

## 2017 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

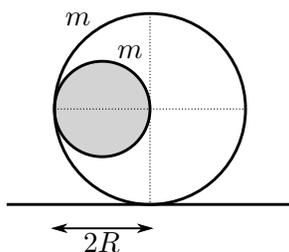
### INSTRUCTIONS

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

- Use  $g = 10 \text{ N/kg}$  throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2017.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

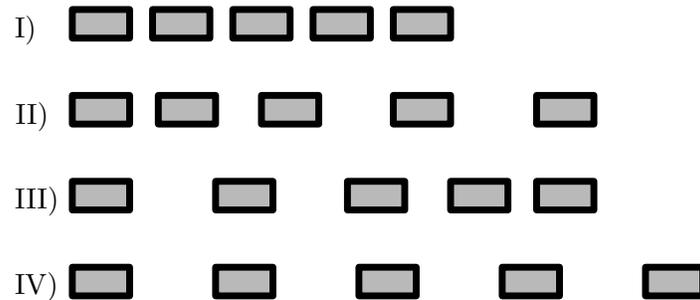
**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

1. A motorcycle rides on the vertical walls around the perimeter of a large circular room. The friction coefficient between the motorcycle tires and the walls is  $\mu$ . How does the minimum  $\mu$  needed to prevent the motorcycle from slipping downwards change with the motorcycle's speed,  $s$ ?
- (A)  $\mu \propto s^0$   
(B)  $\mu \propto s^{-1/2}$   
(C)  $\mu \propto s^{-1}$   
(D)  $\mu \propto s^{-2}$   
(E) none of these
2. A mass  $m$  hangs from a massless spring connected to the roof of a box of mass  $M$ . When the box is held stationary, the mass-spring system oscillates vertically with angular frequency  $\omega$ . If the box is dropped and falls freely under gravity, how will the angular frequency change?
- (A)  $\omega$  will be unchanged  
(B)  $\omega$  will increase  
(C)  $\omega$  will decrease  
(D) Oscillations are impossible under these conditions  
(E)  $\omega$  will increase or decrease depending on the values of  $M$  and  $m$
3. A ball of radius  $R$  and mass  $m$  is magically put inside a thin shell of the same mass and radius  $2R$ . The system is at rest on a horizontal frictionless surface initially. When the ball is, again magically, released inside the shell, it sloshes around in the shell and eventually stops at the bottom of the shell. How far does the shell move from its initial contact point with the surface?



- (A)  $R$ .  
(B)  $R/2$ .  
(C)  $R/4$ .  
(D)  $3R/8$ .  
(E)  $R/8$

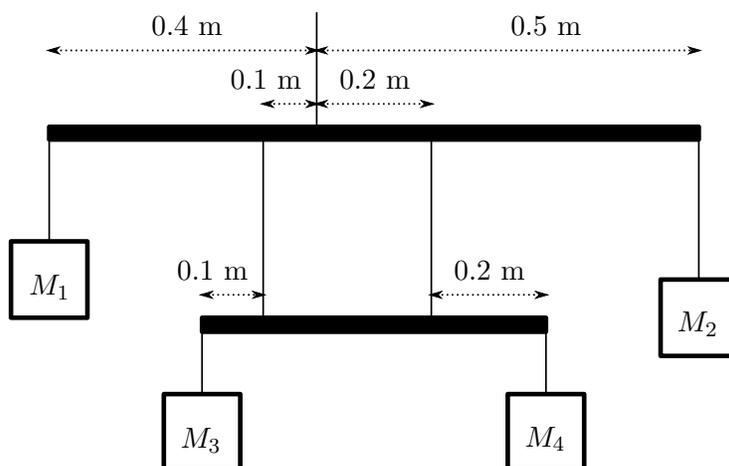
4. Several identical cars are standing at a red light on a one-lane road, one behind the other, with negligible (and equal) distance between adjacent cars. When the green light comes up, the first car takes off to the *right* with constant acceleration. The driver in the second car reacts and does the same 0.2 s later. The third driver starts moving 0.2 s after the second one and so on. All cars accelerate until they reach the speed limit of 45 km/hr, after which they move to the right at a constant speed. Consider the following patterns of cars.



Just before the first car starts accelerating to the right, the car pattern will qualitatively look like the pattern in I. After that, the pattern will qualitatively evolve according to

- (A) First I, then II, and then III.  
 (B) First I, then II, and then IV.  
 (C) First I, and then IV, with neither II nor III as intermediate stage.  
 (D) First I, and then II.  
 (E) First I, and then III.
5. A projectile is launched with speed  $v_0$  off the edge of a cliff of height  $h$ , at an angle  $\theta$  from the horizontal. Air friction is negligible. To maximize the horizontal range of the projectile,  $\theta$  should satisfy
- (A)  $45^\circ < \theta < 90^\circ$   
 (B)  $\theta = 45^\circ$   
 (C)  $0^\circ < \theta < 45^\circ$   
 (D)  $\theta = 0^\circ$   
 (E)  $\theta < 45^\circ$  or  $\theta > 45^\circ$ , depending on the values of  $h$  and  $v_0$ .

6. In the mobile below, the two cross beams and the seven supporting strings are all massless. The hanging objects are  $M_1 = 400$  g,  $M_2 = 200$  g, and  $M_4 = 500$  g. What is the value of  $M_3$  for the system to be in static equilibrium?



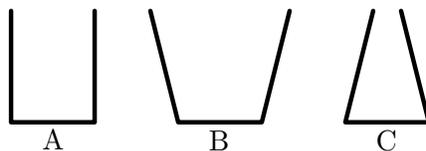
- (A) 300 g  
 (B) 400 g  
 (C) 500 g  
 (D) 600 g  
 (E) 700 g

**The following information applies to questions 7 and 8.**

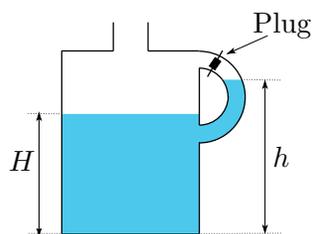
A train, originally of mass  $M$ , is traveling on a frictionless straight horizontal track with constant speed  $v$ . Snow starts to fall vertically and sticks to the train at a rate of  $\rho$ , where  $\rho$  has units of kilograms per second. The train's engine keeps the train moving at constant speed  $v$  as snow accumulates on the train.

7. The rate at which the kinetic energy of the train and snow increases is
- (A) 0  
 (B)  $Mgv$   
 (C)  $\frac{1}{2}Mv^2$   
 (D)  $\frac{1}{2}\rho v^2$   
 (E)  $\rho v^2$
8. The minimum power required from the engine to keep the train traveling at a constant speed  $v$  is
- (A) 0  
 (B)  $Mgv$   
 (C)  $\frac{1}{2}Mv^2$   
 (D)  $\frac{1}{2}\rho v^2$   
 (E)  $\rho v^2$

9. Flasks A, B, and C each have a circular base with a radius of 2 cm. An equal volume of water is poured into each flask, and none overflow. Rank the force of water  $F$  on the base of the flask from greatest to least.



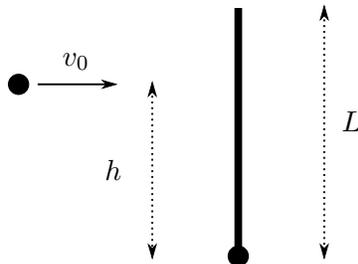
- (A)  $F_A > F_B > F_C$   
 (B)  $F_A > F_C > F_B$   
 (C)  $F_B > F_C > F_A$   
 (D)  $F_C > F_A > F_B$   
 (E)  $F_A = F_B = F_C$
10. The handle of a gallon of milk is plugged by a manufacturing defect. After removing the cap and pouring out some milk, the level of milk in the main part of the jug is lower than in the handle, as shown in the figure. Which statement is true of the gauge pressure  $P$  of the milk at the bottom of the jug?  $\rho$  is the density of the milk.



- (A)  $P = \rho gh$   
 (B)  $P = \rho gH$   
 (C)  $\rho gH < P < \rho gh$   
 (D)  $P > \rho gh$   
 (E)  $P < \rho gH$

The following information applies to questions 11 and 12.

A small hard solid sphere of mass  $m$  and negligible radius is connected to a thin rod of length  $L$  and mass  $2m$ . A second small hard solid sphere, of mass  $M$  and negligible radius, is fired perpendicularly at the rod at a distance  $h$  above the sphere attached to the rod, and sticks to it.

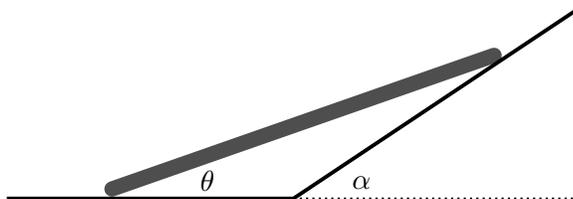


11. In order for the rod not to rotate after the collision, the second sphere should hit the thin rod at
- (A)  $h = 0$
  - (B)  $h = L/3$
  - (C)  $h = L/2$
  - (D)  $h = L$
  - (E) Any location  $L$  will work
12. In order for the rod not to rotate after the collision, the second sphere should have a mass  $M$  given by
- (A)  $M = m$
  - (B)  $M = 1.5m$
  - (C)  $M = 2m$
  - (D)  $M = 3m$
  - (E) Any mass  $M$  will work.
13. A massless rope passes over a frictionless pulley. Particles of mass  $M$  and  $M + m$  are suspended from the two different ends of the rope. If  $m = 0$ , the tension  $T$  in the pulley rope is  $Mg$ . If instead the value  $m$  increases to infinity, the value of the tension
- (A) stays constant
  - (B) decreases, approaching a nonzero constant
  - (C) decreases, approaching zero
  - (D) increases, approaching a finite constant
  - (E) increases to infinity

The following information applies to questions 14 and 15.

An object starting from rest can roll without slipping down an incline.

14. Which of the following four objects, each with mass  $M$  and radius  $R$ , would have the largest acceleration down the incline?
- (A) A uniform solid sphere
  - (B) A uniform solid disk
  - (C) A hollow spherical shell
  - (D) A hoop
  - (E) All objects would have the same acceleration.
15. Which of the following four objects, each a uniform solid sphere released from rest, would have the largest speed after the center of mass has moved through a vertical distance  $h$ ?
- (A) A sphere of mass  $M$  and radius  $R$ .
  - (B) A sphere of mass  $2M$  and radius  $\frac{1}{2}R$ .
  - (C) A sphere of mass  $M/2$  and radius  $2R$ .
  - (D) A sphere of mass  $3M$  and radius  $3R$ .
  - (E) All objects would have the same speed.
16. A rod moves freely between the horizontal floor and the slanted wall. When the end in contact with the floor is moving at  $v$ , what is the speed of the end in contact with the wall?

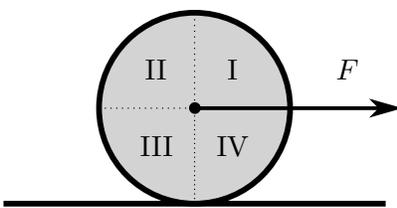


- (A)  $v \frac{\sin \theta}{\cos(\alpha - \theta)}$ .
- (B)  $v \frac{\sin(\alpha - \theta)}{\cos(\alpha + \theta)}$ .
- (C)  $v \frac{\cos(\alpha - \theta)}{\sin(\alpha + \theta)}$ .
- (D)  $v \frac{\cos \theta}{\cos(\alpha - \theta)}$ .
- (E)  $v \frac{\sin \theta}{\cos(\alpha + \theta)}$ .

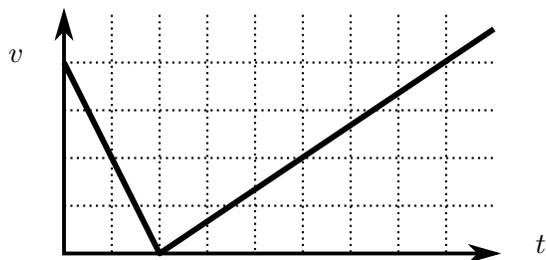
17. An object is thrown directly downward from the top of a 180 meter tall building. It takes 1.0 seconds for the object to fall the last 60 meters. With what initial downward speed was the object thrown from the roof?

(A) 15 m/s  
 (B) 25 m/s  
 (C) 35 m/s  
 (D) 55 m/s  
 (E) insufficient information.

18. A uniform disk is being pulled by a force  $F$  through a string attached to its center of mass. Assume that the disk is rolling smoothly without slipping. At a certain instant of time, in which region of the disk (if any) is there a point moving with zero total acceleration?



- (A) Region I  
 (B) Region II  
 (C) Region III  
 (D) Region IV  
 (E) All points on the disk have non-zero acceleration.
19. A puck is kicked up a ramp, which makes an angle of  $30^\circ$  with the horizontal. The graph below depicts the speed of the puck versus time. What is the coefficient of friction between the puck and the ramp?



(A) 0.07  
 (B) 0.15  
 (C) 0.22  
 (D) 0.29  
 (E) 0.37

The following information applies to questions 20, 21, and 22.

A particle of mass  $m$  moving at speed  $v_0$  collides with a particle of mass  $M$  which is originally at rest. The fractional momentum transfer  $f$  is the absolute value of the final momentum of  $M$  divided by the initial momentum of  $m$ .

20. If the collision is completely *inelastic*, under what condition will the fractional momentum transfer between the two objects be a maximum?

(A)  $m/M \ll 1$   
(B)  $0.5 < m/M < 1$   
(C)  $m = M$   
(D)  $1 < m/M < 2$   
(E)  $m/M \gg 1$

21. If the collision is perfectly *elastic*, what is the maximum possible fractional momentum transfer,  $f_{\max}$ ?

(A)  $0 < f_{\max} < \frac{1}{2}$   
(B)  $f_{\max} = \frac{1}{2}$   
(C)  $\frac{1}{2} < f_{\max} < \frac{3}{2}$   
(D)  $f_{\max} = 2$   
(E)  $f_{\max} \geq 3$

22. The fractional energy transfer is the absolute value of the final kinetic energy of  $M$  divided by the initial kinetic energy of  $m$ .

If the collision is perfectly elastic, under what condition will the fractional energy transfer between the two objects be a maximum?

(A)  $m/M \ll 1$   
(B)  $0.5 < m/M < 1$   
(C)  $m = M$   
(D)  $1 < m/M < 2$   
(E)  $m/M \gg 1$

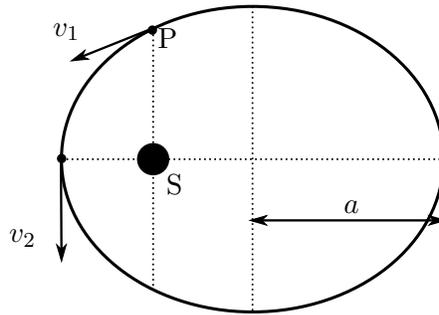
23. A spring has a length of 1.0 meter when there is no tension on it. The spring is then stretched between two points 10 meters apart. A wave pulse travels between the two end points in the spring in a time of 1.0 seconds. The spring is now stretched between two points that are 20 meters apart. The new time it takes for a wave pulse to travel between the ends of the spring is closest to

(A) 0.5 seconds  
(B) 0.7 seconds  
(C) 1 second  
(D) 1.4 seconds  
(E) 2 seconds

24. A ball of mass  $m$  moving at speed  $v$  collides with a massless spring of spring constant  $k$  mounted on a stationary box of mass  $M$  in free space. No mechanical energy is lost in the collision. If the system does not rotate, what is the maximum compression  $x$  of the spring?

- (A)  $x = v\sqrt{\frac{mM}{(m+M)k}}$   
 (B)  $x = v\sqrt{\frac{m}{k}}$   
 (C)  $x = v\sqrt{\frac{M}{k}}$   
 (D)  $x = v\sqrt{\frac{m+M}{k}}$   
 (E)  $x = v\sqrt{\frac{(m+M)^3}{mMk}}$

25. A planet orbits around a star S, as shown in the figure. The semi-major axis of the orbit is  $a$ . The perigee, namely the shortest distance between the planet and the star is  $0.5a$ . When the planet passes point P (on the line through the star and perpendicular to the major axis), its speed is  $v_1$ . What is its speed  $v_2$  when it passes the perigee?



- (A)  $v_2 = \frac{3}{\sqrt{5}}v_1$ .  
 (B)  $v_2 = \frac{3}{\sqrt{7}}v_1$ .  
 (C)  $v_2 = \frac{2}{\sqrt{3}}v_1$ .  
 (D)  $v_2 = \frac{\sqrt{7}}{\sqrt{3}}v_1$ .  
 (E)  $v_2 = 4v_1$ .