AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam A

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2022.**

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We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A projectile is thrown upward with speed v. By the time its speed has decreased to $v/2$, it has risen a height h. Neglecting air resistance, what is the maximum height reached by the projectile?

(A) $\frac{5h}{4}$ (B) $\frac{4h}{3}$ (C) $\frac{3h}{2}$ (D) $2h$ (E) $3h$

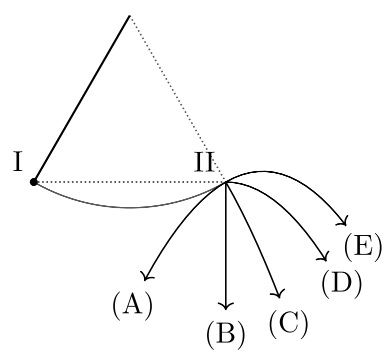
2. A car is moving at 60 miles per hour (mph), when the driver notices an obstacle ahead. Hitting the brakes, the driver decelerates at a constant rate, and manages to come to a stop just barely before hitting the obstacle. If the car had instead been moving at 70 mph, and started decelerating at the same place and at the same rate, with what speed would it have hit the obstacle?

(A) 10 mph
(B) 14 mph
(C) 28 mph
(D) 36 mph
(E) There is not enough information to decide.

3. Two blocks of mass m have an inelastic one-dimensional collision. Initially, the first block is moving with speed 5 m/s, and the second is at rest. After the collision, the first block is moving with speed 2 m/s. What percentage of the system's original kinetic energy was lost during the collision?

(A) 16% (B) 42% (C) 48% (D) 52% (E) 84%

4. A mass on an ideal pendulum is released from rest at point I. It swings over to point II, at which point the string suddenly breaks. Which of the following shows the trajectory of the mass?



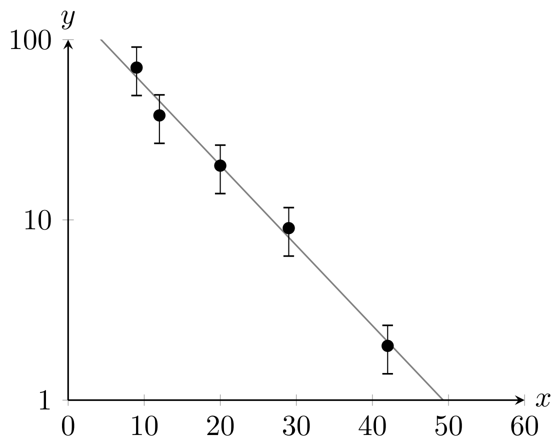
5. A uniform solid ball with mass $m = 1$ kg and radius $R = 10$ cm rolls without slipping on a horizontal plane, so that its center of mass has velocity $v = 1$ m/s. What is the ball's total kinetic energy?

(A) 0.2 J (B) 0.5 J (C) 0.7 J (D) 1 J (E) 1.4 J

6. A bob of mass m hangs from a rigid, massless rod, forming an ideal pendulum. The rod is held horizontally and released from rest. What is its maximum tension during its swing?

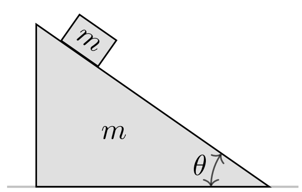
(A) mg (B) $\frac{3}{2}mg$ (C) $2mg$ (D) $3mg$ (E) $4mg$

7. The following graph shows the results of measurements of two physical quantities, y and x. What is the following best describes the functional dependence of y on x? Below, A and B are positive constants.



(A) $y = Ax + B$ (B) $y = -Ax + B$ (C) $y = A/x^B$ (D) $y = Ae^{Bx}$ (E) $y = Ae^{-Bx}$

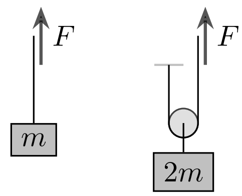
8. A block of mass m is placed on a wedge of mass m, inclined at an angle θ to the horizontal.



The coefficients of friction between the block and wedge, and the wedge and ground, are high enough for both the block and the wedge to remain static. What is the magnitude of the friction force of the ground on the wedge?

(A) $mg\sin\theta$ (B) $mg\cos\theta$ (C) $mg\sin\theta\cos\theta$ (D) $mg\tan\theta$ (E) 0

9. A person is holding a massless rope, on which hangs a mass m, as shown at left. To pull the end of the rope with constant upward velocity v, the person must exert a force F_v. To pull the end of the rope with constant upward acceleration a, the person must exert a force F_a. Now the rope is wrapped around a fixed, massless pulley, and the mass is doubled to $2m$, as shown at right.



Compared to the original setup, how do the forces F_v and F_a needed to pull the end of the rope with a given upward velocity and acceleration change? In both cases, ignore friction and air resistance.

(A) F_v stays the same, and F_a decreases.

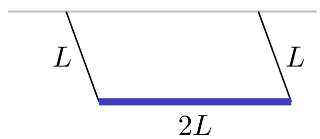
(B) Both F_v and F_a stay the same.

(C) F_v stays the same, and F_a increases.

(D) F_v increases, and F_a stays the same.

(E) Both F_v and F_a increase.

10. The two ends of a uniform rod of length $2L$ are hung on massless strings of length L.



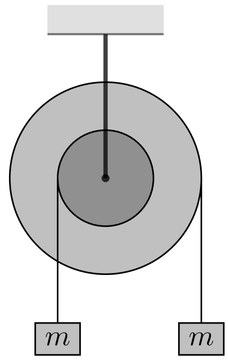
If the strings are attached to the ceiling, and the rod is pulled a small distance horizontally and released as shown, what is the period of oscillation?

(A) $2\pi\sqrt{\frac{L}{g}}$ (B) $2\pi\sqrt{\frac{7L}{6g}}$ (C) $2\pi\sqrt{\frac{4L}{3g}}$ (D) $2\pi\sqrt{\frac{2L}{g}}$ (E) $2\pi\sqrt{\frac{7L}{3g}}$

11. Two identical spherically symmetric planets, each of mass M, are somehow held at rest with respect to each other. Each planet has radius R, and the distance between the centers of the planets is $4R$. If a rocket is launched from the surface of one planet with speed v, what is the minimum speed v so that the rocket can reach the other planet?

(A) $\sqrt{\frac{2GM}{R}}$ (B) $\sqrt{\frac{GM}{R}}$ (C) $\sqrt{\frac{3GM}{4R}}$ (D) $\sqrt{\frac{2GM}{3R}}$ (E) $\sqrt{\frac{GM}{2R}}$

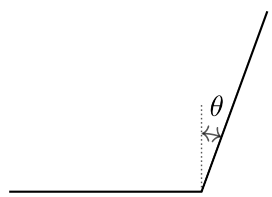
12. A pulley is constructed by attaching two concentric cylinders, with the larger cylinder having twice the radius. Ropes are wrapped around both cylinders, a mass m is hung from each rope, and the system is released from rest.



Neglect the masses of the cylinders and ropes. Each mass experiences both a gravitational and a tension force. If the *net* force experienced by the left mass is F_1, and the net force experienced by the right mass is F_2, what is the ratio F_2/F_1?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) 4

13. Consider a laptop made of two identical uniform plates, each of mass $m/2$, connected by a hinge. The hinge is locked when the screen makes an angle θ to the vertical, as shown, fixing the angle between the two pieces.

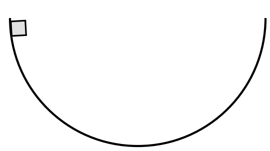


Assuming the laptop does not slip, what is the minimum force that can be exerted on the top of the laptop, in the plane of the page, to cause the bottom of the laptop to lift off the ground?

(A) $\dfrac{mg(1-\sin\theta)}{2}$ (B) $\dfrac{mg(\cos\theta+\sin\theta)}{2}$ (C) $\dfrac{mg(1-\sin\theta)}{4}$

(D) $\dfrac{mg(1+\sin\theta)}{4}$ (E) $\dfrac{mg(\cos\theta+\sin\theta)}{4}$

14. A small block is released from rest on the rim of a fixed, frictionless hemispherical bowl.



From the time the block is released, until it reaches the bottom of the bowl, which of the following is true?

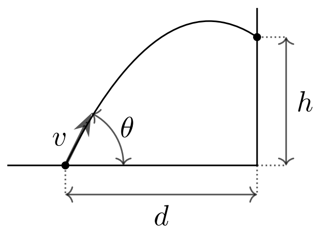
I. The speed of the block never decreases.

II. The magnitude of the horizontal component of the velocity of the block never decreases.

III. The magnitude of the vertical component of the velocity of the block never decreases.

(A) Only I. (B) Only III. (C) I and II. (D) I and III. (E) I, II, and III.

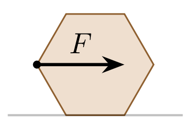
15. An egg is launched with speed v from the ground, a distance d from a vertical wall.



If v is high enough for the egg to hit the wall, which of the following could describe the angle θ that maximizes the height h at which the egg hits the wall?

(A) $\sin\theta = \dfrac{v^2}{gd}$ (B) $\tan\theta = \dfrac{v^2}{gd}$ (C) $\sin 2\theta = \dfrac{gd}{2v^2}$ (D) $\cos\theta = \dfrac{gd}{v^2}$ (E) $\sin 2\theta = \dfrac{v^2}{gd}$

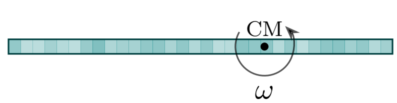
16. A hexagonal pencil of uniform density lies at rest on a horizontal table. It is pushed horizontally with a steadily increasing force halfway up its height, as shown.



What is the minimum value of the coefficient of static friction between the floor and pencil, so that the pencil will eventually begin to roll?

(A) 0 (B) $\dfrac{1}{3}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{\sqrt{3}}{3}$ (E) $\dfrac{\sqrt{3}}{2}$

17. A thin rod has a *nonuniform* density. It is mounted on an axle passing perpendicular to it, through its center of mass, as shown, and is then rotated about the axle.



The axle divides the rod into two parts, one on each side of it. Which of the following must be true, no matter how the mass in the rod is distributed?

(A) The two parts have the same mass.

(B) The magnitudes of the momenta of the two parts are equal.

(C) The magnitudes of the angular momenta of the two parts, about the center of mass, are equal.

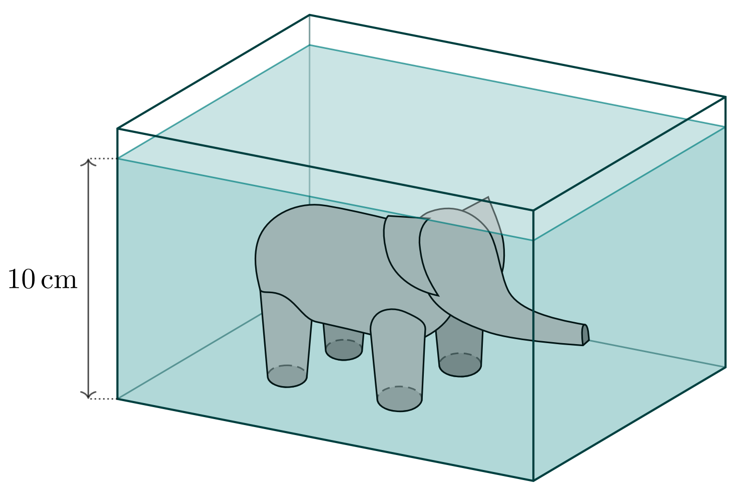
(D) The kinetic energies of the two parts are equal.

(E) At least two of the above are true.

18. A cylindrical piece of cork of density ρ_c, height h_c, and cross-sectional area A_c is in a larger empty cylindrical container of cross-sectional area A_w. Water of density $\rho_w > \rho_c$ is slowly poured into the empty container. What is the height of the water in the container when the cork starts to float?

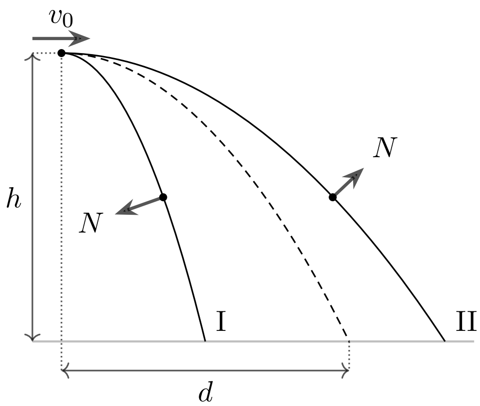
(A) $\dfrac{h_c \rho_c A_c}{\rho_w A_w}$ (B) $\dfrac{h_c \rho_c}{\rho_w}$ (C) $\dfrac{h_c \rho_w}{\rho_c}$ (D) $\dfrac{h_c \rho_c A_c}{\rho_w (A_w - A_c)}$ (E) $\dfrac{h_c \rho_c A_c^2}{\rho_w A_w^2}$

19. A toy elephant is standing on the bottom of a fish tank. The fish tank is filled with water to a depth of 10 cm, completely covering the toy. The elephant's legs are perfectly polished, so that there is no water between the bottom of the legs and the tank's floor, and the total area of contact is $0.16\,\text{cm}^2$. The water has density $\rho = 10^3\,\text{kg/m}^3$, the toy has uniform density 2ρ, the atmospheric pressure is $P_{\text{atm}} = 10^5\,\text{Pa}$, and the toy has total mass 120 g. What is the total hydrostatic force that the water exerts on the toy?



(A) 1 N, down (B) 0.6 N, down (C) 0 N (D) 0.6 N, up (E) 1 N, up

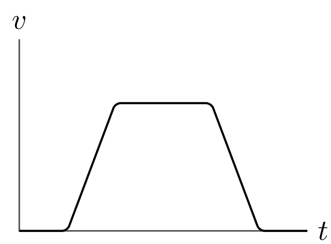
20. A bead is threaded on a frictionless wire and launched horizontally from height h with speed v_0, as shown. If the shape of the wire is steep, as in curve I, then the normal force from the wire on the bead will point inward. If it is shallow, as in curve II, then the normal force will point outward.

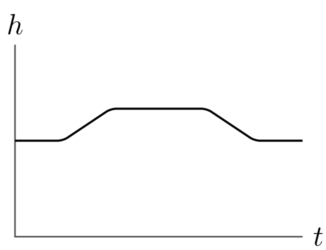


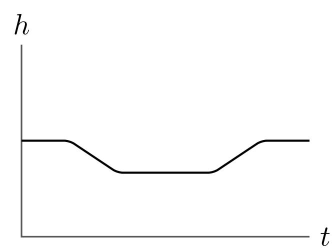
There is exactly one possible shape of wire, shown as a dotted line, for which the normal force of the wire on the bead is *always* equal to zero. What is the horizontal displacement d of the bead when it travels along this wire?

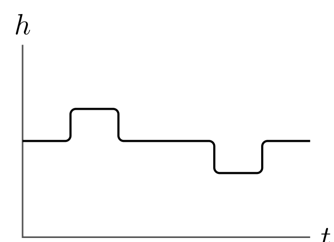
(A) $v_0\sqrt{\dfrac{4g}{h}}$ (B) $v_0\sqrt{\dfrac{2h}{g}}$ (C) $v_0\sqrt{\dfrac{h}{g}}$ (D) $v_0\sqrt{\dfrac{h}{2g}}$ (E) $v_0\sqrt{\dfrac{h}{4g}}$

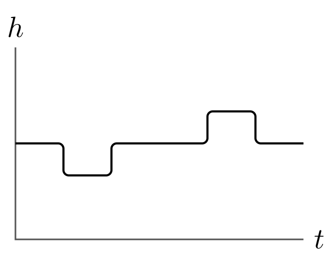
21. A cork floating in a cup filled with a viscous fluid is placed in an elevator. Below is a plot of the velocity v of the elevator as a function of time t. Which of the following plots best describes the height h of the cork in the cup as a function of time? Assume that the fluid is viscous enough to dampen all oscillations, that the fluid does not slosh as the elevator accelerates, and that both the cork and fluid are incompressible.



(A)


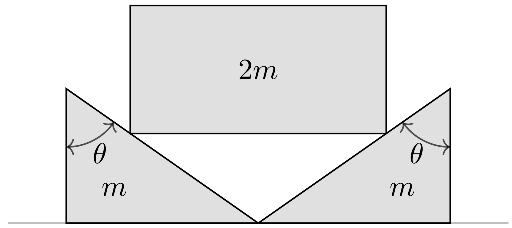
(B)


(C)


(D)


(E)


22. A block of mass $2m$ is placed symmetrically on two identical wedges of mass m, as shown.



All surfaces are frictionless, and the wedges have angle θ to the vertical. If the system is released from rest, what is the downward acceleration of the block?

(A) $g \sin\theta$ (B) $g\sin(2\theta)$ (C) $g\cos\theta$ (D) $g\cos(2\theta)$ (E) $g\cos^2\theta$

23. For objects moving through air, the force of air resistance can be modeled as proportional to the speed ("linear drag") or proportional to the square of the speed ("quadratic drag"), depending on the circumstances. Two identical objects, A and B, are dropped from the same height h simultaneously, but object A is given an initial horizontal velocity v. The objects hit the ground at times t_A and t_B. Accounting for air resistance, which of the following is true?

(A) For both linear drag and quadratic drag, $t_A = t_B$.
(B) For linear drag, $t_A > t_B$, while for quadratic drag, $t_A = t_B$.
(C) For linear drag, $t_A = t_B$, while for quadratic drag, $t_A > t_B$.
(D) For both linear drag and quadratic drag, $t_A > t_B$.
(E) For both linear drag and quadratic drag, the answer depends on v and h.

24. A satellite is in orbit around a planet of mass M. Its maximum distance from the center of the planet is d, and at this point, it is traveling at a speed of $\frac{1}{2}\sqrt{\frac{GM}{d}}$. What is the area of the satellite's orbit?

(A) $\frac{8}{15}\sqrt{\frac{2}{15}}\pi d^2$ (B) $\frac{4}{7}\sqrt{\frac{1}{7}}\pi d^2$ (C) $\frac{1}{3}\sqrt{\frac{2}{3}}\pi d^2$ (D) $\frac{8}{7}\sqrt{\frac{1}{7}}\pi d^2$ (E) $\frac{2}{3}\sqrt{\frac{2}{3}}\pi d^2$

25. A cylinder is placed with its axis vertical, and a rubber band of mass m and tension T is wrapped horizontally around it. What is the minimum coefficient of static friction μ between the rubber band and the cylinder such that the band will not slide down the cylinder?

(A) $\frac{mg}{2\pi T}$ (B) $\frac{mg}{T}$ (C) $\frac{4mg}{T}$ (D) $\frac{2\pi mg}{T}$ (E) $\frac{2m^2g^2}{T^2}$