AAPT UNITED STATES PHYSICS TEAM
AIP 2018

2018 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

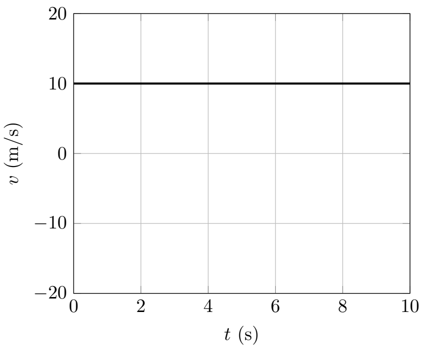
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2018.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

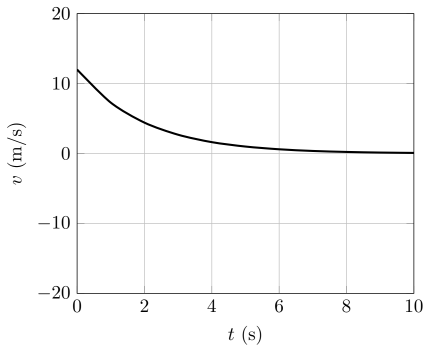
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. Which of the following graphs best shows the velocity versus time of an object originally moving upward in the presence of air friction?

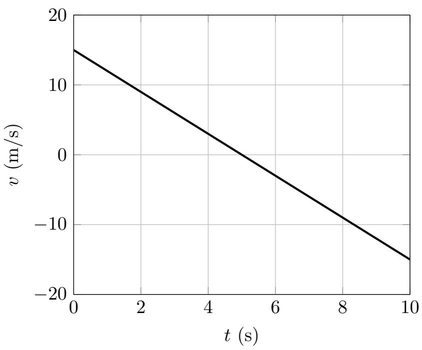
(A)



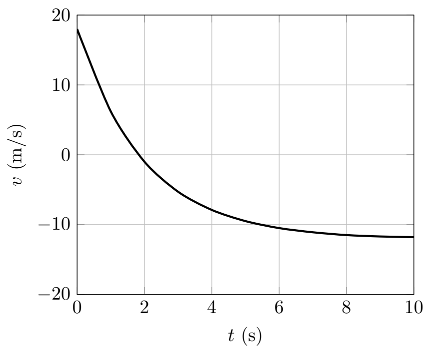
(B)



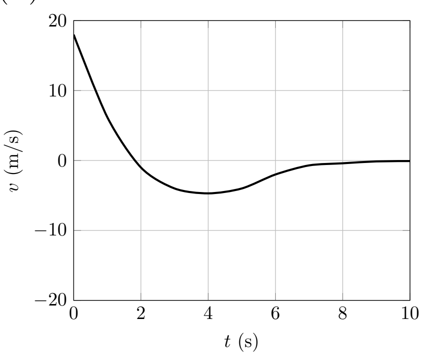
(C)



(D)



(E)



D ← CORRECT

Solution

The terminal velocity should be constant and negative, ruling out everything but choice D. Also note that the magnitude of the acceleration should be decreasing.

2. A 3.0 kg mass moving at 30 m/s to the right collides elastically with a 2.0 kg mass traveling at 20 m/s to the left. After the collision, the center of mass of the system is moving at a speed of

(A) 5 m/s
(B) 10 m/s ← **CORRECT**
(C) 20 m/s
(D) 24 m/s
(E) 26 m/s

Solution

The velocity of the center of mass is conserved by conservation of momentum, so we don't have to account for the collision. Before the collision, the total momentum is 50 kg m/s and the total mass is 5 kg, so the speed of the center of mass is 10 m/s.

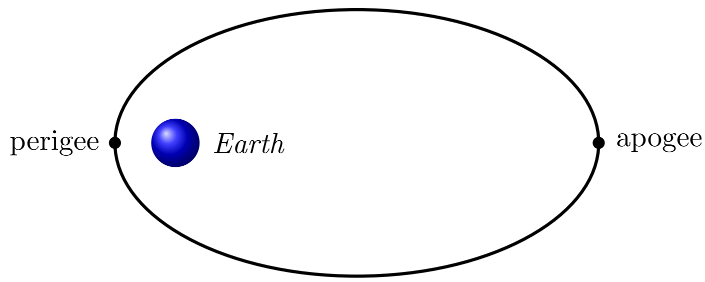
3. Ball 1 traveling in the positive x direction strikes an equal mass ball 2 that is originally at rest. All of the following must be true after the collision, except

(A) The total final momentum in the x direction equals the initial momentum of ball 1.
(B) The total kinetic energy after the collision equals the initial kinetic energy of ball 1. ← **CORRECT**
(C) The final momentum of the two balls in the y direction adds to zero.
(D) The final speed of the center of mass of the two balls is equal to half the initial speed of ball 1.
(E) The balls can't both be at rest after the collision.

Solution

Choices A, C, and E are true by conservation of momentum; choice D is also true, using the result $p_{\text{tot}} = m_{\text{tot}} v_{\text{CM}}$. Choice B is not true since the collision may be partially or completely inelastic.

4. A satellite is following an elliptical orbit around the Earth. Its engines are capable of providing a one-time impulse of a fixed magnitude. In order to maximize the energy of the satellite, the impulse should be



(A) directed along the satellite's velocity and applied when the satellite is in its perigee. ← **CORRECT**

(B) directed along the satellite's velocity and applied when the satellite is in apogee.

(C) directed toward the Earth and applied when the satellite is in perigee.

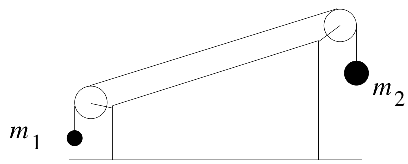
(D) directed toward the Earth and applied when the satellite is in apogee.

(E) directed away from the Earth and applied when the satellite is in apogee.

Solution

The change in the velocity $\Delta\mathbf{v}$ has fixed magnitude. Since the kinetic energy is proportional to v^2, the greatest change is attained when the impulse is parallel to the velocity, when the speed is as large as possible, which occurs at perigee.

5. Two masses are attached with pulleys by a massless rope on an inclined plane as shown. All surfaces are frictionless. If the masses are released from rest, then the inclined plane



(A) accelerates to the left if $m_1 < m_2$

(B) accelerates to the right if $m_1 < m_2$

(C) accelerates to the left regardless of the masses

(D) accelerates to the right regardless of the masses

(E) does not move $\leftarrow$ **CORRECT**

Solution

The inclined plane does not move. There are several ways to see this, but one simple reason is that nothing about this system has horizontal momentum; the masses just move up and down. (The string carries no horizontal momentum since it is massless.)

6. A packing crate with mass $m = 115$ kg is slid up a 5.00 m long ramp which makes an angle of 20.0° with respect to the horizontal by an applied force of $F = 1.00 \times 10^3$ N directed parallel to the ramp's incline. A frictional force of magnitude $f = 4.00 \times 10^2$ N resists the motion. If the crate starts from rest, what is its speed at the top of the ramp?

(A) 4.24 m/s $\leftarrow$ **CORRECT**

(B) 5.11 m/s

(C) 7.22 m/s

(D) 8.26 m/s

(E) 9.33 m/s

Solution

The component of the gravitational force parallel to the incline is given by $-mg\sin\theta = -393$ N. Then the net force is 207 N. For an object with mass 115 kg, the acceleration is 1.8 m/s^2. Then, the velocity is $\sqrt{2ad} = 4.24$ m/s.

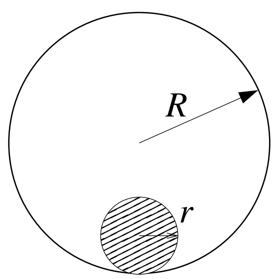
7. A car has a maximum acceleration of a_0 and a minimum acceleration of $-a_0$. The shortest possible time for the car to begin at rest, then arrive at rest at a point a distance d away is

(A) $\sqrt{d/2a_0}$
(B) $\sqrt{d/a_0}$
(C) $\sqrt{2d/a_0}$
(D) $\sqrt{3d/a_0}$
(E) $2\sqrt{d/a_0}$ ← **CORRECT**

Solution

The optimal solution is to 'floor the gas' for the first half and 'floor the brake' for the second half. The first half takes time $\sqrt{d/a_0}$, and the second half takes the same amount of time by symmetry, giving total time $2\sqrt{d/a_0}$.

8. A disk of radius r rolls uniformly without slipping around the inside of a fixed hoop of radius R. If the period of the disc's motion around the hoop is T, what is the instantaneous speed of the point on the disk opposite to the point of contact?

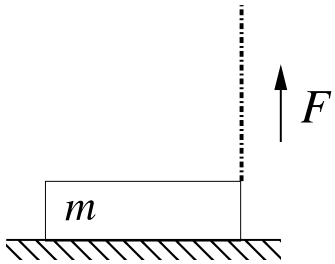


(A) $2\pi(R + r)/T$
(B) $2\pi(R + 2r)/T$
(C) $4\pi(R - 2r)/T$
(D) $4\pi(R - r)/T$ ← **CORRECT**
(E) $4\pi(R + r)/T$

Solution

The center of mass of the disk will be a distance $R - r$ from the center of the hoop. The speed of the disk's center will be $\omega(R - r) = (2\pi/T)(R - r)$. The speed of the point on the disk opposite to the point of contact will be twice the speed of the center of the disk, since the disk instantaneously rotates about the contact point, so $v = 4\pi(R - r)/T$.

9. A uniform stick of mass m is originally on a horizontal surface. One end is attached to a vertical rope, which pulls up with a constant tension force F so that the center of the mass of the stick moves upward with acceleration $a < g$. The normal force N of the ground on the other end of the stick shortly after the right end of the stick leaves the surface satisfies



(A) $N = mg$
(B) $mg > N > mg/2$ ← **CORRECT**
(C) $N = mg/2$
(D) $mg/2 > N > 0$
(E) $N = 0$

Solution

Newton's second law for the vertical motion gives

$$F + N - mg = ma.$$

Now take torques about the left end of the stick. We have

$$\tau = FL - \frac{mgL}{2}, \quad I = \frac{1}{3}mL^2, \quad \alpha = \frac{a}{L/2}.$$

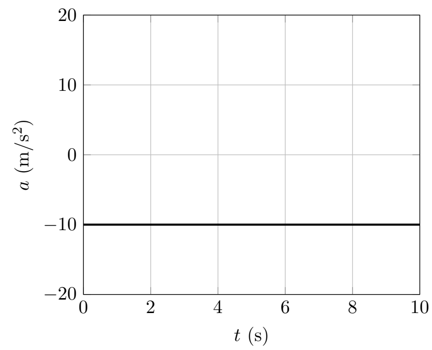
Solving these two equations for N gives

$$N = \frac{1}{3}ma + \frac{1}{2}mg$$

which for the range of a given satisfies choice B.

10. Which of the following graphs best shows the acceleration versus time of an object originally moving upward in the presence of air friction?

(A)



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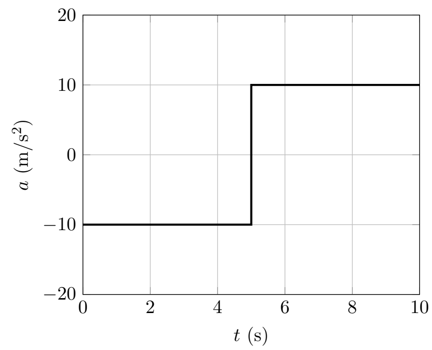

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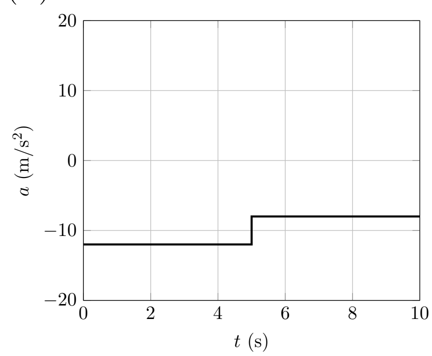

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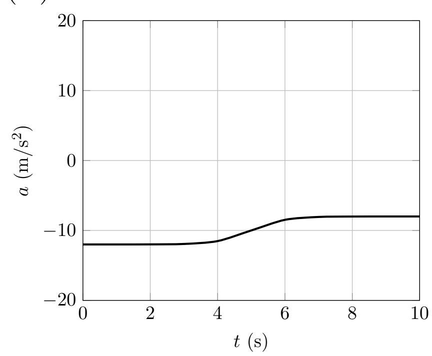

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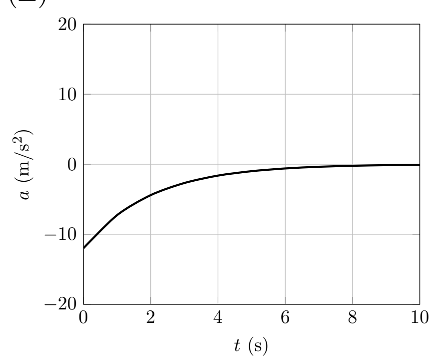


E ← CORRECT

Solution

The acceleration should asymptote to zero, as the object reaches its terminal velocity. It should begin less than $-10\,\mathrm{m/s^2}$, since the air friction acts in the same direction as gravity.

11. A light, uniform, ideal spring is fixed at one end. If a mass is attached to the other end, the system oscillates with angular frequency ω. Now suppose the spring is fixed at the other end, then cut in half. The mass is attached between the two half springs.

The new angular frequency of oscillations is

(A) $\omega/2$

(B) ω

(C) $\sqrt{2}\omega$

(D) 2ω ← **CORRECT**

(E) 4ω

Solution

Cutting a spring in half doubles its spring constant; if a force F would elongate the original spring by an amount x, it elongates the new spring by an amount $x' = x/2$, and using $F = -kx$ gives $k' = 2k$. The new system thus consists of two springs, each of whose spring constant is twice that of the original spring; the effective spring constant has thus increased by a factor of four. Since the angular frequency goes as the square root of the spring constant, the angular frequency increases by a factor of two.

12. A group of students wish to measure the acceleration of gravity with a simple pendulum. They take one length measurement of the pendulum to be $L = 1.00 \pm 0.05$ m. They then measure the period of a single swing to be $T = 2.00 \pm 0.10$ s. Assume that all uncertainties are Gaussian. The computed acceleration of gravity from this experiment illustrating the range of possible values should be recorded as

(A) 9.87 ± 0.10 m/s^2

(B) 9.87 ± 0.15 m/s^2

(C) 9.9 ± 0.25 m/s^2

(D) 9.9 ± 1.1 m/s^2 ← **CORRECT**

(E) 9.9 ± 1.5 m/s^2

Solution

Rearranging the formula for the period of a pendulum gives $g = 4\pi^2 L/T^2$. We then use the standard error propagation rules, which may be derived by identifying the uncertainty with the standard deviation:

(A) If x has relative uncertainty $\Delta x/x$, then x^n has relative uncertainty $\Delta(x^n)/x^n = |n|\Delta x/x$.

(B) If two independent quantities are multiplied, their relative uncertainties add in quadrature,

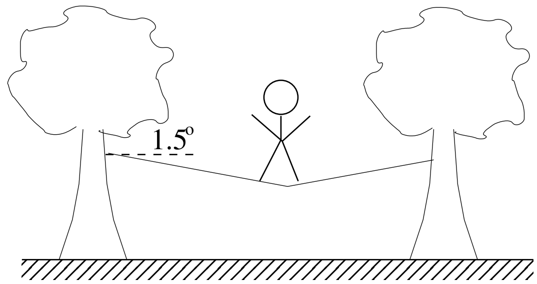
$$\frac{\Delta(xy)}{xy} = \sqrt{\left(\frac{\Delta x}{x}\right)^2 + \left(\frac{\Delta y}{y}\right)^2}$$

.

(C) The relative uncertainty stays the same upon scaling of a variable.

In this case, the relative uncertainty of L is 5%, the relative uncertainty of T is 5%, and the relative uncertainty of $1/T^2$ is 10%. The relative uncertainty of g is $\sqrt{0.1^2 + 0.05^2} \approx 11\%$, giving choice D.

13. A massless cable of diameter 2.54 cm (1 inch) is tied horizontally between two trees 18.0 m apart. A tightrope walker stands at the center of the cable, giving it a tension of 7300 N. The cable stretches and makes an angle of 1.50° with the horizontal.



The Young's modulus is defined as the ratio of stress to strain, where stress is the force applied per unit area and strain is the fractional change in length $\Delta L/L$. The cable's Young's modulus is

(A) 1.5×10^6 N/m^2

(B) 2.0×10^8 N/m^2

(C) 2.2×10^9 N/m^2

(D) 2.4×10^{10} N/m^2

(E) 4.2×10^{10} N/m^2 ← **CORRECT**

Solution

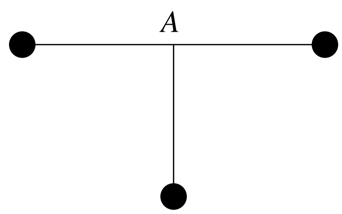
The strain of the cable is

$$\frac{\Delta L}{L} = \frac{L_0/\cos\theta - L_0}{L_0} = \frac{1}{\cos\theta} - 1 \approx \frac{\theta^2}{2}$$

where L_0 is the initial length. Then the Young's modulus is

$$Y = \frac{(7300\,\mathrm{N})}{\pi(0.0127\,\mathrm{m})^2}\left(\frac{(1.50\,\pi/180)^2}{2}\right)^{-1} = 4.2 \times 10^{10}\,\mathrm{N/m^2}$$

14. Three identical masses are connected with identical rigid rods and pivoted at point A. If the lowest mass receives a small horizontal push to the left, it oscillates with period T_1. If it instead receives a small push into the page, it oscillates with period T_2. The ratio T_1/T_2 is



(A) $1/2$

(B) 1

(C) $\sqrt{3}$ ← **CORRECT**

(D) $2\sqrt{2}$

(E) $2\sqrt{5}$

Solution

Both modes are physical pendulums, which have period proportional to $\sqrt{I/Mgx}$ where x is the distance from the pivot to the center of mass, and I is the moment of inertia about the pivot. Since x is the same in both cases, $T_1/T_2 \propto \sqrt{I_1/I_2} = \sqrt{3}$, because in the second case only the bottom mass contributes to the moment of inertia.

15. A satellite is in a circular orbit about the Earth. Over a long period of time, the effects of air resistance decrease the satellite's total energy by 1 J. The kinetic energy of the satellite

(A) increases by 1 J. ← **CORRECT**

(B) remains unchanged.

(C) decreases by $\frac{1}{2}$ J.

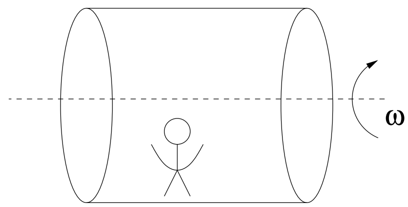
(D) decreases by 1 J.

(E) decreases by 2 J.

Solution

Since the process is slow, the satellite remains in a circular orbit. In a circular orbit, the total energy $E = K + V$ satisfies $E = -K$. Then if the total energy decreases by ΔE, the kinetic energy increases by ΔE.

16. A cylindrical space station produces 'artificial gravity' by rotating with angular frequency ω. Consider working in the reference frame rotating with the space station. In this frame, an astronaut is initially at rest standing on the floor, facing in the direction that the space station is rotating. The astronaut jumps up vertically relative to the floor of the space station, with an initial speed less than that of the speed of the floor. Just after leaving the floor, the motion of the astronaut, relative to the space station floor,



(A) always has a component of acceleration directed toward the floor, and they land at the same point they jumped from.

(B) always has a component of acceleration directed toward the floor, and they land in front of the point they jumped from. ← **CORRECT**

(C) always has a component of acceleration directed toward the floor, and they land behind the point they jumped from.

(D) has a component of acceleration directed away from the floor, and they land behind the point they jumped from.

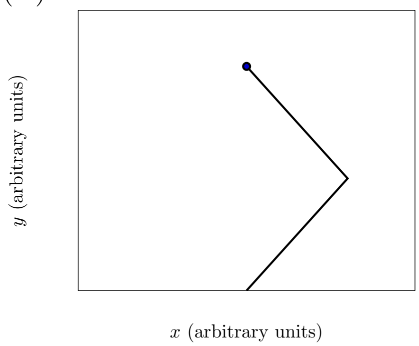
(E) has a zero acceleration relative to the floor, and the astronaut never reaches the floor again.

Solution

The problem can be solved by using the Coriolis force. Alternatively, we can draw the path of the astronaut in the original reference frame, where the path is simply a line. Then the astronaut always lands 'ahead' of the rotation, and hence to the left of where they started in the rotating frame.

17. A stream of sand is dropped out of a helicopter initially moving at a constant speed v to the right. The helicopter suddenly turns and begins moving a constant speed v to the left. Neglecting air resistance on the sand, what is the shape of the stream of sand, as viewed from the ground? The black dot represents the helicopter.

(A)



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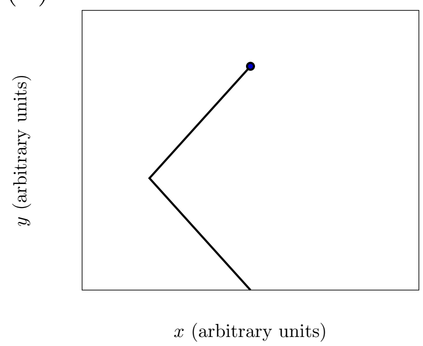

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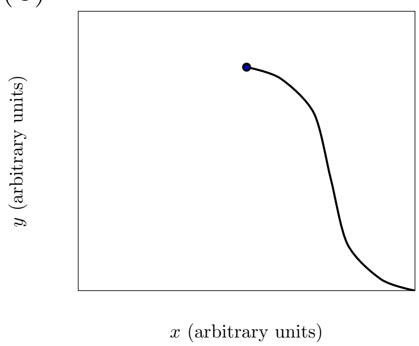
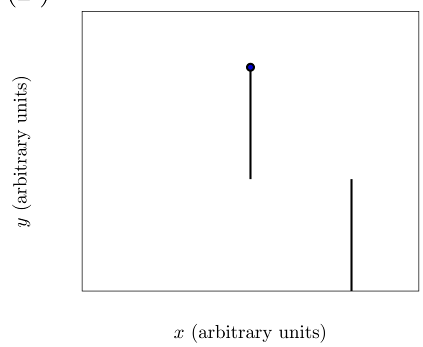


(D)



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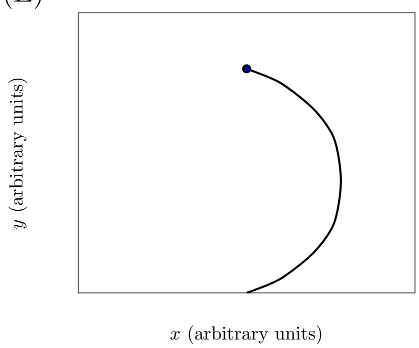


D ← CORRECT

Solution

If sand is dropped out of a helicopter with constant velocity, it always remains directly beneath the helicopter. Thus the path of the sand should be two straight vertical lines.

18. A mass m is attached to a thin rod of length ℓ so that it can freely spin in a vertical circle with period T. The difference in the tensions in the rod when the mass is at the top and the bottom of the circle is

    (A) $6mg^2T^2/\ell$
    (B) $4\pi mg^2T^2/\ell$
    (C) $6mg$ ← **CORRECT**
    (D) $\pi^2 m\ell/T^2$
    (E) $4\pi m\ell/T^2$

Solution

There is a difference of $2mg$ due to gravity acting against tension at the bottom and along tension at the top. The difference in the centripetal force required is $\Delta(mv^2/\ell) = 2\Delta K/\ell = 2(2mg\ell)/\ell = 4mg$. Then the total difference is $6mg$.

19. Raindrops with a number density of n drops per cubic meter and radius r_0 hit the ground with a speed v_0. The resulting pressure on the ground from the rain is P_0. If the number density is doubled, the drop radius is halved, and the speed is halved, the new pressure will be

    (A) P_0
    (B) $P_0/2$
    (C) $P_0/4$
    (D) $P_0/8$
    (E) $P_0/16$ ← **CORRECT**

Solution

Each drop has 1/8 the mass and 1/2 the speed, so 1/16 the momentum. The rate at which they hit the ground is the same, since the number density is doubled but the speed is halved. Thus the pressure is $P_0/16$.

20. A spring stretched to double its unstretched length has a potential energy U_0. If the spring is cut in half, and each half spring is stretched to double its unstretched length, then the total potential energy stored in the two half springs will be

(A) $4U_0$
(B) $2U_0$
(C) U_0 ← **CORRECT**
(D) $U_0/2$
(E) $U_0/4$

Solution

One can compute this directly using $U = kx^2/2$ and the fact that a cut spring has double the spring constant. Alternatively, note that the energy is all stored microscopically, and microscopically, each molecule has no idea whether it's in a full spring or a half-spring. Then the answer must remain the same when we cut the spring.

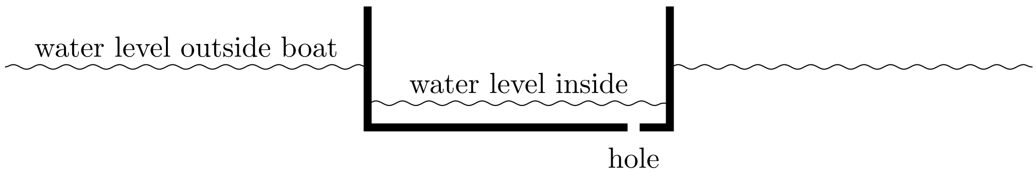
21. A ping-pong ball (a hollow spherical shell) with mass m is placed on the ground with initial velocity v_0 and zero angular velocity at time $t = 0$. The coefficient of friction between the ping-pong ball and the ground is $\mu_s = \mu_k = \mu$. The time the ping-pong ball begins to roll without slipping is

(A) $t = (2/5)v_0/\mu g$ ← **CORRECT**
(B) $t = (2/3)v_0/\mu g$
(C) $t = v_0/\mu g$
(D) $t = (5/3)v_0/\mu g$
(E) $t = (3/2)v_0/\mu g$

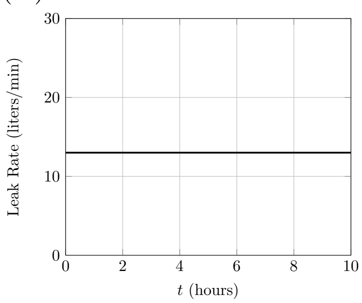
Solution

The friction force is μmg, so the velocity decreases as $\dot{v} = -\mu g$. The torque about the center of mass is μmgR where R is the radius, so the angular velocity increases as $\dot{\omega} = (3/2)\mu g/R$. Rolling without slipping begins once $v = R\omega$, which gives $t = (2/5)v_0/\mu g$.

22. A small hole is punched into the bottom of a rectangular boat, allowing water to enter the boat. As the boat sinks into the water, which of the following graphs best shows how the rate water flows through the hole varies with time? Assume that the boat remains horizontal as it sinks.



(A)



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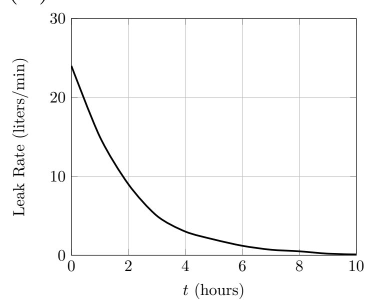

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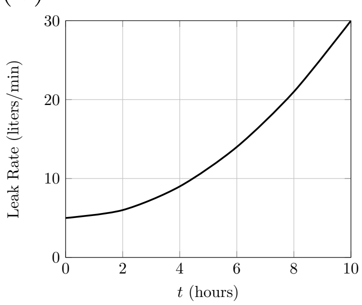

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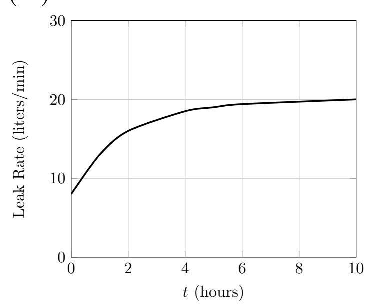

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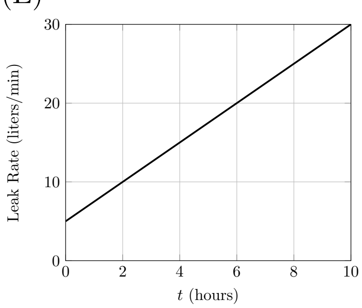


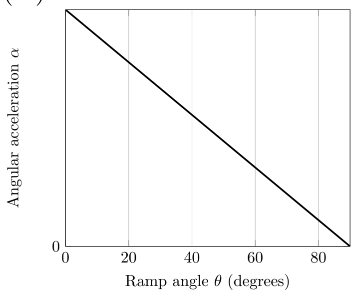
A ← CORRECT

Solution

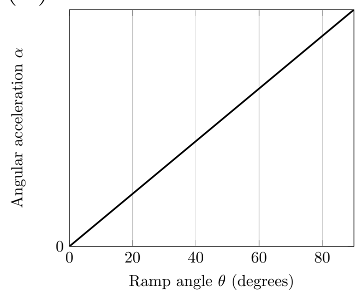
By Archimedes' principle, the difference in the outer and inner water levels always remains the same. Then the amount of pressure forcing the water through the hole remains the same by Bernoulli's principle, so the rate of sinking is constant.

23. The coefficients of static and kinetic friction between a ball and an ramp are $\mu_s = \mu_k = \mu$. The ball is released from rest at the top of the ramp. Which of the following graphs best shows the rotational acceleration of the ball about its center of mass as a function of the angle of the ramp?

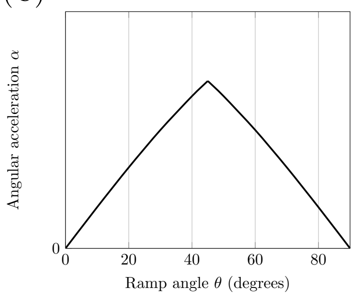
(A)



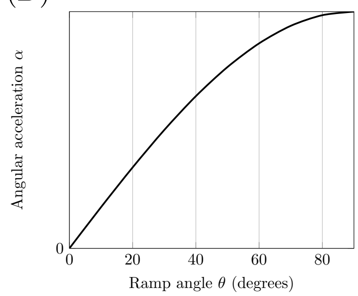
(B)



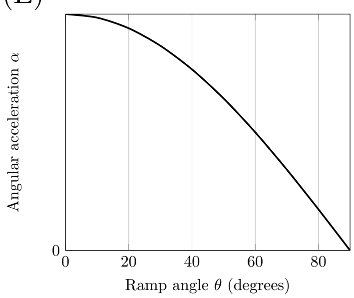
(C)



(D)



(E)



C ← CORRECT

Solution

The problem can be solved by limiting cases: note that the rotational acceleration is zero for a horizontal ramp, since the ball just sits motionless, and zero for a vertical ramp, since there is no friction force.

Alternatively, one can explicitly find the angular accelerations. In the case of rolling without slipping, the acceleration is $g \sin\theta/(1 + I/MR^2) \propto \sin\theta$. When the angle is high enough for slipping to begin, the angular acceleration is proportional to the kinetic friction force, which is proportional to $\cos\theta$. Thus the graph should consist of an increasing and decreasing part, both concave down, with a sharp corner when slipping occurs, matching choice C.

24. A mass is attached to one end of a rigid rod, while the other end of the rod is attached to a fixed horizontal axle. Initially the mass hangs at the end of the rod and the rod is vertical. The mass is given an initial kinetic energy K. If K is very small, the mass behaves like a pendulum, performing small-angle oscillations with period T_0. As K is increased, the period of the motion for the mass

(A) remains the same.

(B) increases, approaching a finite constant.

(C) decreases, approaching a finite non-zero constant.

(D) decreases, approaching zero.

(E) initially increases, then decreases. ← **CORRECT**

Solution

As the amplitude increases to π, the period diverges, so the period initially increases. (Alternatively, note that the quartic term in the expansion for $\cos\theta$ has the opposite sign as the quadratic term. This reduces the restoring force and makes the period longer.) If the energy is further increased, the mass goes around in one direction forever, with a finite and decreasing period.

25. Alice and Bob are working on a lab report. Alice measures the period of a pendulum to be 1.013 ± 0.008 s, while Bob independently measures the period to be 0.997 ± 0.016 s. Alice and Bob can combine their measurements in several ways.

1: Keep Alice's result and ignore Bob's

2: Average Alice's and Bob's results

3: Perform a weighted average of Alice's and Bob's results, with Alice's result weighted 4 times more than Bob's

How are the uncertainties of these results related?

(A) Method 1 has the lowest uncertainty, and method 2 has the highest

(B) Method 3 has the lowest uncertainty, and method 2 has the highest ← **CORRECT**

(C) Method 2 has the lowest uncertainty, and method 1 has the highest

(D) Method 3 has the lowest uncertainty, and method 1 has the highest

(E) Method 1 has the lowest uncertainty, and method 3 has the highest

Solution

Let Alice's measurement have uncertainty Δx, so Bob's has uncertainty $2\Delta x$. The uncertainty obeys the following rules, which can be derived by identifying the uncertainty with the standard deviation:

(A) Scaling: if x has uncertainty Δx, then cx has uncertainty $c\Delta x$.

(B) Addition in quadrature: if x has uncertainty Δx and an independent measurement y has uncertainty Δy, then $x + y$ has uncertainty $\sqrt{(\Delta x)^2 + (\Delta y)^2}$.

Then the uncertainties of the three possibilities are:

1: Δx

2: $\sqrt{(\Delta x)^2 + (2\Delta x)^2}/2 = (\sqrt{5}/2)\Delta x$

3: $\sqrt{(4\Delta x)^2 + (2\Delta x)^2}/5 = (2/\sqrt{5})\Delta x$

The lowest uncertainty is from method 3 (which is in fact optimal); the highest is from method 2.