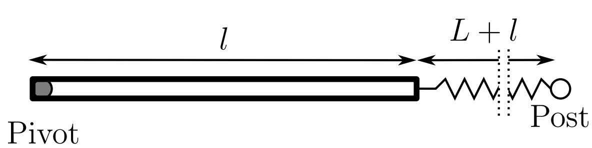
(B) $f/\sqrt{2}$

(C) f

(D) $\sqrt{2}f$

(E) $2f$

15. The spring attachment is moved back to the end of the rod; the post is moved so that it is in line with the rod and the pivot and the spring is unstretched. The post is then moved away from the pivot by an additional amount l. What is the new frequency of oscillation?



(A) $f/3$

(B) $f/\sqrt{3}$

(C) f

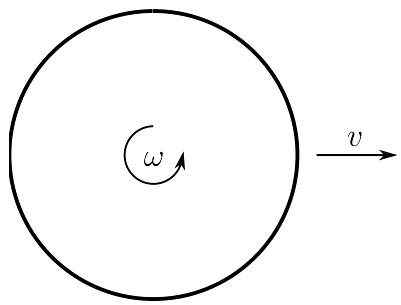
(D) $\sqrt{3}f$

(E) $3f$

16. A ball rolls from the back of a large truck traveling 10.0 m/s to the right. The ball is traveling horizontally at 8.0 m/s to the left relative to an observer in the truck. The ball lands on the roadway 1.25 m below its starting level. How far behind the truck does it land?

    (A) 0.50 m
    (B) 1.0 m
    (C) 4.0 m
    (D) 5.0 m
    (E) 9.0 m

17. As shown in the figure, a ping-pong ball with mass m with initial horizontal velocity v and angular velocity ω comes into contact with the ground. Friction is not negligible, so both the velocity and angular velocity of the ping-pong ball changes. What is the critical velocity v_c such that the ping-pong will stop and remain stopped? Treat the ping-pong ball as a hollow sphere.

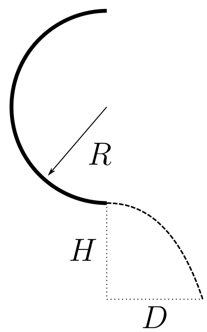


    (A) $v = \frac{2}{3}R\omega$
    (B) $v = \frac{2}{5}R\omega$
    (C) $v = R\omega$
    (D) $v = \frac{3}{5}R\omega$
    (E) $v = \frac{5}{3}R\omega$

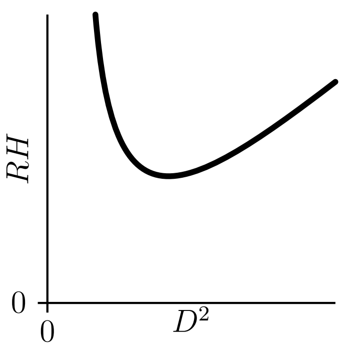
18. A spinning object begins from rest and accelerates to an angular velocity of $\omega = \pi/15$ rad/s with an angular acceleration of $\alpha = \pi/75$ rad/s^2. It remains spinning at that constant angular velocity and then stops with an angular acceleration of the same magnitude as it previously accelerated. The object made a total of 3 complete rotations during the entire motion. How much time did the motion take?

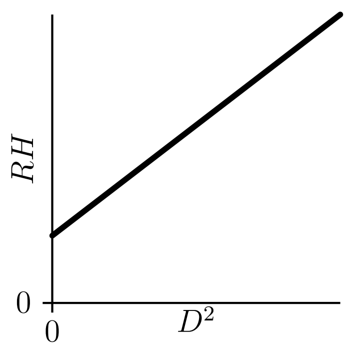
    (A) 75 s
    (B) 80 s
    (C) 85 s
    (D) 90 s
    (E) 95 s

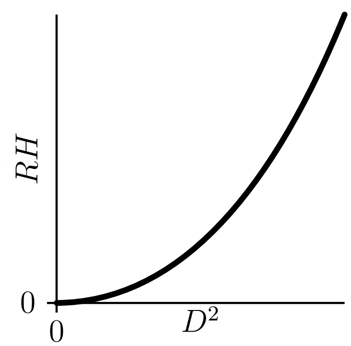
19. A semicircular wire of radius R is oriented vertically. A small bead is released from rest at the top of the wire, it slides without friction under the influence of gravity to the bottom, where is then leaves the wire horizontally and falls a distance H to the ground. The bead lands a horizontal distance D away from where it was launched.

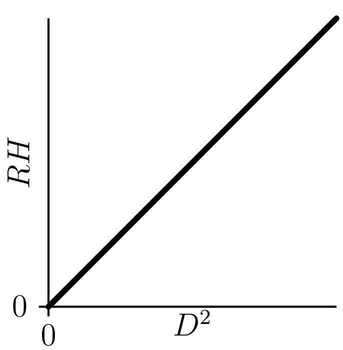


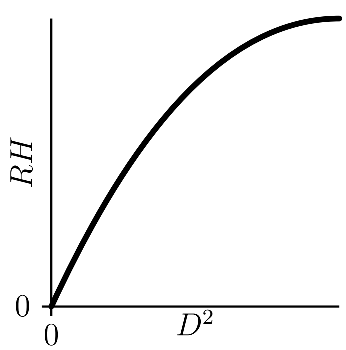
Which of the following is a correct graph of RH against D^2?

(A) 

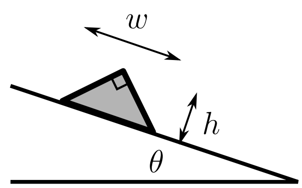
(B) 

(C) 

(D) 

(E) 

20. A uniform solid right prism whose cross section is an isosceles right triangle with height h and width $w = 2h$ is placed on an incline that has a variable angle with the horizontal θ. What is the minimum coefficient of static friction so that the prism topples before it begins sliding as θ is slowly increased from zero?



(A) 0.71
(B) 1.41
(C) 1.50
(D) 1.73
(E) 3.00

The following information applies to questions 21 and 22.

A small ball of mass $3m$ is at rest on the ground. A second small ball of mass m is positioned above the ground by a vertical massless rod of length L that is also attached to the ball on the ground. The original orientation of the rod is directly vertical, and the top ball is given a small horizontal nudge. There is no friction; assume that everything happens in a single plane.

21. Determine the horizontal displacement x of the second ball just before it hits the ground.

(A) $x = \frac{3}{4}L$
(B) $x = \frac{3}{5}L$
(C) $x = \frac{1}{4}L$
(D) $x = \frac{1}{3}L$
(E) $x = \frac{2}{5}L$

22. Determine the speed v of the second (originally top) ball just before it hits the ground.

(A) $v = \sqrt{2gL}$
(B) $v = \sqrt{gL}$
(C) $v = \sqrt{2gL/3}$
(D) $v = \sqrt{3gL/2}$
(E) $v = \sqrt{gL/4}$

23. A uniform thin circular rubber band of mass M and spring constant k has an original radius R. Now it is tossed into the air. Assume it remains circular when stabilized in air and rotates at angular speed ω about its center uniformly. Which of the following gives the new radius of the rubber band?

(A) $(2\pi kR)/(2\pi k - M\omega^2)$
(B) $(4\pi kR)/(4\pi k - M\omega^2)$
(C) $(8\pi^2 kR)/(8\pi^2 k - M\omega^2)$
(D) $(4\pi^2 kR)/(4\pi^2 k - M\omega^2)$
(E) $(4\pi kR)/(2\pi k - M\omega^2)$

24. The moment of inertia of a uniform equilateral triangle with mass m and side length a about an axis through one of its sides and parallel to that side is $(1/8)ma^2$. What is the moment of inertia of a uniform regular hexagon of mass m and side length a about an axis through two opposite vertices?

(A) $(1/6)ma^2$
(B) $(5/24)ma^2$
(C) $(17/72)ma^2$
(D) $(19/72)ma^2$
(E) $(9/32)ma^2$

25. Three students make measurements of the length of a 1.50 meter rod. Each student reports an uncertainty estimate representing an independent random error applicable to the measurement.

Alice: A single measurement using a 2.0 meter long tape measure, to within ± 2 mm.
Bob: Two measurements using a wooden meter stick, each to within ± 2 mm, which he adds together.
Christina: Two measurements using a machinist's meter ruler, each to within ± 1 mm, which she adds together.

The students' teacher prefers measurements that are likely to have less error. Which is the correct order of preference?

(A) Christina's is preferable to Alice's, which is preferable to Bob's
(B) Alice's is preferable to Christina's, which is preferable to Bob's
(C) Alice's and Christina's are equally preferable; both are preferable to Bob's
(D) Christina's is preferable to both Alice's and to Bob's, which are equally preferable
(E) Alice's is preferable to Bob's and Christina's, which are equally preferable