AAPT UNITED STATES PHYSICS TEAM
AIP 2022

2022 $F = ma$ Exam B

25 QUESTIONS - 75 MINUTES

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Use $g = 10$ N/kg throughout this contest.
- You may write in this booklet of questions. However, you will not receive any credit for anything written in this booklet. You may only use the scratch paper provided by the proctor.
- This test contains 25 multiple choice questions. Select the answer that provides the best response to each question. Please be sure to use a No.2 pencil and completely fill the box corresponding to your choice. If you change an answer, the previous mark must be completely erased. Only the boxes preceded by numbers 1 through 25 are to be used on the answer sheet.
- All questions are equally weighted, but are not necessarily of the same level of difficulty.
- Correct answers will be awarded one point; incorrect answers or leaving an answer blank will be awarded zero points. There is no additional penalty for incorrect answers.
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **In order to maintain exam security, do not communicate any information about the questions (or their answers or solutions) on this contest until after February 25, 2022.**

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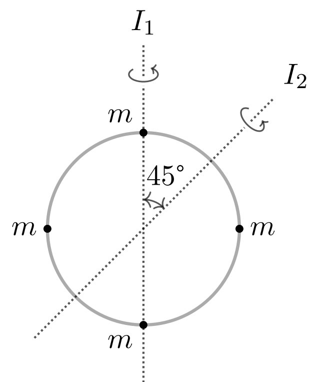
We acknowledge the following people for their contributions to this year's exams (in alphabetical order):

Tengiz Bibilashvili, Abi Krishnan, Andrew Lin, Kris Lui, Kye Shi, Brian Skinner, Mike Winer and Kevin Zhou

1. A ball is held a height h above a slope, which is at an angle 45° from the horizontal. The ball is dropped from rest. Assume the ball bounces off the slope perfectly elastically. What is the distance between its first and second impact points?

(A) $2h$ (B) $2\sqrt{2}\,h$ (C) $4h$ (D) $4\sqrt{2}\,h$ (E) $8h$

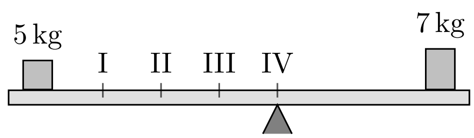
2. A massless wheel of radius R has four small masses M placed evenly along its rim. Let the moment of inertia for rotations about the center of wheel, in the plane of the wheel, be I_0.



Now consider rotations about the two axes shown, with corresponding moments of inertia I_1 and I_2. Which of the following is true?

(A) $I_1 = I_2 = I_0/2$ (B) $I_1 = I_2 = I_0/\sqrt{2}$ (C) $I_1 = I_2 = I_0$
(D) $I_1 = I_0/2, I_2 = I_0/\sqrt{2}$ (E) $I_1 = I_0, I_2 = I_0/\sqrt{2}$

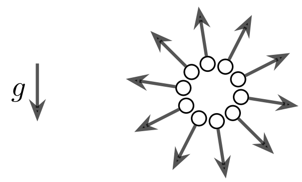
3. Four evenly spaced points are marked on a massless seesaw, as shown.



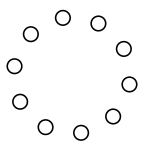
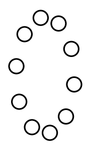
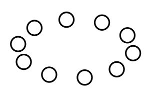
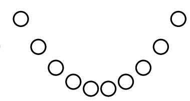
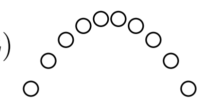
Two blocks, of masses 5 kg and 7 kg, are placed on the seesaw, so that it balances when the fulcrum is at point IV. (The diagram is not drawn to scale.) Now suppose the fulcrum is moved to point II. How much mass should be placed at point I so that the seesaw again balances?

(A) 12 kg
(B) 18 kg
(C) 24 kg
(D) 36 kg
(E) There is not enough information to decide.

4. A firework explodes, sending shells in all directions in a vertical plane, as shown.



Suppose the shells are all launched with the same speed, and ignore air resistance, but not gravity. A long time later, but before any of the shells hit the ground, what shape do the shells make?

(A)  (B)  (C)  (D)  (E) 

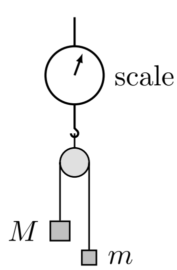
5. A swimmer swims at speed v relative to still water. A river flows from a pier to a lake, with speed u. If the swimmer swims downstream from the pier to the lake, then back upstream, what was their average speed during the trip?

(A) v (B) $\sqrt{v^2 - u^2}$ (C) $\frac{(v-u)^2}{v}$ (D) $\frac{(v+u)^2}{v}$ (E) $\frac{v^2 - u^2}{v}$

6. A block of mass m is at rest on a frictionless table. It is pushed horizontally by a constant force F, and has total momentum p when it reaches the end of the table. If a block of mass $2m$ is pushed across the table in the same way, also starting from rest, what is its momentum when it reaches the end of the table?

(A) $\frac{p}{2}$ (B) $\frac{p}{\sqrt{2}}$ (C) p (D) $\sqrt{2}\,p$ (E) $2p$

7. Two blocks of unequal mass M and m are hung from the ends of a massless, frictionless pulley, as shown. The blocks are held in place, and the entire pulley is mounted on a sensitive scale.



After the blocks are released from rest, but before either has fallen off the pulley, what is the scale reading?

(A) It is less than $(M + m)g$.

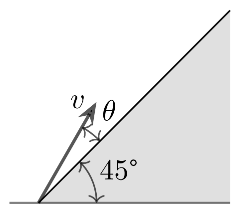
(B) It is equal to $(M + m)g$.

(C) It is more than $(M + m)g$.

(D) It is initially less than $(M + m)g$, then approaches $(M + m)g$.

(E) It is initially more than $(M + m)g$, then approaches $(M + m)g$.

8. A ball is launched with speed v at an angle θ to a fixed ramp, which itself makes a 45° angle with the horizontal.



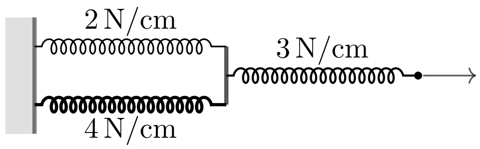
Assume that when the ball hits the ramp, it bounces elastically and frictionlessly. For what value of θ will the ball bounce off the ramp two times, and then return to its original launch point?

(A) 9.5° (B) 11.3° (C) 14.0° (D) 14.5° (E) 18.4°

9. Billy is leaning on a box of mass 30.0 kg, exerting a force 35.0° below the horizontal. If the coefficient of static friction of the box on the ground is $\mu_s = 0.400$, what is the minimum force needed for the box to slide?

(A) 120 N (B) 147 N (C) 203 N (D) 224 N (E) 342 N

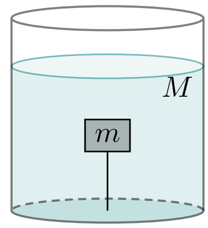
10. Two springs with spring constants 2 N/cm and 4 N/cm are connected in parallel. They are both connected in series to a spring of constant 3 N/cm, as shown.



What force must be exerted at the end to extend the system by 1 cm from its relaxed length?

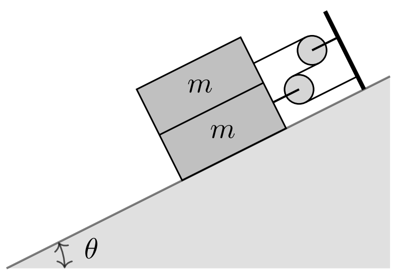
(A) 0.5 N (B) 2 N (C) 3 N (D) 6 N (E) 9 N

11. Water with total mass M is poured into a cup of cross-sectional area A. A block of mass m, whose density is half that of water, is tied to a thin string. The string is attached to the bottom of the cup, and the block floats in the water as shown. The atmospheric pressure is P_{atm}. What is the total pressure force that the water exerts on the bottom of the cup?



(A) Mg (B) $(M+m)g$ (C) $P_{\text{atm}}A + Mg$
(D) $P_{\text{atm}}A + (M+m)g$ (E) $P_{\text{atm}}A + (M+2m)g$

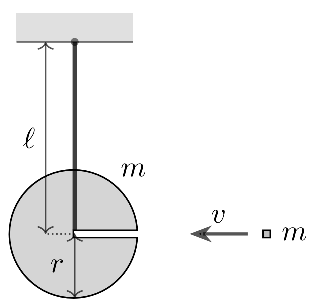
12. Two identical bricks of mass m are attached to two frictionless pulleys by a massless string, on a fixed slope of angle θ to the horizontal, as shown.



Suppose that the coefficient of static friction between the lower block and the slope is μ, and all other surfaces are frictionless. What is the minimum value of μ so that the blocks can stay static?

(A) $\frac{1}{2}\tan\theta$ (B) $\frac{2}{3}\tan\theta$ (C) $\tan\theta$ (D) $\frac{3}{2}\tan\theta$ (E) $2\tan\theta$

13. A ballistic pendulum is designed as shown below.



A uniform ball of mass m and radius r is attached to a massless rigid rod, so that its center is a distance ℓ from the ceiling. The ball has a small tunnel hollowed out. A small block of mass m and speed v goes into the tunnel and collides at the center of the ball, perfectly inelastically. Subsequently, to what maximum height does the block rise?

(A) $\dfrac{v^2}{8g}$ (B) $\dfrac{v^2}{8g(1+r^2/5\ell^2)^2}$ (C) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)^2}$
(D) $\dfrac{v^2}{8g(1+r^2/5\ell^2)}$ (E) $\dfrac{v^2}{8g(1+2r^2/5\ell^2)}$

14. A cylinder of water is suspended in a space station. Under the influence of surface tension, the cylinder splits into droplets. After a short time, viscosity causes the droplets to settle into static shapes. Neglect the effect of evaporation. Compared to the original cylinder, the final set of droplets will have

(A) Almost exactly the same volume and surface area

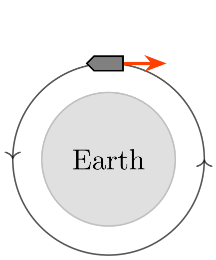
(B) More volume, but almost exactly the same surface area

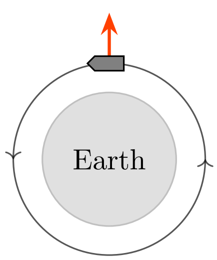
(C) Less volume, but almost exactly the same surface area

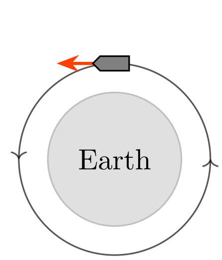
(D) More surface area, but almost exactly the same volume

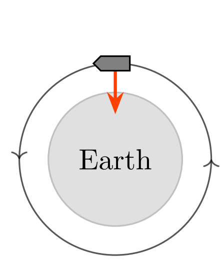
(E) Less surface area, but almost exactly the same volume

15. Five astronauts are orbiting Earth in a low circular orbit. They decided to go around the planet following the same orbit, but much faster, to set a new record for orbiting Earth. Four of them thought that they could quickly increase the velocity, then use a jet engine to maintain an orbit of the same shape; in the first four choices below, the red arrow denotes the direction that fuel will flow out of the jet nozzle. The fifth astronaut was skeptical about such a possibility. Who is right?

(A) 

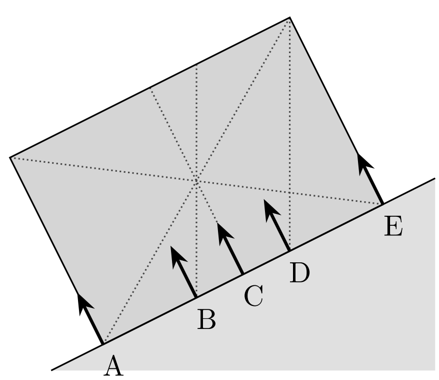
(B) 

(C) 

(D) 

(E) It is impossible to follow the same orbit faster.

16. A uniform rectangular block is in static equilibrium on an inclined plane, and experiences gravity, static friction, and the normal force from the plane. Though gravity acts on the entire volume of the block, for the purposes of torque balance, it is equivalent to a single resultant force acting at the block's center of mass. Similarly, while the normal force acts on the entire bottom surface of the block, its torque is equivalent to a resultant force acting at a single point. Which of the following best shows this point?



(A) Point A, at the lowest point of the bottom side

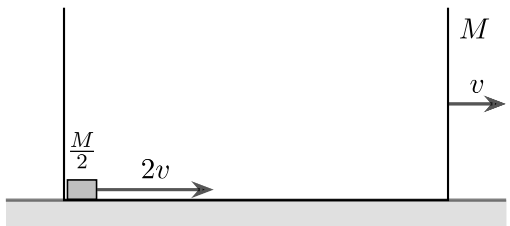
(B) Point B, directly below the center of mass

(C) Point C, at the midpoint of the bottom side

(D) Point D, directly below the top corner

(E) Point E, at the highest point of the bottom side

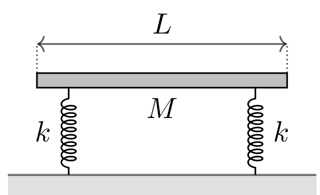
17. A box of mass M is sliding to the right with velocity v on a frictionless table. A small puck of mass $M/2$ slides frictionlessly inside the box. Initially, the puck is at the left wall of the box, with a rightward velocity of $2v$ with respect to the table. After a time T, the puck collides elastically with the right wall of the box. How much longer will it take until the puck hits the left wall of the box again?



(A) $T/2$ (B) T (C) $2T$ (D) $3T$ (E) $4T$

18. **The following information applies to problems 18 and 19.**

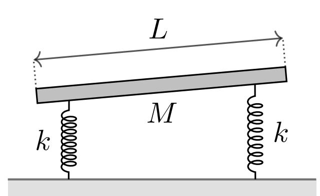
A uniform rod of length L and mass M is placed with its ends resting on two identical springs of spring constant k, as shown.



The rod is initially in equilibrium. If the rod is uniformly displaced downward and released from rest, what is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{4M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$

19. Next, the rod is brought back to equilibrium. It is slightly rotated about its center of mass, then released from rest.



What is the frequency f of its oscillations?

(A) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{3k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{4k}{M}}$ (E) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$

20. A uniform, hollow sphere is initially at rest on a horizontal plane. The plane begins to accelerate horizontally to the right, with acceleration a. If friction is high enough to prevent the sphere from slipping, what is its translational acceleration?

(A) $2a/3$ to the left (B) $a/3$ to the left (C) zero
(D) $2a/5$ to the right (E) $2a/3$ to the right

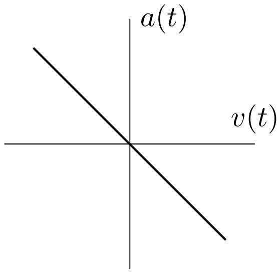
21. Amora and Bronko are given a long, thin rectangle of sheet metal. (It has been machined very precisely, so they can assume it is perfectly rectangular.) Using calipers, Amora measures the width of the rectangle as 1 cm with 1% uncertainty. Using a tape measure, Bronko independently measures its length as 100 cm with 0.1% uncertainty. Which of the following are closest to the relative uncertainties they should report for the area and the perimeter of the rectangle, respectively?

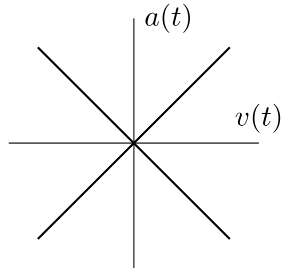
(A) 0.1%; 0.2% (B) 1%; 0.1% (C) 1%; 0.2% (D) 1%; 1% (E) 1%; 2%

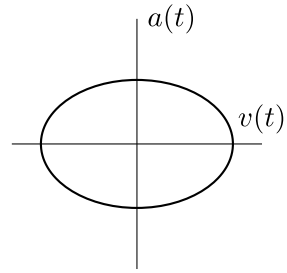
22. A cannon just above the surface of a spherical planet with mass M and radius R launches a particle with speed v, where $\sqrt{GM/R} < v < \sqrt{2GM/R}$. The initial velocity of the particle makes an angle θ with the vertical direction. Neglect drag. For what θ will the particle never collide with the planet?

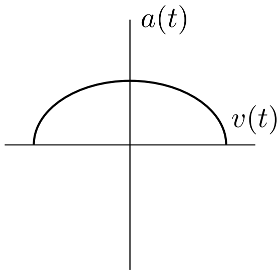
(A) All possible launch angles, $0° \leq \theta \leq 90°$
(B) $\cos^{-1}\left(\sqrt{\frac{v^2R}{GM}-1}\right) \leq \theta \leq 90°$
(C) $\sin^{-1}\left(\sqrt{\frac{v^2R}{GM}-1}\right) \leq \theta \leq 90°$
(D) Only $\theta = 90°$
(E) A collision will occur for any value of θ

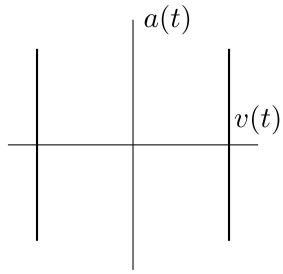
23. A mass attached to a spring is performing simple harmonic motion, with velocity $v(t)$ and acceleration $a(t)$. Which of the following could be a graph of the curve $(v(t), a(t))$ over a complete oscillation?

(A) 

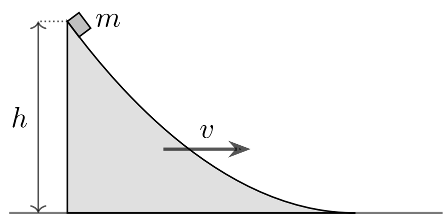
(B) 

(C) 

(D) 

(E) 

24. A ramp with height h is moving with fixed, uniform speed v to the right. A small block of mass m is placed at the top of the ramp, and is released at rest with respect to the ramp.



The block slides smoothly to the bottom of the ramp and onto the floor. How much kinetic energy does it gain in this process? Neglect friction.

(A) mgh (B) $mgh + mv^2/2$ (C) $mgh + mv\sqrt{2gh}$
(D) $mgh + mv\sqrt{gh} + mv^2/2$ (E) $mgh + mv\sqrt{2gh} + mv^2$

25. Two masses are initially at rest, separated by a distance r, and attract each other gravitationally. If their masses are m and $2m$, then they will collide after a time T. How long would they take to collide if they both had mass $2m$?

(A) $\left(\frac{2}{3}\right)^{3/2} T$ (B) $\frac{3}{4} T$ (C) $\sqrt{\frac{2}{3}}\, T$ (D) $\sqrt{\frac{3}{4}}\, T$ (E) $\sqrt{\frac{8}{9}}\, T$