AAPT UNITED STATES PHYSICS TEAM
AIP 2018

2018 $F = ma$ Contest

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

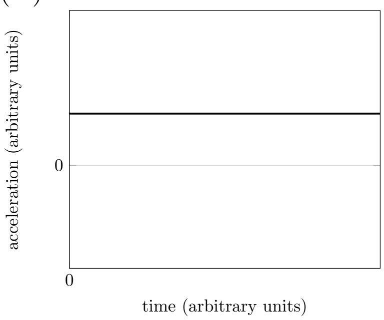
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet.
- Your answer to each question must be marked on the optical mark answer sheet.
- Select the single answer that provides the best response to each question. Please be sure to use a No. 2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased.
- Correct answers will be awarded one point; incorrect answers and leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- This test contains 25 multiple choice questions. Your answer to each question must be marked on the optical mark answer sheet that accompanies the test. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily the same level of difficulty.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 20, 2018.**
- The question booklet and answer sheet will be collected at the end of this exam. You may not use scratch paper.

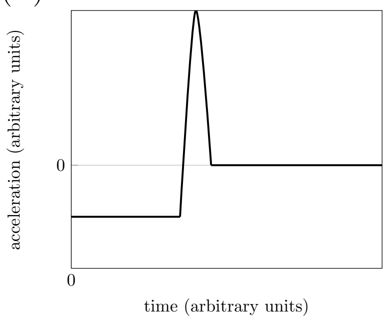
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

1. A large lump of clay is dropped off of a wall and lands on the ground. Which graph best represents the acceleration of the center of mass of the clay as a function of time?

(A)



(B)



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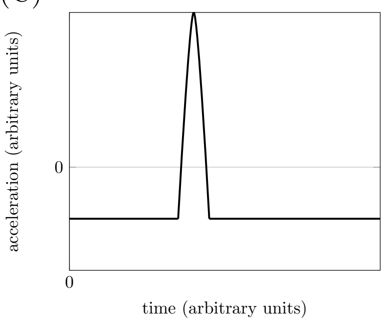

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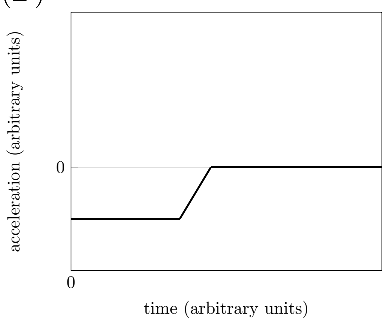

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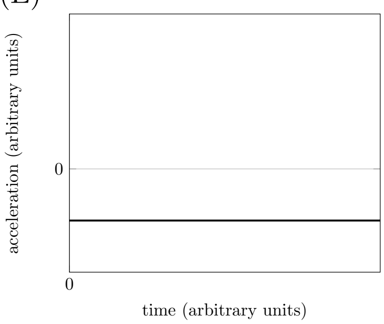


B ← CORRECT

Solution

During freefall, the acceleration is constant and approximately $-10\,\mathrm{m/s^2}$. It spikes up during the landing and remains zero afterward.

2. A uniform block of mass 10 kg is released at rest from the top of an incline with length 10 m and inclination 30°, and slides to the bottom. The coefficients of static and kinetic friction are $\mu_s = \mu_k = 0.1$. How much energy is dissipated due to friction?

   (A) 0 J
   (B) 22 J
   (C) 43 J
   (D) 87 J ← **CORRECT**
   (E) 164 J

Solution

The normal force is $(100)(\sqrt{3}/2)$ N, so the friction force is $5\sqrt{3}$ N, applied over a distance of 10m. Then the energy dissipated is $50\sqrt{3}$ J.

3. A 3.0 kg mass moving at 40 m/s to the right collides with and sticks to a 2.0 kg mass traveling at 20 m/s to the right. After the collision, the kinetic energy of the system is closest to

   (A) 600 J
   (B) 1200 J
   (C) 2600 J ← **CORRECT**
   (D) 2800 J
   (E) 3400 J

Solution

The initial velocity of the center of mass is $(3 \cdot 40 + 2 \cdot 20)/5 = 32\,\mathrm{m/s}$. Then the kinetic energy is $m_{\mathrm{tot}} v^2/2 \approx 2600\,\mathrm{J}$.

4. A basketball is released from rest and bounces on the ground. Considering only the ball just before and just after the bounce, which of the following statements must be true?

   (A) The momentum and the total energy of the ball are conserved.

   (B) The momentum of the ball is conserved, but not the kinetic energy.

   (C) The total energy of the ball is conserved, but not the momentum.

   (D) The kinetic energy of the ball is conserved, but not the momentum.

   (E) Neither the kinetic energy of the ball nor the momentum is conserved. ← **CORRECT**

Solution

The momentum of the ball isn't conserved, since it experiences an external force from the ground. The kinetic energy isn't conserved either, as some of it is dissipated into sound waves, vibration of the ground, etc.

5. The hard disk in a computer will spin up to speed within 10 rotations, but when turned off will spin through 50 rotations before coming to a stop. Assuming the hard disk has constant angular acceleration α_1 and angular deceleration α_2, the ratio α_1/α_2 is

   (A) $1/5$

   (B) $1/\sqrt{5}$

   (C) $\sqrt{5}$

   (D) 5 ← **CORRECT**

   (E) 25

Solution

Since $\omega_f^2 - \omega_i^2 = 2\alpha\theta$, the quantity $\alpha\theta$ is the same for both cases, so the acceleration is five times greater.

6. A massless beam of length L is fixed on one end. A downward force F is applied to the free end of the beam, deflecting the beam downward by a distance x. The deflection x is linear in F and is inversely proportional to the cross-section moment I, which has units m^4. The deflection is also dependent on Young's modulus E, which has units $\mathrm{N/m}^2$. Then x depends on L according to

   (A) $x \propto \sqrt{L}$
   (B) $x \propto L$
   (C) $x \propto L^2$
   (D) $x \propto L^3$ $\leftarrow$ **CORRECT**
   (E) $x \propto L^4$

Solution

Everything except for F or E only has length units, so to cancel out the other units from the force, we need $x \propto F/EI$ which has dimensions of m^{-3}. Then $x \propto L^3$. The exact formula is $x = FL^3/3EI$.

7. A pendulum of length L oscillates inside a box. A person picks up the box and gently shakes it vertically with frequency ω and a fixed amplitude for a fixed time. To maximize the final amplitude of the pendulum, ω should satisfy

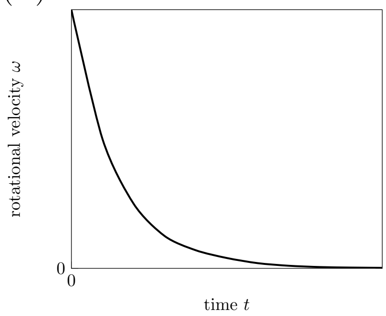
   (A) $\omega = \sqrt{4g/L}$ $\leftarrow$ **CORRECT**
   (B) $\omega = \sqrt{2g/L}$
   (C) $\omega = \sqrt{g/L}$
   (D) $\omega = \sqrt{g/4L}$
   (E) there will be no significant effect on the pendulum amplitude for any value of ω

Solution

In the frame of the box, vertical shaking is equivalent to having g oscillate with frequency ω. To increase the amplitude, gravity should be strong when the pendulum is moving down and weak when it is moving up, so there must be two oscillations per oscillation of the pendulum, so $\omega = 2\sqrt{g/L}$.

8. The coefficients of static and kinetic friction between a ball and a horizontal plane are $\mu_s = \mu_k = \mu$. The ball is given a horizontal speed but no rotational velocity about its center of mass. Which of the following graphs best shows the rotational velocity of the ball about its center of mass as a function of time?

(A)



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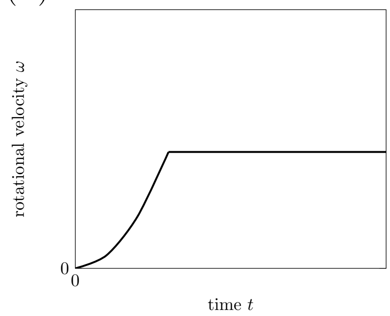

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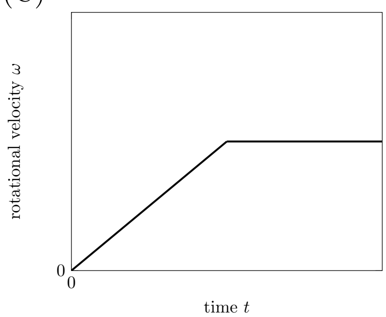
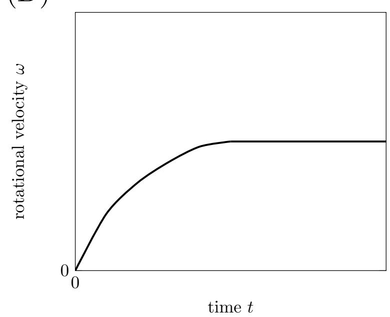
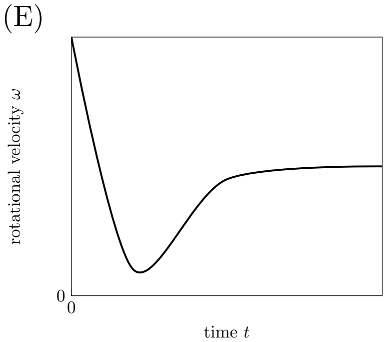


(D)



(E)



C ← CORRECT

Solution

The kinetic friction force μN applies a constant torque, so the angular velocity initially increases linearly. After it begins rolling without slipping, the angular velocity remains constant.

9. A 3.0 kg ball moving at 10 m/s east collides elastically with a 2.0 kg ball moving 15 m/s west. Which of the following statements could be true after the collision?

   (A) The two balls are both moving directly east.
   (B) The 3.0 kg ball moves directly west at 15 m/s.
   (C) The 2.0 kg ball moves directly north at 10 m/s.
   (D) The 3.0 kg ball is at rest.
   (E) The 2.0 kg ball moves directly south at 15 m/s. ← **CORRECT**

Solution

Note that we're in the center-of-mass frame where the total momentum is zero. Then the final momenta have to be equal and opposite, and to satisfy energy conservation the final speeds of each ball have to be the same as their initial speeds, ruling out all but choice E.

10. A balloon filled with air submerged in water at a depth h experiences a buoyant force B_0. The balloon is moved to a depth of $2h$, where it experiences a buoyant force B. Assuming the water is incompressible and the balloon and air are compressible, the buoyant force B satisfies

   (A) $B \geq 2B_0$
   (B) $B_0 < B < 2B_0$
   (C) $B = B_0$
   (D) $B < B_0$ ← **CORRECT**
   (E) it depends on the compressibility of the balloon and air

Solution

The buoyant force is ρV where ρ is the density of water and V is the volume of water displaced. Since the water is incompressible, ρ is constant, while V decreases since the balloon and air are compressed.

11. A circle of rope is spinning in outer space with an angular velocity ω_0. Transverse waves on the rope have speed v_0, as measured in a rotating reference frame where the rope is at rest. If the angular velocity of the rope is doubled, the new speed of transverse waves, as measured in a rotating reference frame where the rope is at rest, will be

(A) v_0

(B) $\sqrt{2}v_0$

(C) $2v_0$ ← **CORRECT**

(D) $4v_0$

(E) $8v_0$

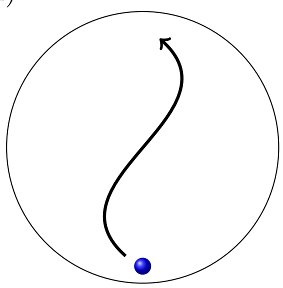
Solution

Consider a small segment of rope. The tension forces on each end of the rope provide the centripetal force, so $T \propto \omega^2$. The wave speed is $v = \sqrt{T/\mu} \propto \omega$, giving choice C.

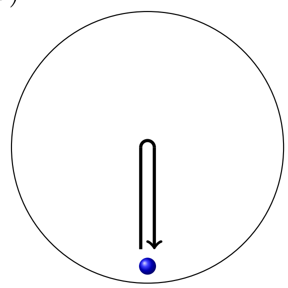
This problem can also be solved by dimensional analysis. The only length scale is the length of the rope, and the only frequency scale is the angular velocity. Then we must have $v \propto \omega L$ where L is the length of the rope.

12. A child in a circular, rotating space station tosses a ball in such a way so that once the station has rotated through one half rotation, the child catches the ball. From the child's point of view, which plot shows the trajectory of the ball? The child is at the bottom of the space station in the diagrams below, but only the initial location of the ball is shown.

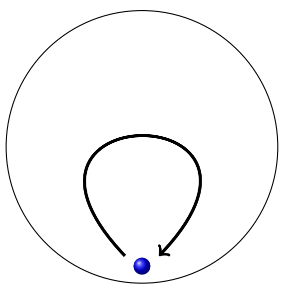
(A)



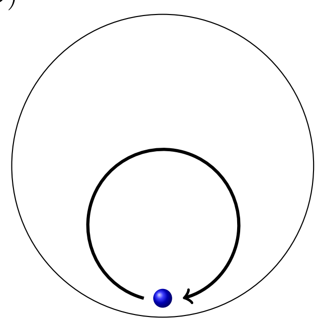
(B)



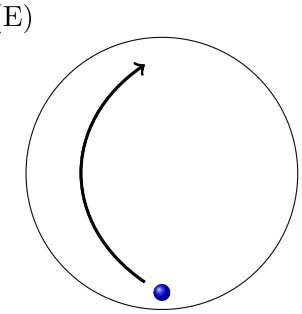
(C)



(D)



(E)



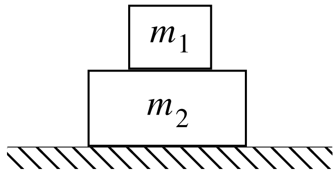
C ← CORRECT

Solution

The child catches the ball, so the path must start and end at the child, ruling out choices A and E. To decide between choices B, C, and D, consider the velocity the child observes at the moment the ball is thrown. At this moment, the ball is moving directly upward while the child is moving directly to the right, so the relative velocity is directed both upward and leftward, giving choice C.

It's also possible to solve the problem exactly, though this wasn't necessary. In polar coordinates, the radius r of the ball is linear in time, and the angle θ of the ball is also linear in time, because it is constant in the lab frame but the child's frame rotates uniformly. Explicitly, $r \propto \theta - \pi/2$ for $\theta \in [0, \pi]$ and plotting this gives choice C.

13. Two blocks of masses $m_1 = 2.0$ kg and $m_2 = 1.0$ kg are stacked together on top of a frictionless table as shown. The coefficient of static friction between the blocks is $\mu_s = 0.20$. What is the minimum horizontal force that must be applied to the top block to make it slide across the bottom block?



(A) 4.0 N

(B) 6.0 N

(C) 8.0 N

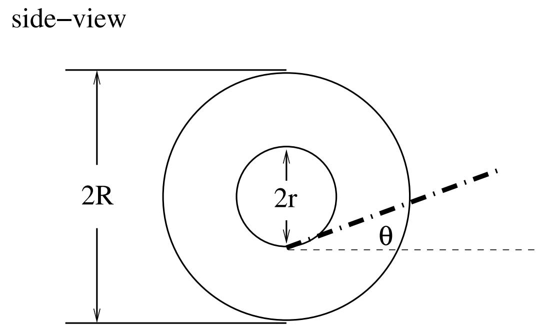
(D) 12.0 N ← **CORRECT**

(E) The top block will not slide across the bottom block

Solution

Suppose that m_1 is just about to slide. Then the friction force is the maximum possible, 4 N, while m_1 and m_2 have the same acceleration. Then $F - 4\,\mathrm{N} = 8\,\mathrm{N}$, so $F = 12\,\mathrm{N}$.

14. A spool is made of a cylinder with a thin disc attached to either end of the cylinder, as shown. The cylinder has radius $r = 0.75$ cm and the discs each have radius $R = 1.00$ cm. A string is attached to the cylinder and wound around the cylinder a few times. At what angle above the horizontal can the string be pulled so that the spool will slip without rotating?



(A) 31.2°
(B) 41.4° ← **CORRECT**
(C) 54.0°
(D) 60.8°
(E) 81.5°

Solution

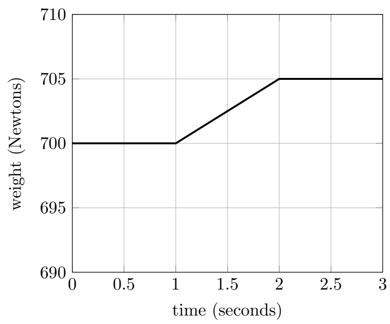
For the spool to avoid rolling, there must be zero torque about the spool's contact point with the ground. This occurs when the string lies on a line that intersects the contact point. Drawing a right triangle gives

$$\sin(90^\circ - \theta) = \frac{r}{R}$$

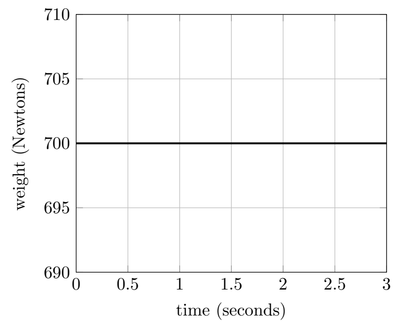
and plugging in numbers gives $\theta = 41.4^\circ$.

15. You are standing on a weight scale that reads 700 Newtons while holding a large physics textbook that is originally at rest. At time $t = 1$ seconds you begin moving the textbook upward so that by time $t = 2$ seconds the textbook is now half a meter higher and once again at rest. Which of the following graphs best illustrates how the reading on the scale might vary with time?

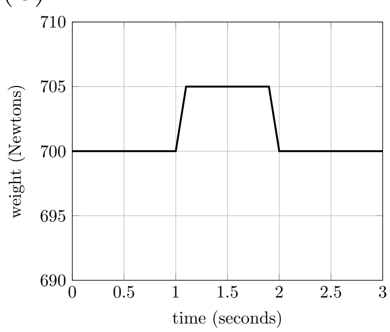
(A)



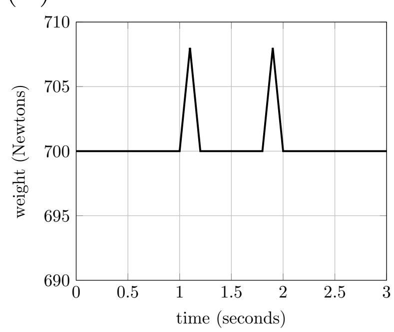
(B)



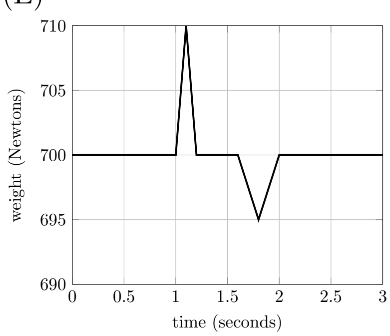
(C)



(D)



(E)



E ← CORRECT

Solution

Note that the center of mass of you and the textbook begins and ends with zero velocity; therefore the average reading on the scale must be equal to the combined weight 700 N. There is an upward spike in the beginning, because you exert a sudden upward force on the book to get it to start moving, which thus exerts a downward force on you; similarly there is a downward spike at the end.

16. A plane can fly by tilting the trailing edge of the wings downward by a small angle θ, called the angle of attack. In still air, a plane with ground speed v will have a lift force proportional to $v^2\theta$ and a drag force is proportional to v^2.

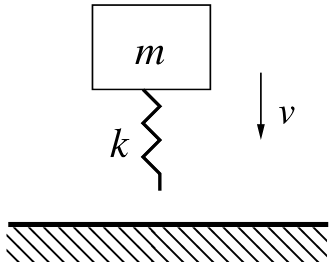
Consider a plane initially in level flight in still air with constant ground speed v. If the plane enters a region with a tailwind with speed $w < v$ (the wind is blowing in the same direction that the plane wants to fly), how must the engine power and the angle of attack change for the plane to maintain level flight at the same ground speed?

(A) The engine power decreases and the angle of attack decreases
(B) The engine power decreases and the angle of attack stays the same
(C) The engine power decreases and the angle of attack increases ← **CORRECT**
(D) The engine power increases and the angle of attack decreases
(E) The engine power increases and the angle of attack increases

Solution

Let u be the relative velocity between the plane and the air. To keep the lift force the same, if u decreases, then θ increases. Since the drag force is proportional to u^2, the power supplied by the engines is proportional to u^3, which decreases.

17. A pogo stick is modeled as a massless spring of spring constant k attached to the bottom of a block of mass m. The pogo stick is dropped with the spring pointing downward and hits the ground with speed v. At the moment of the collision, the free end of the spring sticks permanently to the ground.



During the subsequent oscillations, the maximum speed of the block is

(A) v

(B) $v + 2mg^2/kv$

(C) $v + mg^2/kv$

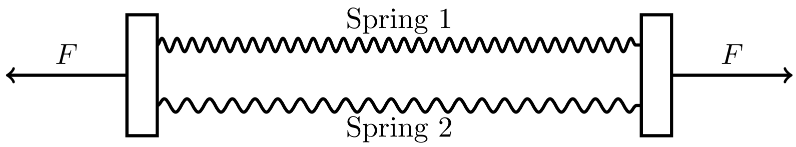
(D) $\sqrt{v^2 + 2mg^2/k}$

(E) $\sqrt{v^2 + mg^2/k}$ ← **CORRECT**

Solution

A vertical spring-mass system is equivalent to a horizontal spring-mass system with the equilibrium point shifted by $\Delta x = mg/k$. Therefore, the initial energy is $E = mv^2/2 + k(\Delta x)^2/2$. The maximum velocity satisfies $mv_{\text{max}}^2/2 = E$. Solving gives $v_{\text{max}} = \sqrt{v^2 + mg^2/k}$.

18. A spring of relaxed length ℓ_1 and spring constant k_1 is placed 'in parallel' with a spring of relaxed length ℓ_2 and spring constant k_2. A force F is applied to each end.



The combination of the springs acts like a single spring with spring constant k and relaxed length ℓ where

(A) $k = k_1 + k_2$ and $\ell = \ell_1\ell_2/(\ell_1 + \ell_2)$
(B) $k = k_1 + k_2$ and $\ell = (\ell_1 k_1 + \ell_2 k_2)/(k_1 + k_2)$ ← **CORRECT**
(C) $k = k_1 + k_2$ and $\ell = (\ell_1 k_2 + \ell_2 k_1)/(k_1 + k_2)$
(D) $k = (\ell_1 k_1 + \ell_2 k_2)/(\ell_1 + \ell_2)$ and $\ell = (\ell_1 k_1 + \ell_2 k_2)/(k_1 + k_2)$
(E) $k = (\ell_2 k_1 + \ell_1 k_2)/(\ell_1 + \ell_2)$ and $\ell = (\ell_1 k_2 + \ell_2 k_1)/(k_1 + k_2)$

Solution

The restoring force is $F = k_1(x - \ell_1) + k_2(x - \ell_2) = k(x - \ell)$, giving the result after some algebra.

19. In an experiment to determine the speed of sound, a student measured the distance that a sound wave traveled to be 75.0 ± 2.0 cm, and found the time it took the sound wave to travel this distance to be 2.15 ± 0.10 ms. Assume the uncertainties are Gaussian. The computed speed of sound should be recorded as

(A) 348.8 ± 0.5 m/s
(B) 348.8 ± 0.8 m/s
(C) 349 ± 8 m/s
(D) 349 ± 15 m/s
(E) 349 ± 19 m/s ← **CORRECT**

Solution

In this solution we use fractional uncertainties (see solutions to $F = ma$ A for more explanation). Since $v \propto d/t$, we add the fractional uncertainties of the measured quantities in quadrature, giving

$$\sqrt{\left(\frac{2.0}{75.0}\right)^2 + \left(\frac{0.10}{2.15}\right)^2} = 5.4\%.$$

Then the absolute uncertainty is $(0.054)(349\,\text{m/s}) = 19\,\text{m/s}$.

20. A massive, uniform, flexible string of length L is placed on a horizontal table of length $L/3$ that has a coefficient of friction $\mu_s = 1/7$, so equal lengths $L/3$ of string hang freely from both sides of the table. The string passes over the edges of the table on smooth, frictionless, curved surfaces.

Now suppose that one of the hanging ends of the string is pulled a distance x downward, then released at rest. Neither end of the string ever touches the ground in this problem. The maximum value of x so that the string does not slip off of the table is

(A) $L/42$ ← **CORRECT**

(B) $L/21$

(C) $L/14$

(D) $2L/21$

(E) $3L/14$

Solution

The flat part of the table at the top is the only place where friction acts. The mass of the rope on this portion is $M/3$, so the frictional force that must be overcome is

$$f = \mu m g = \frac{1}{7}\frac{Mg}{3}.$$

The force difference between the two hanging sections is given by

$$\Delta F = 2x\frac{Mg}{L}$$

so the condition for slipping is $\Delta f > f$, or $x > L/42$.

21. A uniform bar of length L and mass M is supported by a fixed pivot a distance x from its center. The bar is released from rest from a horizontal position. The period of the resulting oscillations is minimal when

(A) $x = L/2$

(B) $x = L/2\sqrt{3}$ ← **CORRECT**

(C) $x = L/4$

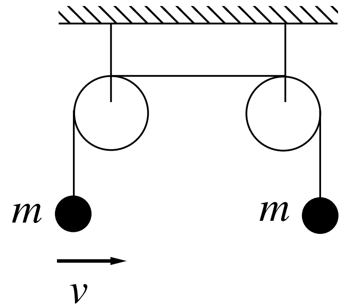
(D) $x = L/4\sqrt{3}$

(E) $x = L/12$

Solution

The bar is a physical pendulum, so its period is proportional to $\sqrt{I/Mgx}$ where $I = ML^2/12 + Mx^2$ by the parallel axis theorem. Then x should be chosen to minimize $(L^2/12 + x^2)/x$. This can be done by either calculus or using the equality case in the AM-GM inequality, either of which yields $x = L/2\sqrt{3}$.

22. Two particles of mass m are connected by pulleys as shown.



The mass on the left is given a small horizontal velocity, and oscillates back and forth. The mass on the right

(A) remains at rest

(B) oscillates vertically, and with a net upward motion ← **CORRECT**

(C) oscillates vertically, and with a net downward motion

(D) oscillates vertically, with no net motion

(E) oscillates horizontally, with no net motion

Solution

The mass on the right will oscillate because the tension in the string oscillates; however, it cannot oscillate horizontally because there is no torque exerted on it. Suppose it oscillates vertically with no net motion. Then the average y-component of the tension acting on the mass on the left is equal to mg, but the x-component of the tension is nonzero; then the average magnitude of the tension is greater than mg, a contradiction. Thus the right mass must instead oscillate and move upward.

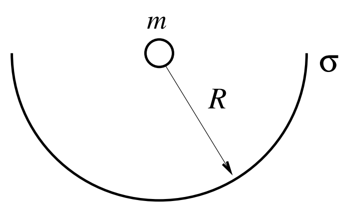
23. Two particles with mass m_1 and m_2 are connected by a massless rigid rod of length L and placed on a horizontal frictionless table. At time $t = 0$, the first mass receives an impulse perpendicular to the rod, giving it speed v. At this moment, the second mass is at rest. The next time the second mass is at rest is

(A) $t = 2\pi L/v$ ← **CORRECT**
(B) $t = \pi(m_1 + m_2)L/m_2 v$
(C) $t = 2\pi m_2 L/(m_1 + m_2)v$
(D) $t = 2\pi m_1 m_2 L/(m_1 + m_2)^2 v$
(E) $t = 2\pi m_1 L/(m_1 + m_2)v$

Solution

The motion is the superposition of two motions: uniform translation of both masses with speed $m_1 v/(m_1 + m_2)$ and circular motion about the common center of mass, where the first mass has speed $m_2 v/(m_1 + m_2)$. The second mass comes to rest again after one period of the circular motion. The radius is $Lm_2/(m_1 + m_2)$, giving a period of $t = 2\pi L/v$.

24. A particle of mass m is placed at the center of a hemispherical shell of radius R and mass density σ, where σ has dimensions of kg/m^2.



The gravitational force of the shell on the particle is

(A) $(1/3)(\pi G m \sigma)$
(B) $(2/3)(\pi G m \sigma)$
(C) $(1/\sqrt{2})(\pi G m \sigma)$
(D) $(3/4)(\pi G m \sigma)$
(E) $\pi G m \sigma$ ← **CORRECT**

Solution

By Newton's third law, it is equivalent to find the force of the particle on the shell. The pressure exerted on the shell is $P = Gm\sigma/R^2$. Now consider a closed hemisphere filled with a gas of pressure P. Since the hemisphere cannot move on its own, the net force on the flat face is equal to the net force on the curved face, which is $\pi R^2 P$. Then the force is $\pi(Gm\sigma)$.

25. A student is measuring the surface area of a cylindrical wire. The student measures the radius of the wire to be 1 ± 0.1 cm using a ruler and the length of the wire to be 1.00 ± 0.01 m using a meter-stick. The precision of their result can be increased in several ways.

1: Upgrade the ruler to a caliper with uncertainty 0.01 cm.

2: Upgrade the meter-stick to a tape measure with uncertainty 0.001 m.

3: Repeat the measurement independently ten times and average the results.

How do the resulting uncertainties of the measurements compare?

(A) Method 3 has the lowest uncertainty, while methods 1 and 2 have the same uncertainty

(B) Method 3 has the highest uncertainty, while methods 1 and 2 have the same uncertainty

(C) Method 1 has the highest uncertainty, and method 2 has the lowest

(D) Method 2 has the highest uncertainty, and method 1 has the lowest $\leftarrow$ **CORRECT**

(E) Method 2 has the highest uncertainty, and method 3 has the lowest

Solution

In this solution we use fractional uncertainties (see solutions to $F = ma$ A for more explanation). By the rules explained there, the fractional uncertainty of an average of N independent trials is decreased by a factor of $1/\sqrt{N}$. The surface area is proportional to the radius times the length, so we add their fractional uncertainties in quadrature. Then we have

1: $\sqrt{(1\%)^2 + (1\%)^2} \approx 1.4\%$

2: $\sqrt{(0.1\%)^2 + (10\%)^2} \approx 10\%$

3: $\sqrt{(1\%)^2 + (10\%)^2}/\sqrt{10} \approx 3.3\%.$

Then method 2 has the highest uncertainty and method 1 has the lowest.