

2018 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

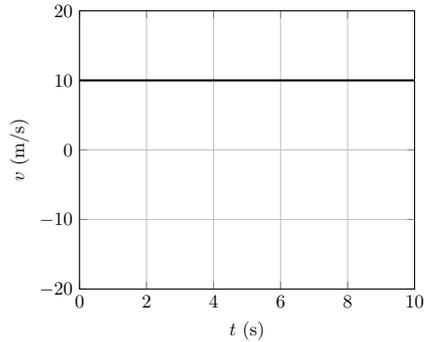
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10 \text{ N/kg}$ throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2018.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

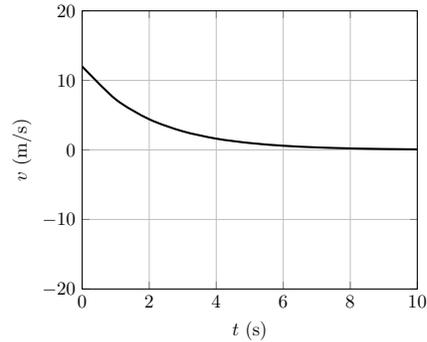
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1. Which of the following graphs best shows the velocity versus time of an object originally moving upward in the presence of air friction?

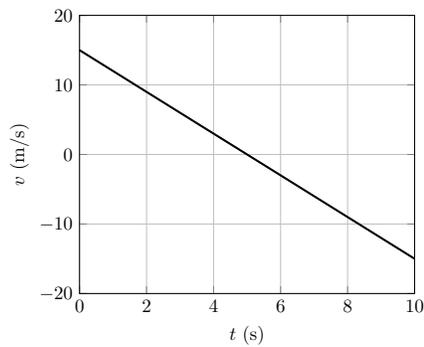
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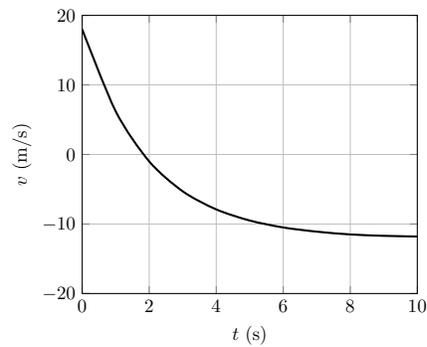
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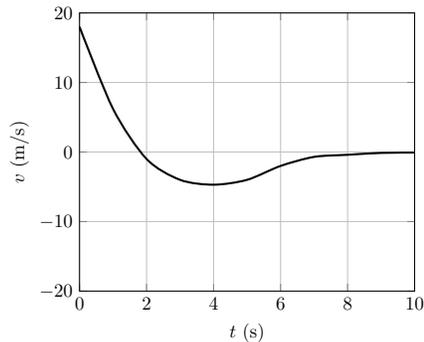
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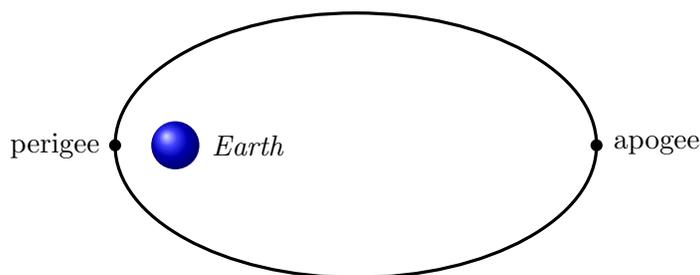
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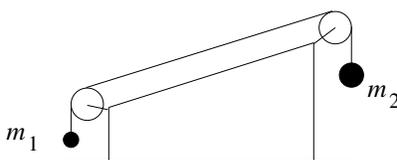
2. A 3.0 kg mass moving at 30 m/s to the right collides elastically with a 2.0 kg mass traveling at 20 m/s to the left. After the collision, the center of mass of the system is moving at a speed of

- (A) 5 m/s
 (B) 10 m/s
 (C) 20 m/s
 (D) 24 m/s
 (E) 26 m/s

3. Ball 1 traveling in the positive x direction strikes an equal mass ball 2 that is originally at rest. All of the following must be true after the collision, except
- (A) The total final momentum in the x direction equals the initial momentum of ball 1.
 - (B) The total kinetic energy after the collision equals the initial kinetic energy of ball 1.
 - (C) The final momentum of the two balls in the y direction adds to zero.
 - (D) The final speed of the center of mass of the two balls is equal to half the initial speed of ball 1.
 - (E) The balls can't both be at rest after the collision.
4. A satellite is following an elliptical orbit around the Earth. Its engines are capable of providing a one-time impulse of a fixed magnitude. In order to maximize the energy of the satellite, the impulse should be

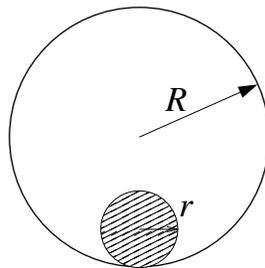


- (A) directed along the satellite's velocity and applied when the satellite is in its perigee.
 - (B) directed along the satellite's velocity and applied when the satellite is in apogee.
 - (C) directed toward the Earth and applied when the satellite is in perigee.
 - (D) directed toward the Earth and applied when the satellite is in apogee.
 - (E) directed away from the Earth and applied when the satellite is in apogee.
5. Two masses are attached with pulleys by a massless rope on an inclined plane as shown. All surfaces are frictionless. If the masses are released from rest, then the inclined plane



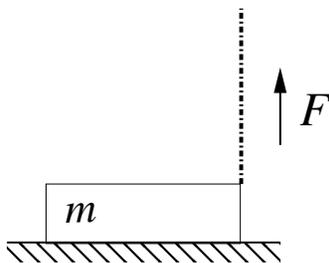
- (A) accelerates to the left if $m_1 < m_2$
- (B) accelerates to the right if $m_1 < m_2$
- (C) accelerates to the left regardless of the masses
- (D) accelerates to the right regardless of the masses
- (E) does not move

6. A packing crate with mass $m = 115$ kg is slid up a 5.00 m long ramp which makes an angle of 20.0° with respect to the horizontal by an applied force of $F = 1.00 \times 10^3$ N directed parallel to the ramp's incline. A frictional force of magnitude $f = 4.00 \times 10^2$ N resists the motion. If the crate starts from rest, what is its speed at the top of the ramp?
- (A) 4.24 m/s
(B) 5.11 m/s
(C) 7.22 m/s
(D) 8.26 m/s
(E) 9.33 m/s
7. A car has a maximum acceleration of a_0 and a minimum acceleration of $-a_0$. The shortest possible time for the car to begin at rest, then arrive at rest at a point a distance d away is
- (A) $\sqrt{d/2a_0}$
(B) $\sqrt{d/a_0}$
(C) $\sqrt{2d/a_0}$
(D) $\sqrt{3d/a_0}$
(E) $2\sqrt{d/a_0}$
8. A disk of radius r rolls uniformly without slipping around the inside of a fixed hoop of radius R . If the period of the disc's motion around the hoop is T , what is the instantaneous speed of the point on the disk opposite to the point of contact?



- (A) $2\pi(R + r)/T$
(B) $2\pi(R + 2r)/T$
(C) $4\pi(R - 2r)/T$
(D) $4\pi(R - r)/T$
(E) $4\pi(R + r)/T$

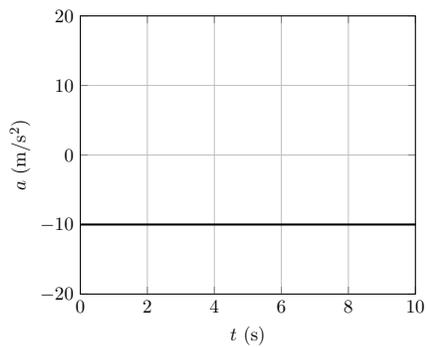
9. A uniform stick of mass m is originally on a horizontal surface. One end is attached to a vertical rope, which pulls up with a constant tension force F so that the center of the mass of the stick moves upward with acceleration $a < g$. The normal force N of the ground on the other end of the stick shortly after the right end of the stick leaves the surface satisfies



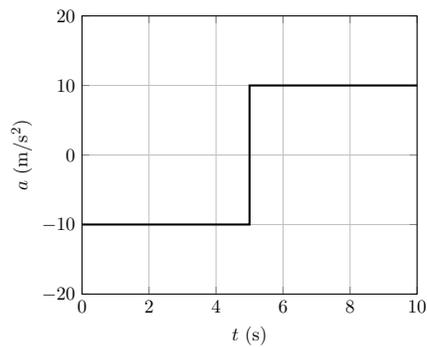
- (A) $N = mg$
(B) $mg > N > mg/2$
(C) $N = mg/2$
(D) $mg/2 > N > 0$
(E) $N = 0$

10. Which of the following graphs best shows the acceleration versus time of an object originally moving upward in the presence of air friction?

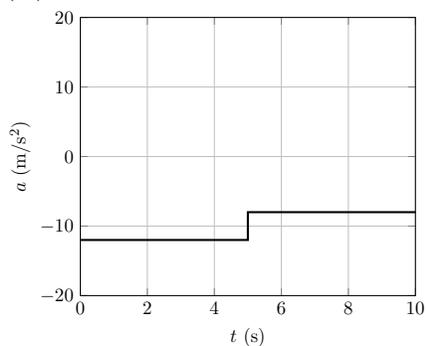
(A)



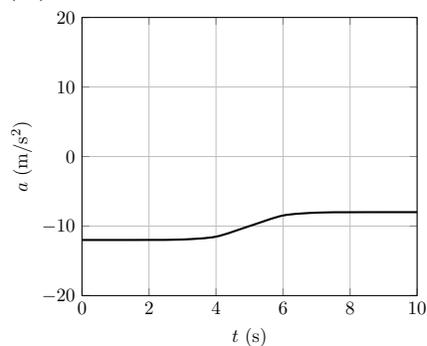
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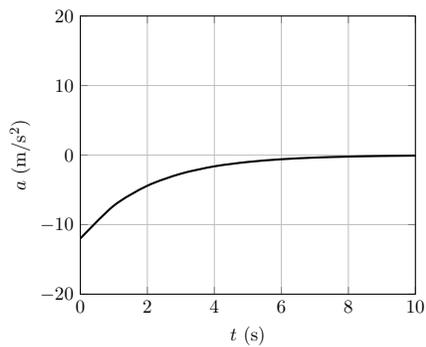
(C)



(D)



(E)



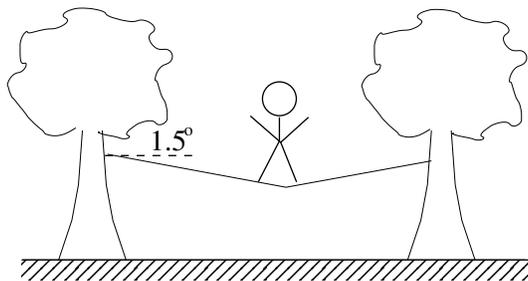
11. A light, uniform, ideal spring is fixed at one end. If a mass is attached to the other end, the system oscillates with angular frequency ω . Now suppose the spring is fixed at the other end, then cut in half. The mass is attached between the two half springs.



The new angular frequency of oscillations is

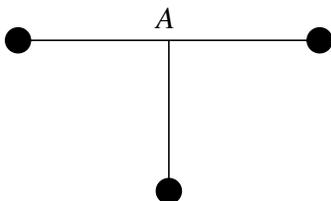
- (A) $\omega/2$
(B) ω
(C) $\sqrt{2}\omega$
(D) 2ω
(E) 4ω
12. A group of students wish to measure the acceleration of gravity with a simple pendulum. They take one length measurement of the pendulum to be $L = 1.00 \pm 0.05$ m. They then measure the period of a single swing to be $T = 2.00 \pm 0.10$ s. Assume that all uncertainties are Gaussian. The computed acceleration of gravity from this experiment illustrating the range of possible values should be recorded as
- (A) 9.87 ± 0.10 m/s²
(B) 9.87 ± 0.15 m/s²
(C) 9.9 ± 0.25 m/s²
(D) 9.9 ± 1.1 m/s²
(E) 9.9 ± 1.5 m/s²

13. A massless cable of diameter 2.54 cm (1 inch) is tied horizontally between two trees 18.0 m apart. A tightrope walker stands at the center of the cable, giving it a tension of 7300 N. The cable stretches and makes an angle of 1.50° with the horizontal.



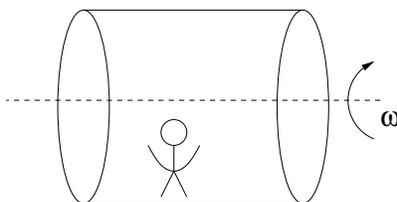
The Young's modulus is defined as the ratio of stress to strain, where stress is the force applied per unit area and strain is the fractional change in length $\Delta L/L$. The cable's Young's modulus is

- (A) $1.5 \times 10^6 \text{ N/m}^2$
(B) $2.0 \times 10^8 \text{ N/m}^2$
(C) $2.2 \times 10^9 \text{ N/m}^2$
(D) $2.4 \times 10^{10} \text{ N/m}^2$
(E) $4.2 \times 10^{10} \text{ N/m}^2$
14. Three identical masses are connected with identical rigid rods and pivoted at point A . If the lowest mass receives a small horizontal push to the left, it oscillates with period T_1 . If it instead receives a small push into the page, it oscillates with period T_2 . The ratio T_1/T_2 is



- (A) $1/2$
(B) 1
(C) $\sqrt{3}$
(D) $2\sqrt{2}$
(E) $2\sqrt{5}$

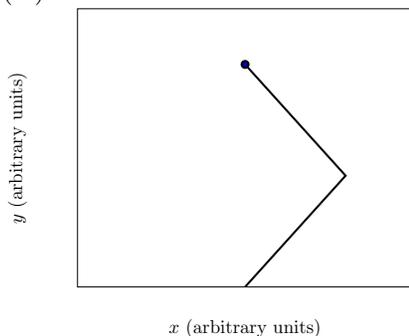
15. A satellite is in a circular orbit about the Earth. Over a long period of time, the effects of air resistance decrease the satellite's total energy by 1 J. The kinetic energy of the satellite
- (A) increases by 1 J.
 - (B) remains unchanged.
 - (C) decreases by $\frac{1}{2}$ J.
 - (D) decreases by 1 J.
 - (E) decreases by 2 J.
16. A cylindrical space station produces 'artificial gravity' by rotating with angular frequency ω . Consider working in the reference frame rotating with the space station. In this frame, an astronaut is initially at rest standing on the floor, facing in the direction that the space station is rotating. The astronaut jumps up vertically relative to the floor of the space station, with an initial speed less than that of the speed of the floor. Just after leaving the floor, the motion of the astronaut, relative to the space station floor,



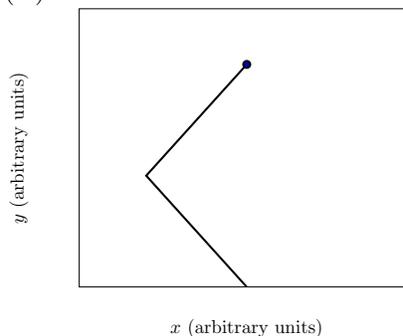
- (A) always has a component of acceleration directed toward the floor, and they land at the same point they jumped from.
- (B) always has a component of acceleration directed toward the floor, and they land in front of the point they jumped from.
- (C) always has a component of acceleration directed toward the floor, and they land behind the point they jumped from.
- (D) has a component of acceleration directed away from the floor, and they land behind the point they jumped from.
- (E) has a zero acceleration relative to the floor, and the astronaut never reaches the floor again.

17. A stream of sand is dropped out of a helicopter initially moving at a constant speed v to the right. The helicopter suddenly turns and begins moving a constant speed v to the left. Neglecting air resistance on the sand, what is the shape of the stream of sand, as viewed from the ground? The black dot represents the helicopter.

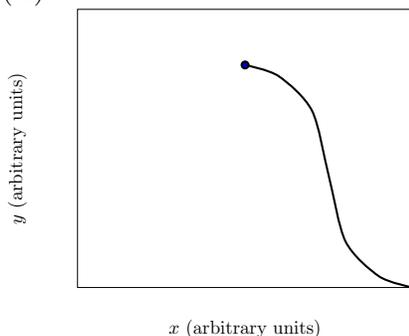
(A)



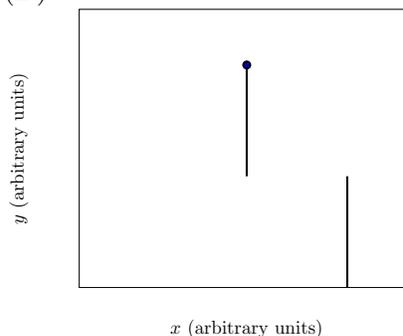
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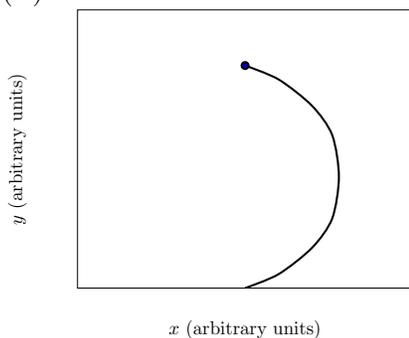
(C)



(D)



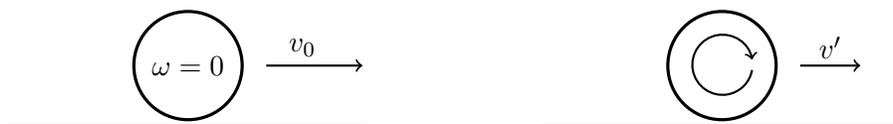
(E)



18. A mass m is attached to a thin rod of length ℓ so that it can freely spin in a vertical circle with period T . The difference in the tensions in the rod when the mass is at the top and the bottom of the circle is

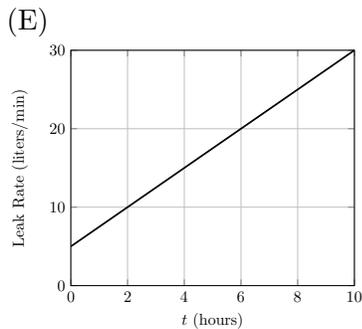
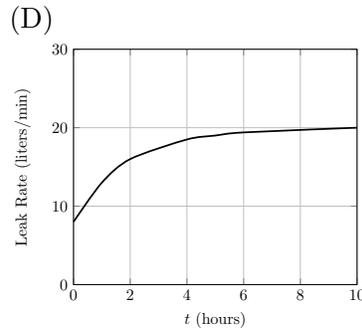
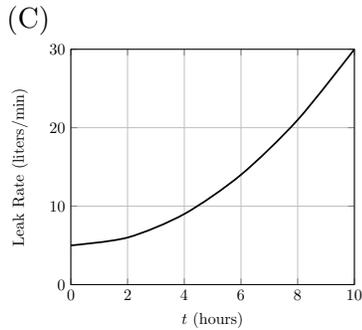
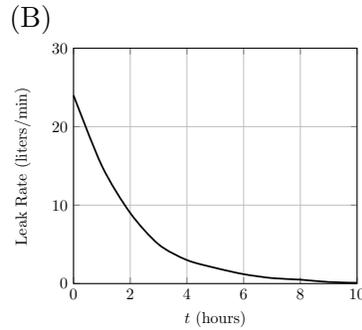
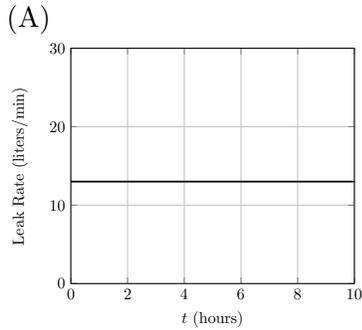
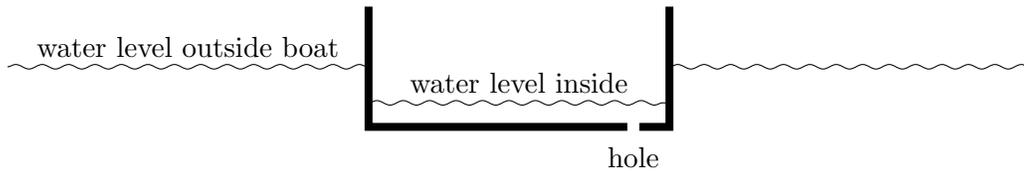
- (A) $6mg^2T^2/\ell$
- (B) $4\pi mg^2T^2/\ell$
- (C) $6mg$
- (D) $\pi^2 m\ell/T^2$
- (E) $4\pi m\ell/T^2$

19. Raindrops with a number density of n drops per cubic meter and radius r_0 hit the ground with a speed v_0 . The resulting pressure on the ground from the rain is P_0 . If the number density is doubled, the drop radius is halved, and the speed is halved, the new pressure will be
- (A) P_0
 (B) $P_0/2$
 (C) $P_0/4$
 (D) $P_0/8$
 (E) $P_0/16$
20. A spring stretched to double its unstretched length has a potential energy U_0 . If the spring is cut in half, and each half spring is stretched to double its unstretched length, then the total potential energy stored in the two half springs will be
- (A) $4U_0$
 (B) $2U_0$
 (C) U_0
 (D) $U_0/2$
 (E) $U_0/4$
21. A ping-pong ball (a hollow spherical shell) with mass m is placed on the ground with initial velocity v_0 and zero angular velocity at time $t = 0$. The coefficient of friction between the ping-pong ball and the ground is $\mu_s = \mu_k = \mu$. The time the ping-pong ball begins to roll without slipping is

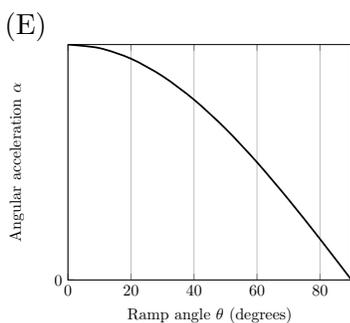
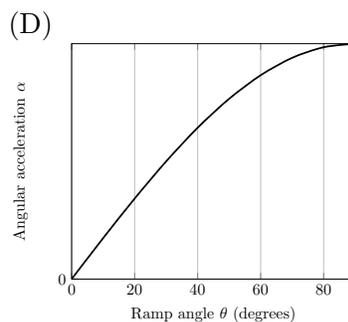
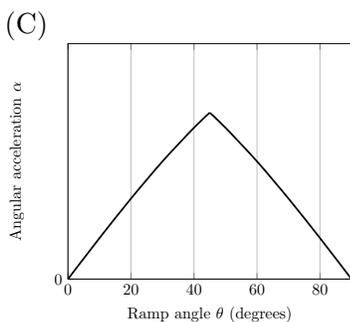
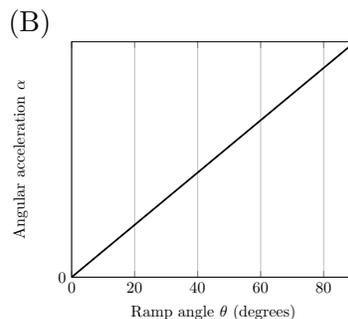
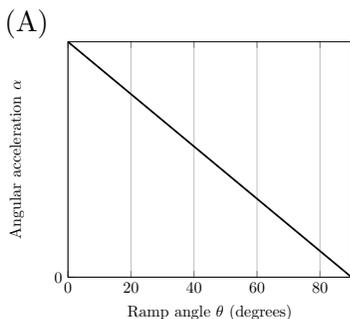


- (A) $t = (2/5)v_0/\mu g$
 (B) $t = (2/3)v_0/\mu g$
 (C) $t = v_0/\mu g$
 (D) $t = (5/3)v_0/\mu g$
 (E) $t = (3/2)v_0/\mu g$

22. A small hole is punched into the bottom of a rectangular boat, allowing water to enter the boat. As the boat sinks into the water, which of the following graphs best shows how the rate water flows through the hole varies with time? Assume that the boat remains horizontal as it sinks.



23. The coefficients of static and kinetic friction between a ball and an ramp are $\mu_s = \mu_k = \mu$. The ball is released from rest at the top of the ramp. Which of the following graphs best shows the rotational acceleration of the ball about its center of mass as a function of the angle of the ramp?



24. A mass is attached to one end of a rigid rod, while the other end of the rod is attached to a fixed horizontal axle. Initially the mass hangs at the end of the rod and the rod is vertical. The mass is given an initial kinetic energy K . If K is very small, the mass behaves like a pendulum, performing small-angle oscillations with period T_0 . As K is increased, the period of the motion for the mass

- (A) remains the same.
 (B) increases, approaching a finite constant.
 (C) decreases, approaching a finite non-zero constant.
 (D) decreases, approaching zero.
 (E) initially increases, then decreases.

25. Alice and Bob are working on a lab report. Alice measures the period of a pendulum to be 1.013 ± 0.008 s, while Bob independently measures the period to be 0.997 ± 0.016 s. Alice and Bob can combine their measurements in several ways.

- 1: Keep Alice's result and ignore Bob's
- 2: Average Alice's and Bob's results
- 3: Perform a weighted average of Alice's and Bob's results, with Alice's result weighted 4 times more than Bob's

How are the uncertainties of these results related?

- (A) Method 1 has the lowest uncertainty, and method 2 has the highest
- (B) Method 3 has the lowest uncertainty, and method 2 has the highest
- (C) Method 2 has the lowest uncertainty, and method 1 has the highest
- (D) Method 3 has the lowest uncertainty, and method 1 has the highest
- (E) Method 1 has the lowest uncertainty, and method 3 has the highest