AAPT UNITED STATES PHYSICS TEAM
AIP 2016

2016 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2016.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. A car drives anticlockwise (counterclockwise) around a flat, circular curve at constant speed, so that the left, front wheel traces out a circular path of radius $R = 9.60$ m. If the width of the car is 1.74 m, what is the ratio of the angular velocity about its axle of the left, front wheel to that of the right, front wheel, of the car as it moves through the curve? Assume the wheels roll without slipping.

   (A) 0.331
   (B) 0.630
   (C) 0.708
   (D) 0.815
   (E) 0.847 ← **CORRECT**

Solution

At constant speed, we can use the average angular velocity: $\omega = \Delta\theta/\Delta t$.
The two wheels will share the same Δt, but $\Delta\theta$ can be found through

$$\Delta\theta = S/C,$$

where S is the arc length of the wheel's path, and C is the circumference of the wheel. Let us consider a convenient arc length of π radians, so that $S = \pi R$. Then, for the left wheel,

$$\Delta\theta_l = \frac{\pi R}{2\pi r},$$

and for the right wheel,

$$\Delta\theta_r = \frac{\pi(R + dR)}{2\pi r},$$

where r is the same radius of the two wheels. We then take the ratio,

$$\frac{\omega_l}{\omega_r} = \frac{\frac{\pi R}{2\pi r\Delta t}}{\frac{\pi(R+dR)}{2\pi r\Delta t}},$$

$$\frac{\omega_l}{\omega_r} = \frac{R}{R + dR},$$

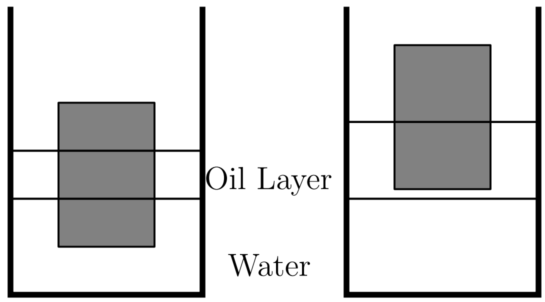
$$\frac{\omega_l}{\omega_r} = \frac{1}{1 + \frac{dR}{R}}.$$

Plugging in the numbers,

$$\frac{\omega_l}{\omega_r} = \frac{1}{1 + \frac{1.74\,\mathrm{m}}{9.60\,\mathrm{m}}},$$

$$\frac{\omega_l}{\omega_r} = 0.847.$$

2. A 3.0 cm thick layer of oil with density $\rho_o = 800$ kg/m^3 is floating above water that has density $\rho_w = 1000$ kg/m^3. A solid cylinder is floating so that 1/3 is in the water, 1/3 is in the oil, and 1/3 is in the air. Additional oil is added until the cylinder is floating only in oil. What fraction of the cylinder is in the oil?



(A) 3/5

(B) 3/4 ← **CORRECT**

(C) 2/3

(D) 8/9

(E) 4/5

Solution

Assume the cylinder is 3 cubic meters. Then the displaced mass of water is 1000 kg, and the displaced mass of oil is 800 kg. Ignore the displaced air. The puts the mass of the cylinder at 1800 kg.

When floating only in oil, it must displace 1800 kg, or a volume of oil given by

$$V_o = (1800 \text{ kg})/(800 \text{ kg/m}^3) = \frac{9}{4} \text{ m}^3$$

The fraction beneath the surface is then

$$(\frac{9}{4} \text{ m}^3)/(3 \text{ m}^3) = \frac{3}{4}$$

3. An introductory physics student, elated by a first semester grade, celebrates by dropping a textbook from a balcony into a deep layer of soft snow which is 3.00 m below. Upon hitting the snow the book sinks a further 1.00 m into it before coming to a stop. The mass of the book is 5.0 kg. Assuming a constant retarding force, what is the force from the snow on the book?

   (A) 85 N
   (B) 100 N
   (C) 120 N
   (D) 150 N
   (E) 200 N ← **CORRECT**

Solution

Assign some variables: $H = 3.00$ m; $h = 1.00$ m; v is the speed when the book hits in the snow.

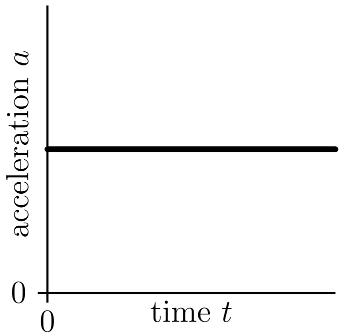
By kinematics

$$2gH = v^2 = 2ah$$

where a is the magnitude of the acceleration in the snow.

Then $a = 3g$, so the upward force of the snow must be $4mg$.

4. A small bead slides from rest along a wire that is shaped like a vertical uniform helix (spring). Which graph below shows the magnitude of the acceleration a as a function of time?

(A)


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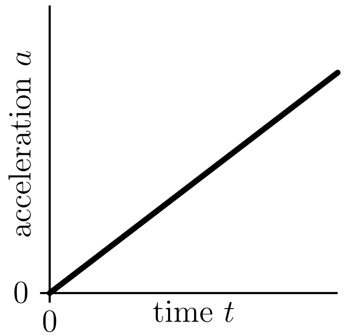

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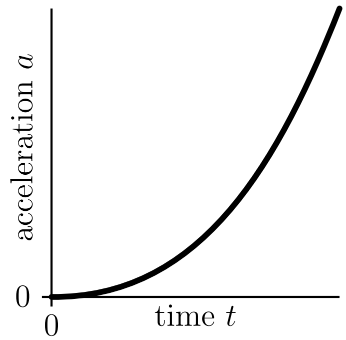

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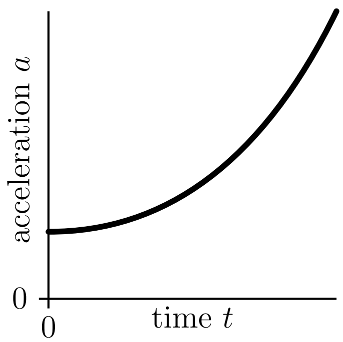

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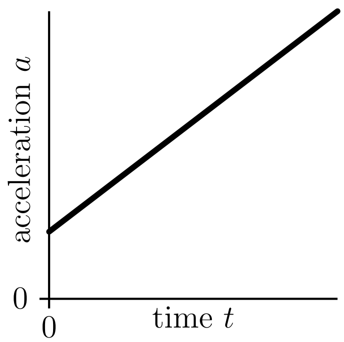


D ← CORRECT

Solution

The acceleration tangent to the incline of the helix is given by

$$a_t = g \sin\theta,$$

where θ is the angle of the incline of the helix. As it accelerates, it moves faster, according to

$$v = a_t t.$$

Since this is motion in a circle, then there is a radial acceleration given by

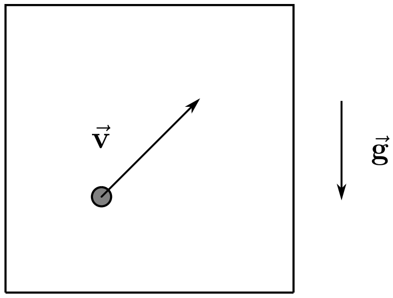
$$a_r = \frac{v^2}{r} = \frac{{a_t}^2}{r} t^2$$

The magnitude of the acceleration is then

$$a = \sqrt{{a_r}^2 + {a_t}^2} = a_t \sqrt{{a_t}^2 \frac{t^4}{r^2} + 1}$$

The following information applies to questions 5 and 6.

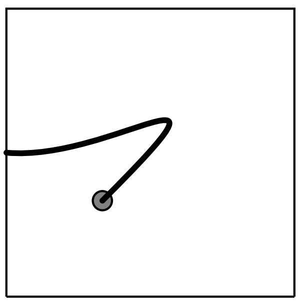
Consider a particle in a box where the force of gravity is down as shown in the figure.



The particle has an initial velocity as shown, and the box has a constant acceleration to the right.

5. In the frame of the box, which of the following is a possible path followed by the particle?

(A)



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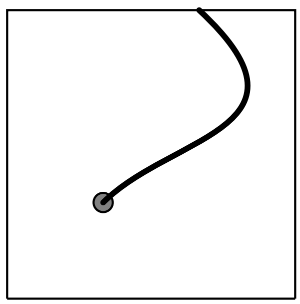

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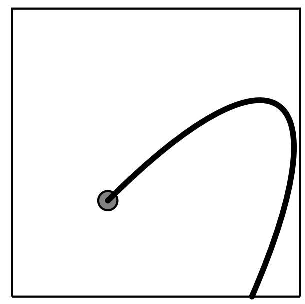

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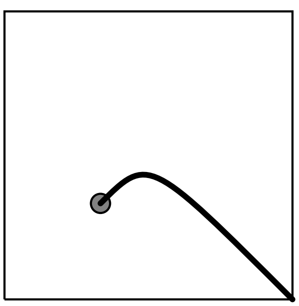

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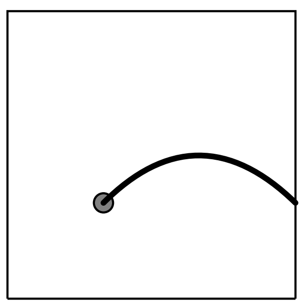
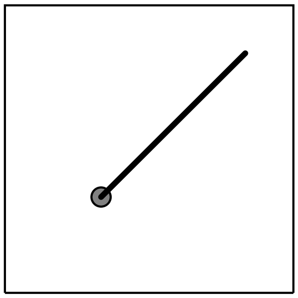


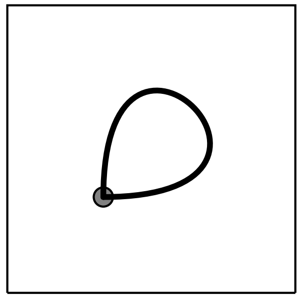
C ← CORRECT

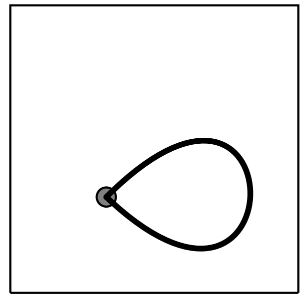
Solution

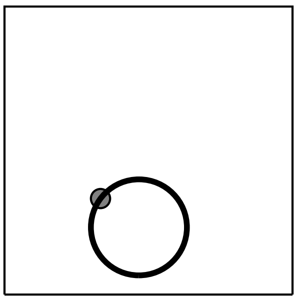
Uniformly accelerated motion is just parabolic; E is wrong because it only works if there is no horizontal acceleration.

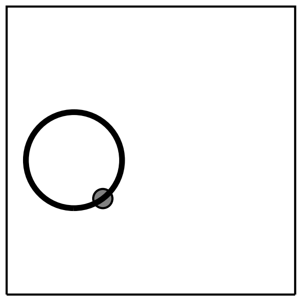
6. If the magnitude of the acceleration of the box is chosen correctly, the launched particle will follow a path that returns to the point that it was launched. In the frame of the box, which path is followed by the particle?

(A) 

(B) 

(C) 

(D) 

(E) 

A ← CORRECT

Solution

See previous solution.

7. A mass on a frictionless table is attached to the midpoint of an originally unstretched spring fixed at the ends. If the mass is displaced a distance A parallel to the table surface but perpendicular to the spring, it exhibits oscillations. The period T of the oscillations

   (A) does not depend on A.

   (B) increases as A increases, approaching a fixed value.

   (C) decreases as A increases, approaching a fixed value. ← **CORRECT**

   (D) is approximately constant for small values of A, then increases without bound.

   (E) is approximately constant for small values of A, then decreases without bound.

Solution

For large values of A, the mass performs simple harmonic motion, so the period is constant. For small values of A, the potential is quartic, and increases more slowly than the corresponding quadratic, so the period is larger than the limiting value.

8. Kepler's Laws state that

   I. the orbits of planets are elliptical with one focus at the sun,

   II. a line connecting the sun and a planet sweeps out equal areas in equal times, and

   III. the square the period of a planet's orbit is proportional to the cube of its semimajor axis.

   Which of these laws would remain true if the force of gravity were proportional to $1/r^3$ rather than $1/r^2$?

   (A) Only I.

   (B) Only II. ← **CORRECT**

   (C) Only III.

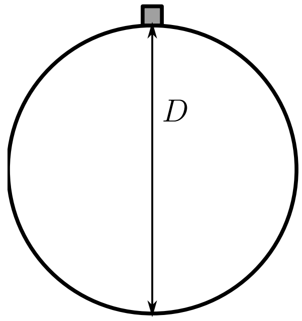
   (D) Both II and III.

   (E) None of the above.

Solution

The second law is a consequence of conservation of angular momentum, which is true for any central potential. The other two are specific to the inverse-square potential.

9. A small bead is placed on the top of a frictionless glass sphere of diameter D as shown. The bead is given a slight push and starts sliding down along the sphere. Find the speed v of the bead at the point at which the which the bead leaves the sphere.



(A) $v = \sqrt{gD}$

(B) $v = \sqrt{4gD/5}$

(C) $v = \sqrt{2gD/3}$

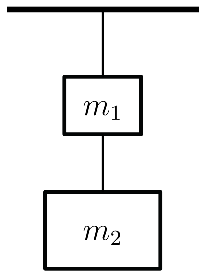
(D) $v = \sqrt{gD/2}$

(E) $v = \sqrt{gD/3}$ $\leftarrow$ **CORRECT**

Solution

$$mgh = \frac{1}{2}mv^2 mg\sin\theta = m\frac{v^2}{R}$$

10. Two blocks are suspended by two massless elastic strings to the ceiling as shown in the figure. The masses of the upper and lower block are $m_1 = 2$ kg and $m_2 = 4$ kg respectively. If the upper string is suddenly cut just above the top block what are the accelerations of the two blocks at the moment when the top block begins to fall?



(A) upper: 10 m/s^2; lower: 0

(B) upper: 10 m/s^2; lower: 10 m/s^2

(C) upper: 20 m/s^2; lower: 10 m/s^2

(D) upper: 30 m/s^2; lower: 0 ← **CORRECT**

(E) upper: 30 m/s^2; lower: 10 m/s^2

Solution

Draw free body diagrams for object 1 and 2 before the string is cut. These diagrams still apply immediately after the string is cut, except that the cut string no longer contributes a force in the free body diagram for object 1. Use the resulting free body diagrams to find acceleration of the two blocks.

11. The power output from a certain experimental car design to be shaped like a cube is proportional to the mass m of the car. The force of air friction on the car is proportional to Av^2, where v is the speed of the car and A the cross sectional area. On a level surface the car has a maximum speed v_{max}. Assuming that all versions of this design have the same density, then which of the following is true?

    (A) $v_{\text{max}} \propto m^{1/9}$ ← **CORRECT**

    (B) $v_{\text{max}} \propto m^{1/7}$

    (C) $v_{\text{max}} \propto m^{1/3}$

    (D) $v_{\text{max}} \propto m^{2/3}$

    (E) $v_{\text{max}} \propto m^{3/4}$

Solution

The size of the cube is $L \propto m^{1/3}$, so $A \propto m^{2/3}$ since the density is constant.

Constant speed requires

$$m \propto P = Fv \propto Av^3 \propto m^{2/3}v^3$$

Therefore

$$m \propto v^9$$

12. A block floats partially submerged in a container of liquid. When the entire container is accelerated upward, which of the following happens? Assume that both the liquid and the block are incompressible.

    (A) The block descends down lower into the liquid.

    (B) The block ascends up higher in the liquid.

    (C) The block does not ascend nor descend in the liquid. ← **CORRECT**

    (D) The answer depends on the direction of motion of the container.

    (E) The answer depends on the rate of change of the acceleration

Solution

Check out the videos from the University of Maryland Physics IQ test (which might be the Physics is Phun site now).

13. An object of mass m_1 initially moving at speed v_0 collides with an originally stationary object of mass $m_2 = \alpha m_1$, where $\alpha < 1$. The collision could be completely elastic, completely inelastic, or partially inelastic. After the collision the two objects move at speeds v_1 and v_2. Assume that the collision is one dimensional, and that object one cannot pass through object two.

After the collision, the speed ratio $r_2 = v_2/v_0$ of object 2 is bounded by

(A) $(1-\alpha)/(1+\alpha) \leq r_2 \leq 1$

(B) $(1-\alpha)/(1+\alpha) \leq r_2 \leq 1/(1+\alpha)$

(C) $\alpha/(1+\alpha) \leq r_2 \leq 1$

(D) $0 \leq r_2 \leq 2\alpha/(1+\alpha)$

(E) $1/(1+\alpha) \leq r_2 \leq 2/(1+\alpha)$ ← **CORRECT**

Solution

Conserving momentum and kinetic energy in the completely elastic collision yields the following quantities.

$$r_1 = \frac{v_1}{v_0} = \frac{1-\alpha}{(1+\alpha)},$$

$$r_2 = \frac{v_2}{v_0} = \frac{2}{(1+\alpha)},$$

Since $\alpha < 1$, object 1 is tricking a less massive object, and therefore continues to move forward.

Conserving momentum in a completely inelastic collision yields

$$r_1 = r_2 = \frac{1}{(1+\alpha)},$$

Note that object 2 will always be moving forward, and since object 1 can't pass through it, object 2 must always move with a more positive (or equal) velocity than object 1. Consequently,

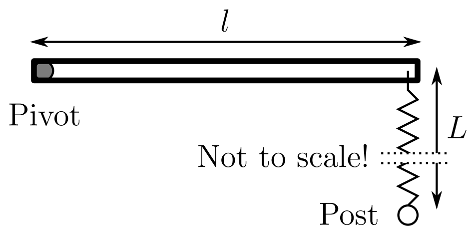
$$1/(1+\alpha) \leq r_2 \leq 2/(1+\alpha)$$

Sorting out object 1 is a little harder, since under certain circumstances it can bounce backward. But in this case, since $\alpha < 1$, it retains a forward velocity after the collision. Then

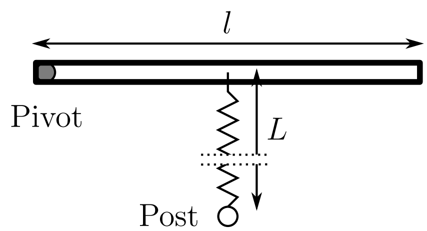
$$(1-\alpha)/(1+\alpha) \leq r_1 \leq 1/(1+\alpha)$$

The following information applies to questions 14 and 15.

A uniform rod of length l lies on a frictionless horizontal surface. One end of the rod is attached to a pivot. An un-stretched spring of length $L \gg l$ lies on the surface perpendicular to the rod; one end of the spring is attached to the movable end of the rod, and the other end is attached to a fixed post. When the rod is rotated slightly about the pivot, it oscillates at frequency f.



14. The spring attachment is moved to the midpoint of the rod, and the post is moved so the spring remains unstretched and perpendicular to the rod. The system is again set into small oscillations. What is the new frequency of oscillation?



(A) $f/2$ ← **CORRECT**

(B) $f/\sqrt{2}$

(C) f

(D) $\sqrt{2}f$

(E) $2f$

Solution

Note that the spring remains perpendicular to the rod to first order.

Suppose that the moment of inertia of the rod about the pivot is I, the distance from the pivot to the spring attachment is r, and the constant of the spring is k. When the rod is displaced by an angle θ, the spring is extended by an amount θr, and the force exerted by the spring is $k\theta r$. The lever arm is r. The equation of motion is therefore

$$\tau = I\alpha$$

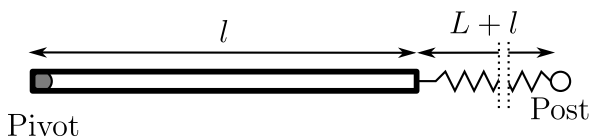
$$-kr^2\theta = I\ddot{\theta}$$

This is a standard simple harmonic oscillator with frequency

$$f = \frac{1}{2\pi}\omega = \frac{1}{2\pi}\sqrt{\frac{kr^2}{I}}$$

The frequency is thus proportional to the distance r; the effect of halving the distance is to halve the frequency.

15. The spring attachment is moved back to the end of the rod; the post is moved so that it is in line with the rod and the pivot and the spring is unstretched. The post is then moved away from the pivot by an additional amount l. What is the new frequency of oscillation?



(A) $f/3$

(B) $f/\sqrt{3}$

(C) f ← **CORRECT**

(D) $\sqrt{3}f$

(E) $3f$

Solution

Note that the tension in the spring can be taken as a constant. Then (using the same symbols as in the previous part) the force exerted by the spring is kl and the lever arm is $l\theta$; the equation of motion is then

$$-kl^2\theta = I\ddot{\theta}$$

This matches the previous equation of motion (with $r = l$), so the frequency remains f.

16. A ball rolls from the back of a large truck traveling 10.0 m/s to the right. The ball is traveling horizontally at 8.0 m/s to the left relative to an observer in the truck. The ball lands on the roadway 1.25 m below its starting level. How far behind the truck does it land?

(A) 0.50 m

(B) 1.0 m

(C) 4.0 m ← **CORRECT**

(D) 5.0 m

(E) 9.0 m

Solution

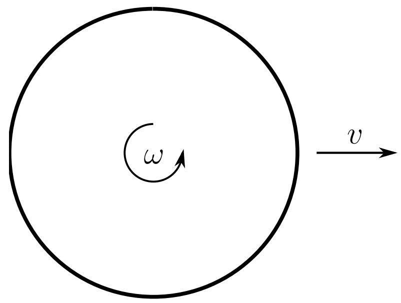
Use

$$d = \frac{1}{2}at^2$$

and find that the ball falls for 0.5 s.

Relative to the truck it is then (8.0 m/s)(0.5 s) = 4.0 m behind the truck.

17. As shown in the figure, a ping-pong ball with mass m with initial horizontal velocity v and angular velocity ω comes into contact with the ground. Friction is not negligible, so both the velocity and angular velocity of the ping-pong ball changes. What is the critical velocity v_c such that the ping-pong will stop and remain stopped? Treat the ping-pong ball as a hollow sphere.



(A) $v = \frac{2}{3}R\omega$ ← **CORRECT**

(B) $v = \frac{2}{5}R\omega$

(C) $v = R\omega$

(D) $v = \frac{3}{5}R\omega$

(E) $v = \frac{5}{3}R\omega$

Solution

The ball initial momentum is mv, the initial angular momentum about the center is $I\omega = \frac{2}{3}mR^2\omega$.

When the ball comes into contact with he ground there is a frictional impulse $F\delta t$ and a frictional torque $FR\delta t$. So

$$F\delta t = mv$$

and

$$Fr\delta t = \frac{2}{3}mr^2\omega,$$

which yields, when dividing one expression by the other,

$$R = \frac{2}{3}R^2\frac{\omega}{v}.$$

18. A spinning object begins from rest and accelerates to an angular velocity of $\omega = \pi/15$ rad/s with an angular acceleration of $\alpha = \pi/75$ rad/s^2. It remains spinning at that constant angular velocity and then stops with an angular acceleration of the same magnitude as it previously accelerated. The object made a total of 3 complete rotations during the entire motion. How much time did the motion take?

    (A) 75 s

    (B) 80 s

    (C) 85 s

    (D) 90 s

    (E) 95 s ← **CORRECT**

Solution

There are three motions. Spinning up to speed, constant speed, and slowing down. The changing speed regions are symmetric, so the distance $\theta = 6\pi$ is

$$\theta = \alpha t_1^2 + \omega t_2$$

but

$$\omega = \alpha t_1$$

so

$$\theta = \alpha \left(\frac{\omega}{\alpha}\right)^2 + \omega t_2$$

which gives

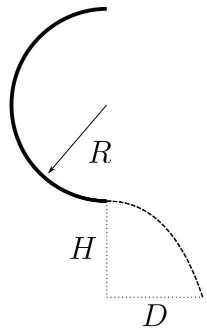
$$t_2 = 85 \text{ s}$$

and

$$t_1 = 5 \text{ s}$$

Note that t_1 happens twice!

19. A semicircular wire of radius R is oriented vertically. A small bead is released from rest at the top of the wire, it slides without friction under the influence of gravity to the bottom, where is then leaves the wire horizontally and falls a distance H to the ground. The bead lands a horizontal distance D away from where it was launched.



Which of the following is a correct graph of RH against D^2?

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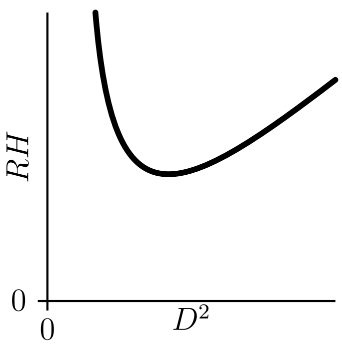

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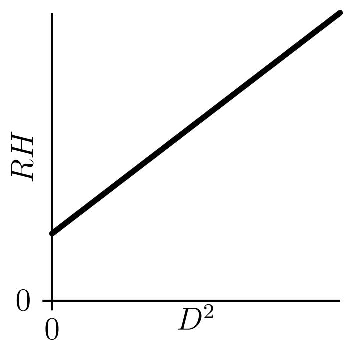

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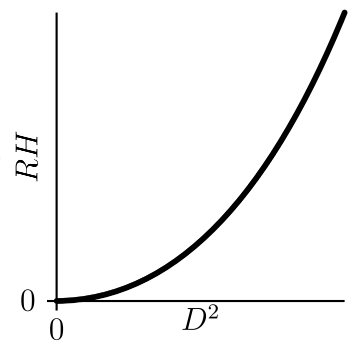

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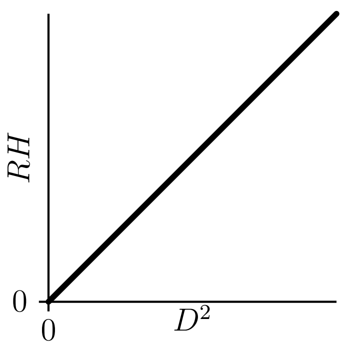
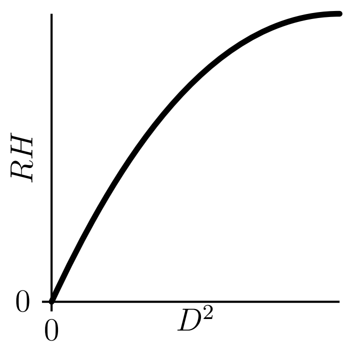


(E)


D ← CORRECT

Solution

Let v be the speed the bead leaves the wire. Then by conservation of energy,

$$\frac{1}{2}v^2 = 2gR$$

Let t be the time the bead is a projectile. Then

$$D = vt$$

and

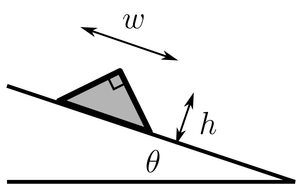
$$H = \frac{1}{2}gt^2 = \frac{1}{2}g\left(\frac{D}{v}\right)^2$$

Combine, and

$$H = \frac{1}{8}\frac{D^2}{R}$$

so that a graph of HR against D^2 is a straight line.

20. A uniform solid right prism whose cross section is an isosceles right triangle with height h and width $w = 2h$ is placed on an incline that has a variable angle with the horizontal θ. What is the minimum coefficient of static friction so that the prism topples before it begins sliding as θ is slowly increased from zero?



(A) 0.71

(B) 1.41

(C) 1.50

(D) 1.73

(E) 3.00 ← **CORRECT**

Solution

Toppling occurs when the center of mass of prism is above right corner of the base. This reduces the problems to trigonometry. Sliding occurs if

$$\tan\theta > \mu,$$

so equate.

The following information applies to questions 21 and 22.

A small ball of mass $3m$ is at rest on the ground. A second small ball of mass m is positioned above the ground by a vertical massless rod of length L that is also attached to the ball on the ground. The original orientation of the rod is directly vertical, and the top ball is given a small horizontal nudge. There is no friction; assume that everything happens in a single plane.

21. Determine the horizontal displacement x of the second ball just before it hits the ground.

(A) $x = \frac{3}{4}L$ $\leftarrow$ **CORRECT**

(B) $x = \frac{3}{5}L$

(C) $x = \frac{1}{4}L$

(D) $x = \frac{1}{3}L$

(E) $x = \frac{2}{5}L$

Solution

The center of mass of the system can't move in the horizontal direction, since there are no horizontal forces.

As such, $x = \frac{3}{4}L$.

22. Determine the speed v of the second (originally top) ball just before it hits the ground.

(A) $v = \sqrt{2gL}$ ← **CORRECT**

(B) $v = \sqrt{gL}$

(C) $v = \sqrt{2gL/3}$

(D) $v = \sqrt{3gL/2}$

(E) $v = \sqrt{gL/4}$

Solution

The second ball will be moving vertically just before it hits the ground, while the first ball will have moved, but now be at rest. energy is conserved, so all of the potential energy of the center of mass of the system will now be in the second ball, so

$$U = 4mgh$$

where h is the height of the center of mass, which is

$$h = \frac{1}{4}L$$

so

$$U = mgL$$

But, of course!

That is converted to kinetic energy of the second ball, so

$$mgL = \frac{1}{2}mv^2$$

or

$$v = \sqrt{2gL}$$

23. A uniform thin circular rubber band of mass M and spring constant k has an original radius R. Now it is tossed into the air. Assume it remains circular when stabilized in air and rotates at angular speed ω about its center uniformly. Which of the following gives the new radius of the rubber band?

(A) $(2\pi kR)/(2\pi k - M\omega^2)$

(B) $(4\pi kR)/(4\pi k - M\omega^2)$

(C) $(8\pi^2 kR)/(8\pi^2 k - M\omega^2)$

(D) $(4\pi^2 kR)/(4\pi^2 k - M\omega^2)$ ← **CORRECT**

(E) $(4\pi kR)/(2\pi k - M\omega^2)$

Solution

$$\text{For a mass element } \Delta m : 2T \sin\frac{\Delta\theta}{2} = \Delta m \omega^2 R'; \sin\frac{\Delta\theta}{2} \approx \frac{\Delta\theta}{2} \quad (1)$$

$$\rightarrow \quad T = \frac{M}{2\pi}\omega^2 R'$$

$$\text{Meanwhile, } T = k2\pi(R' - R) \quad (2)$$

$$\rightarrow \quad R' = \frac{4\pi^2 kR}{4\pi^2 k - M\omega^2} \quad (3)$$

24. The moment of inertia of a uniform equilateral triangle with mass m and side length a about an axis through one of its sides and parallel to that side is $(1/8)ma^2$. What is the moment of inertia of a uniform regular hexagon of mass m and side length a about an axis through two opposite vertices?

(A) $(1/6)ma^2$

(B) $(5/24)ma^2$ ← **CORRECT**

(C) $(17/72)ma^2$

(D) $(19/72)ma^2$

(E) $(9/32)ma^2$

Solution

The hexagon can be decomposed into six equilateral triangles of mass m. Four of them contribute $(1/8)(m/6)a^2$, and two of them contribute $((1/24 + 1/3)a^2)(m/6)$ by the parallel axis theorem. The total is $(5/4)(m/6)a^2 = (5/24)ma^2$.

25. Three students make measurements of the length of a 1.50 meter rod. Each student reports an uncertainty estimate representing an independent random error applicable to the measurement.

Alice: A single measurement using a 2.0 meter long tape measure, to within ±2 mm.

Bob: Two measurements using a wooden meter stick, each to within ±2 mm, which he adds together.

Christina: Two measurements using a machinist's meter ruler, each to within ±1 mm, which she adds together.

The students' teacher prefers measurements that are likely to have less error. Which is the correct order of preference?

(A) Christina's is preferable to Alice's, which is preferable to Bob's ← **CORRECT**

(B) Alice's is preferable to Christina's, which is preferable to Bob's

(C) Alice's and Christina's are equally preferable; both are preferable to Bob's

(D) Christina's is preferable to both Alice's and to Bob's, which are equally preferable

(E) Alice's is preferable to Bob's and Christina's, which are equally preferable