

## 2025 $F = ma$ Exam

25 QUESTIONS - 75 MINUTES

### INSTRUCTIONS

**DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN**

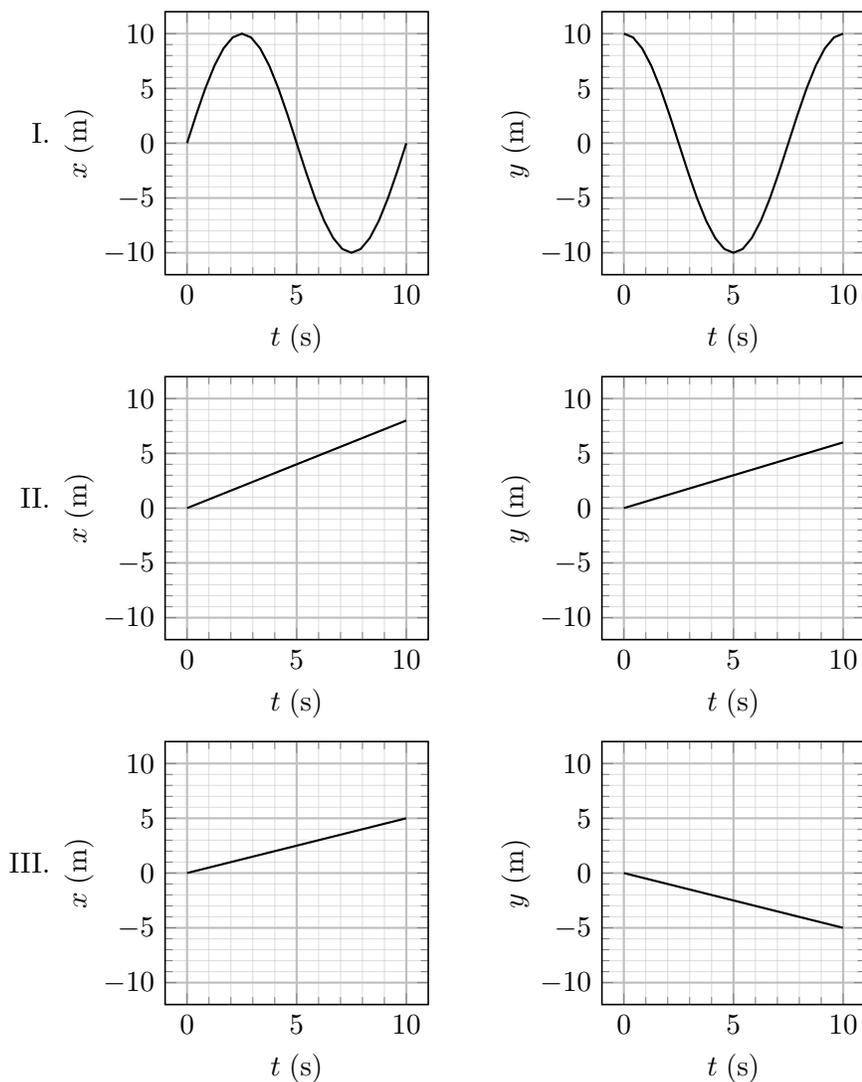
- Use  $g = 10 \text{ N/kg}$  throughout, unless otherwise specified.
- You may write in this question booklet and the scratch paper provided by the proctor.
- This test has 25 multiple choice questions. Select the best response to each question, and use a No.2 pencil to completely fill the box corresponding to your choice. If you change an answer, completely erase the previous mark. Only use the boxes numbered 1 through 25 on the answer sheet.
- All questions are equally weighted, but are not necessarily equally difficult.
- You will receive one point for each correct answer, and zero points for each incorrect or blank answer. There is no additional penalty for incorrect answers.
- You may use a hand-held calculator. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any external references, such as books or formula sheets.
- The question booklet, answer sheet and scratch paper will be collected at the end of this exam.
- **To maintain exam security, do not communicate any information about the questions or their solutions until after February 13, 2025.**

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We acknowledge The US Physics Team coaches and other contributors to this year's exams:

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1. A particle is moving on a plane at a constant speed of 1 m/s, but not necessarily in a straight line. Which of the following plot pairs could describe the particle's position over time, in rectilinear coordinates?

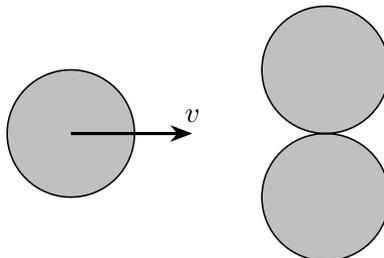


- (A) I only  
 (B) II only  
 (C) III only  
 (D) II and III only  
 (E) All three plots could describe the particle's position over time

All three plots describe the particle moving at a constant speed. In plot I, the particle is moving in a circle of radius 10 m, and traversing the full circle in 10 seconds, so its speed is approximately 63 m/s. (The  $x$  component alone gets to 10 m in 2.5 seconds.) In plot II, the particle moves at a speed of  $\sqrt{(0.8 \text{ m/s})^2 + (0.6 \text{ m/s})^2} = 1 \text{ m/s}$ . In plot III, the particle moves at a speed of  $\sqrt{(0.5 \text{ m/s})^2 + (-0.5 \text{ m/s})^2} < 1 \text{ m/s}$ .

**The following information applies to problems 2 and 3.**

Three identical disks are placed on a frictionless table. Initially, two of the disks are at rest and in contact with each other. The third disk is launched with speed  $v$  directly toward the midpoint of the two stationary disks along a path perpendicular to the line connecting their centers, as shown in the diagram. Analyze the motion of the disks after the collision, assuming all interactions are perfectly elastic and that when the disks collide, there is no friction or inelastic energy loss.



2. Assume that all three disks collide simultaneously. What is the final velocity of the third disk?

- (A)  $v/3$  in the opposite direction to the initial velocity  
 (B)  $v/3$  in the same direction as the initial velocity  
 (C)  $\vec{0}$   
 (D)  $v/5$  in the opposite direction to the initial velocity  
 (E)  $v/5$  in the same direction as the initial velocity

By symmetry, the third disk continues to move along the same axis. The first two disks experience only a normal force from the third disk, which is at an angle  $30^\circ$  to the third disk's original velocity. Thus, if the third disk has final velocity  $v_f$ , and the other two disks have final speed  $u$ , conservation of energy gives

$$v^2 = v_f^2 + 2u^2$$

and conservation of momentum gives

$$v = v_f + \sqrt{3}u.$$

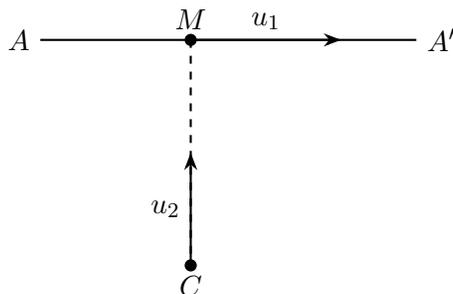
Solving the system gives  $v_f = -v/5$  and  $u = (2\sqrt{3}/5)v$ .

3. Assume that there is a little imperfection in disks' initial alignment so when the disks collide two collisions happen one at a time, rather than all three disks colliding simultaneously. What is the final speed of the third disk?

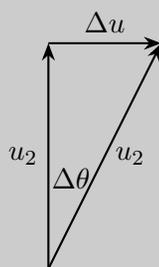
- (A)  $v/2$                       (B)  $v/3$                        (C)  $v/4$                       (D)  $v/5$                       (E) 0

After the first collision, the disk that was at rest gains speed  $v \cos 30^\circ$  and the third disk has speed  $v \sin 30^\circ$ , in the direction that's perpendicular to the line between the disks' centers. (The component of the first disk's velocity along the line connecting their centers is transferred to the second disk; the perpendicular component is unaffected.) The third disk is now moving in a direction that's  $60^\circ$  off from the line connecting its center with the center of the disk that's still at rest, so after the second collision, the disk that was at rest will gain speed of  $v \cos 30^\circ \sin 30^\circ$  and the third disk will have speed  $v \sin^2 30^\circ = v/4$ .

4. A mouse  $M$  is running from  $A$  to  $A'$  with constant speed  $u_1$ . A cat  $C$  is chasing the mouse with constant speed  $u_2$  and direction always toward the mouse. At a certain time  $MC \perp AA'$  and the length of  $MC = L$ . What is the magnitude of the acceleration of the cat  $C$ ?



- (A) 0  
 (B)  $(u_1 - u_2)^2 / (2\pi L)$   
 (C)  $u_1 u_2 / L$   
 (D)  $u_1 u_2 / (2\pi L)$   
 (E) none of the above



For small  $\Delta\theta$ ,  $a = \frac{\Delta u}{\Delta t} = \frac{u_2 \Delta\theta}{\Delta t} = u_2 \omega = \frac{u_2 u_1}{L}$

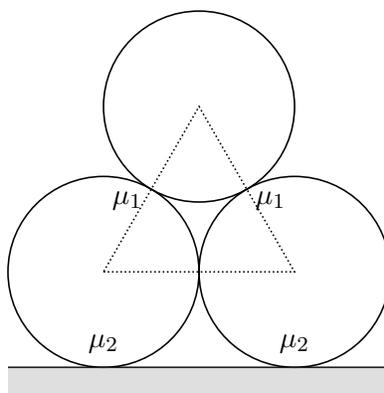
5. Three identical cylinders are used in this setup. Two of them are placed side by side on a horizontal surface, with a negligible distance between their surfaces so they do not touch. The third identical cylinder is placed on top of the first two, such that their centers form an equilateral triangle, as shown in the figure below.

The coefficients of friction are:

- $\mu_1$ : the coefficient of friction between the cylinders, and
- $\mu_2$ : the coefficient of friction between the cylinders and the ground.

For which of the following pairs  $(\mu_1, \mu_2)$  will the system remain in equilibrium?

Pair 1:  $(\frac{1}{2}, \frac{1}{12})$ . Pair 2:  $(\frac{1}{3}, \frac{1}{10})$ . Pair 3:  $(\frac{1}{4}, \frac{1}{8})$ .



- (A) Pair 1 only                      **B** Pair 2 only                      (C) Pair 3 only  
 (D) Pairs 1 and 2 only              (E) Pairs 1, Pair 2, and Pair 3

From the force analysis diagrams, we get a system of 4 equations in order to balance all forces and torques:

$$\begin{aligned} f_1 + \sqrt{3}N_1 &= mg \\ \frac{f_1}{2} + \frac{\sqrt{3}}{2}N_1 - N_2 &= -mg \\ \frac{N_1}{2} - \frac{\sqrt{3}}{2}f_1 - f_2 &= 0 \\ f_1 &= f_2 \end{aligned}$$

which solves to

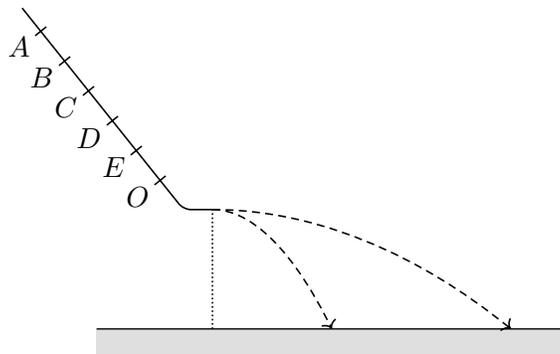
$$\begin{aligned} N_1 &= \frac{1}{2}mg \\ N_2 &= \frac{3}{2}mg \\ f_1 = f_2 &= \frac{mg}{2(2 + \sqrt{3})} \end{aligned}$$

Requiring  $f_1 \leq \mu_1 N_1$  and  $f_2 \leq \mu_2 N_2$  implies

$$\begin{aligned} \mu_1 &\geq \frac{1}{2 + \sqrt{3}} \\ \mu_2 &\geq \frac{1}{3(2 + \sqrt{3})} \end{aligned}$$

Knowing  $1.5 < \sqrt{3} < 2$  is sufficient to choose the correct answer.

6. A ball rolls without slipping down a ramp, which turns horizontal at the bottom; at the bottom of the ramp, the ball falls through the air, as in the diagram. If the ball starts from the position marked  $O$ , it lands 10 cm away from the bottom of the ramp. Which starting position will get the ball to land closest to 25 cm away?

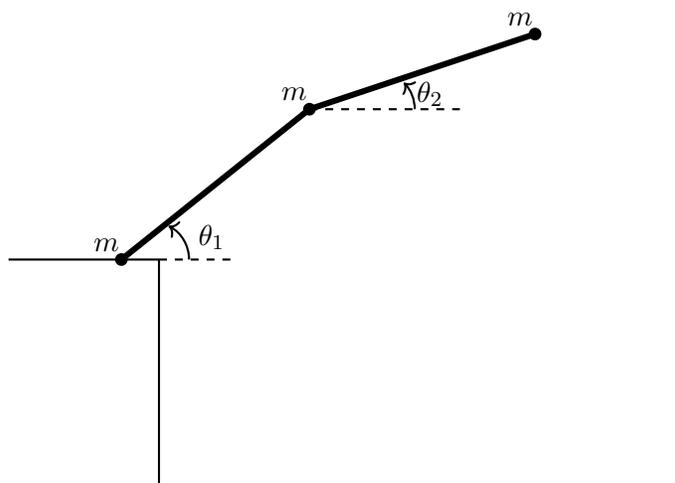


- A** Position A              (B) Position B              (C) Position C              (D) Position D              (E) Position E

The kinetic energy of the ball at the bottom of the slide is proportional to the height drop down the slide, so the horizontal velocity is proportional to the square root of the height drop. The time to fall is constant, so we need the velocity to scale up by 2.5, which means the height drop has to scale up by 6.25. This is choice A.

7. A mechanism consists of three point masses, each of mass  $m$ , connected by two massless rods of length  $l$  and a torsion spring acting as a hinge. The potential energy of the torsion spring is given by  $U_s$ . This system is designed to “walk” down a set of stairs, as shown in the figure. The angles  $\theta_1$  and  $\theta_2$  (see figure) represent the orientation of the rods, and their rates of change,  $\omega_1$  and  $\omega_2$ , are the corresponding angular velocities. Assume that the mass on the surface is instantaneously at rest.

Which equation correctly describes the total energy of the system?



- (A)  $E = ml^2\omega_1^2 + \frac{1}{2}ml^2\omega_2^2 + ml^2\omega_1\omega_2 \cos(\theta_1 + \theta_2) + 2mgl \sin \theta_1 + mgl \sin \theta_2 + U_s$   
 (B)  $E = \frac{1}{2}ml^2\omega_1^2 + \frac{1}{2}ml^2\omega_2^2 + 2mgl \sin \theta_1 + mgl \sin \theta_2 - U_s$   
 (C)  $E = ml^2\omega_1^2 + \frac{1}{2}ml^2\omega_2^2 + ml^2\omega_1\omega_2 \cos(\theta_1 - \theta_2) + 2mgl \sin \theta_1 + mgl \sin \theta_2 + U_s$   
 (D)  $E = ml^2\omega_1^2 + \frac{1}{2}ml^2\omega_2^2 - ml^2\omega_1\omega_2 \cos(\theta_1 - \theta_2) + 2mgl \sin \theta_1 + mgl \sin \theta_2 + U_s$   
 (E)  $E = \frac{1}{2}ml^2\omega_1^2 + \frac{1}{2}ml^2\omega_2^2 + 2mgl \sin \theta_1 + mgl \sin \theta_2 + U_s$

Using expressions for velocities and positions we set up the energy as a sum of kinetic energy, gravitational potential energy, and torsional potential energy terms.  $v_{1x} = -l\omega_1 \sin \theta_1$ ,  $v_{1y} = l\omega_1 \cos \theta_1$ ,  $v_{2x} = -l\omega_1 \sin \theta_1 - l\omega_2 \sin \theta_2$ ,  $v_{2y} = l\omega_1 \cos \theta_1 + l\omega_2 \cos \theta_2$ .  $E = \frac{1}{2}m(v_{1x}^2 + v_{1y}^2 + v_{2x}^2 + v_{2y}^2) + mg[l \sin \theta_1 + (l \sin \theta_1 + l \sin \theta_2)] + U_s$ .

8. A symmetric spinning top, rotating clockwise at an angular frequency  $\omega$ , is placed upright in the center of a frictionless circular plate. The plate then begins to rotate counterclockwise at a constant angular velocity  $\omega$ . Assume the top's axis remains perfectly vertical and stable without any precession. From the perspective of an observer rotating with the plate, how does the top appear to rotate?
- (A) The top appears stationary without any rotation.  
 (B) The top appears to rotate in the clockwise direction at an angular frequency  $\omega$ .  
 (C) The top appears to rotate in the clockwise direction at an angular frequency  $2\omega$ .  
 (D) The top appears to rotate in the counterclockwise direction at an angular frequency  $\omega$ .  
 (E) The top appears to rotate in the counterclockwise direction at an angular frequency  $2\omega$ .

When the plate begins to rotate, it does not exert any torque on the spinning top as the plate is frictionless, thus there are no external forces acting on the top that could change its angular momentum. As a result, the top's angular velocity in the lab frame remains unchanged. From the perspective of an observer who is rotating along with the plate at angular velocity  $\omega$ , the apparent angular velocity of the top is given by the transformation:

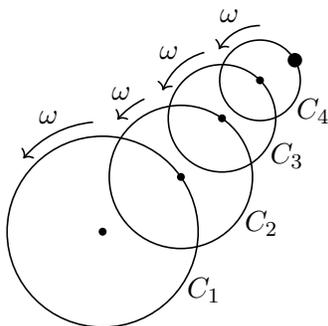
$$\omega_{\text{top,rotating}} = \omega_{\text{top,inertial}} - \omega_{\text{plate}}$$

Plugging in the frequencies:

$$\omega_{\text{top,rotating}} = (-\omega) - (+\omega) = -2\omega$$

Since the negative sign indicates clockwise rotation, the observer on the rotating plate perceives the top to be spinning in the clockwise direction at twice its original frequency.

9.  $N$  circles in a plane,  $C_i$ , each rotate with frequency  $\omega$  relative to an inertial frame. The center of  $C_1$  is fixed in the inertial frame, and the center of  $C_i$  is fixed on  $C_{i-1}$  (for  $i = 2, \dots, N$ ), as shown in the figure. Each circle has radius  $r_i = \lambda r_{i-1}$ , where  $0 < \lambda < 1$ . A mass is fixed on  $C_N$ . The position of the mass relative to the center of  $C_1$  is  $R(t)$ . For the  $N = 4$  case shown, which of the following statements is true?



During the time interval from  $0$  to  $2\pi/\omega$ , the magnitude of acceleration of mass on  $C_4$

- (A) reached its maximum and minimum more than once.  
 (B) reached its maximum and minimum exactly once.  
 (C) reached its maximum only once but the minimum more than once.  
 (D) reached its minimum only once but the maximum more than once.  
 (E) was constant.

The orbit angular velocity and the rotation velocity are the same, which means it will always have the same phase (similar to Moon and Earth, where the Moon always faces the same way relative to Earth). Therefore, the entire structure actually rotates as a rigid body, and the mass is simply moving in a circle at constant speed.

**The following information applies to problems 10 and 11.**

When two objects of very different masses collide, it is difficult to transfer a substantial fraction of the energy of one to the other. Consider two objects, of mass  $m$  and  $M \gg m$ .

10. If the lighter object is initially at rest, and the heavier object collides elastically with it, what is the approximate maximum fraction of the heavier object's kinetic energy that could be transferred to the lighter object?
- (A)  $m/M$       (B)  $2m/M$        (C)  $4m/M$       (D)  $m^2/M^2$       (E)  $2m^2/M^2$

Let the heavier object have initial speed  $v$ . In such an elastic collision, the final speed of the light object is approximately  $2v$ , so its kinetic energy is

$$E = \frac{1}{2}m(2v)^2 = \frac{1}{2}Mv^2 \frac{4m}{M}.$$

Thus, the maximum fraction is approximately  $4m/M$ .

11. Now suppose that instead, the heavier object is initially at rest, and the lighter object collides elastically with it. What is the approximate maximum fraction of the lighter object's kinetic energy that could be transferred to the heavier object?

- (A)  $m/M$       (B)  $2m/M$       **(C)  $4m/M$**       (D)  $m^2/M^2$       (E)  $2m^2/M^2$

Let the lighter object have initial speed  $v$ . Since  $M \gg m$ , its final speed is also approximately  $v$ , but the direction of the velocity is reversed, so the impulse given to the heavier object is about  $2mv$ . Then the kinetic energy of the heavier object is

$$E = \frac{p^2}{2M} \approx \frac{2m^2v^2}{M} = \frac{1}{2}mv^2 \frac{4m}{M}.$$

Thus, the maximum fraction is approximately  $4m/M$ .

12. A 50 g piece of clay is thrown horizontally with a velocity of 20 m/s striking the bob of a stationary pendulum with length  $l = 1$  m and a bob mass of 200 g. Upon impact, the clay sticks to the pendulum weight and the pendulum starts to swing. What is the maximum change in angle of the pendulum?

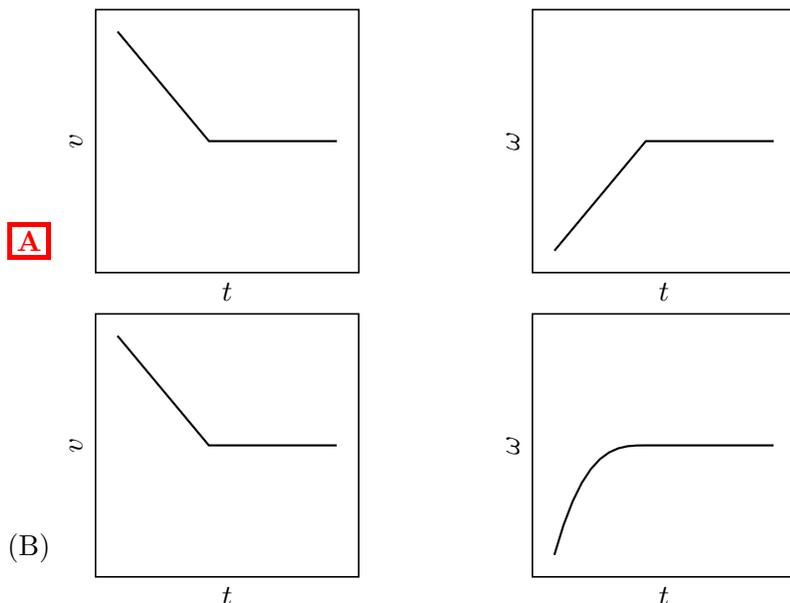
- (A)  $\arccos(1/5)$**       (B)  $\arcsin(7/10)$       (C)  $\arccos(2/3)$       (D)  $\arcsin(3/10)$       (E)  $\arctan(4/5)$

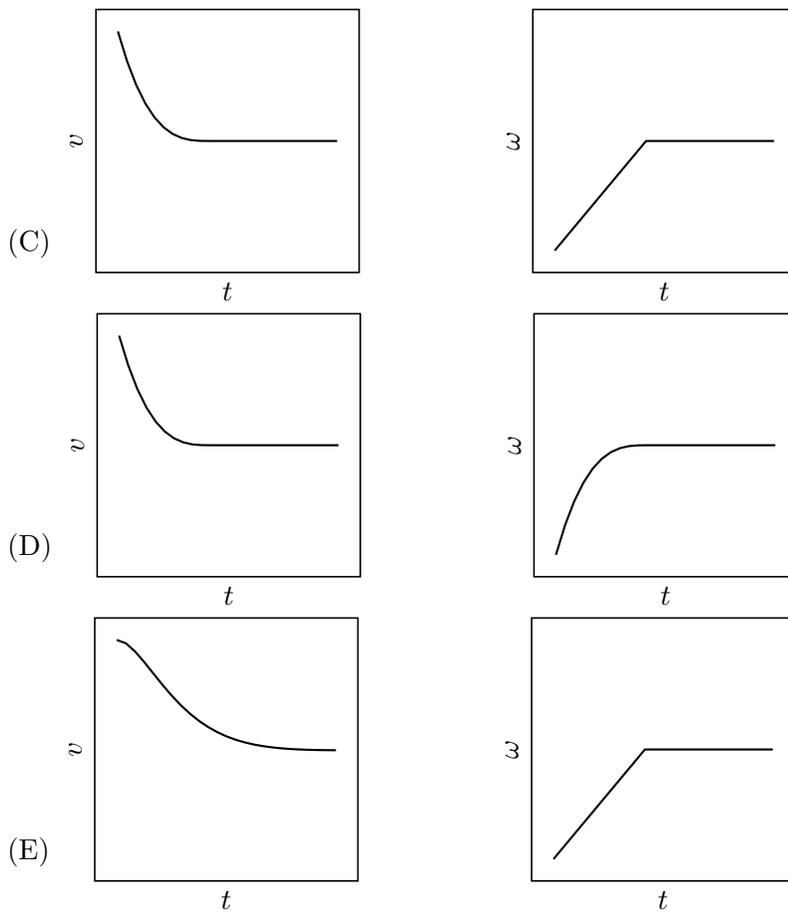
Use the equation for inelastic collision  $m_0v_0 = (m_0+m_b)v_b$  to get the velocity of the bob is 4m/s. Then use energy conservation to equate  $mgh = 1/2mv^2$  to get the height of .8. From there it is just trigonometry to get the angle.

**The following information applies to problems 13 and 14.**

Angela the puppy loves chasing tennis balls, so her owners built a tennis ball launcher. It fires balls along the floor at some initial speed, applying no rotation to them. The balls initially slip along the floor, then start rolling without slipping. Ignore the potential deformation of the ball and floor during this process, as well as air resistance.

13. Which of the following plot pairs could show the linear speed  $v$  and rotational speed  $\omega$  of one of the balls over time? Assume the floor has a constant roughness.





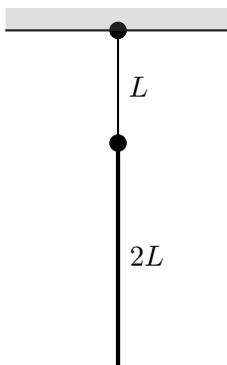
As the ball is rolling while slipping, the friction force acts to reduce its linear momentum and to increase its angular momentum. The change in the angular momentum is proportional to the change in the linear momentum. The ratio depends only on the moment of inertia of the ball.

The friction force will be constant until it drops to zero: this means that the plot of the linear speed should decrease linearly until it hits a constant level, and the plot of the rotational speed should increase linearly until it hits a constant level (at the same time).

14. There are three kinds of balls that can be launched in this set-up, all having the same radius  $R$ :
- I. a regular tennis ball (a thin spherical shell of rubber) of mass  $m_1$
  - II. a solid wooden ball of mass  $m_2$
  - III. a solid rubber ball of mass  $m_3$
- where  $m_1 < m_2 < m_3$ . All three types of ball emerge from the launcher with the same velocity. For which ball will the final velocity be highest?
- (A) Ball I
  - (B) Ball II
  - (C) Ball III
  - D** Balls II and III
  - (E) The final velocity will be the same for all three balls

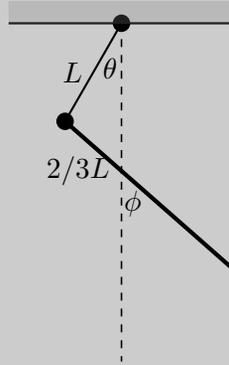
The frictional force provides a linear deceleration of  $dv/dt = -\mu g$ , where  $\mu$  is the coefficient of friction. It provides an angular acceleration of  $d\omega/dt = \mu g R(m/I)$ , where  $m$  is the mass of the ball and  $I$  is its moment of inertia. We're waiting until  $\omega R = v$ , which means that we care about  $d(\omega R)/dt = \mu g(mR^2/I)$ . The quantity  $(mR^2/I)$  is larger for objects where the mass is close to the axis than for objects where it is far away, so it is larger for a solid ball than for a spherical shell. This means that the solid balls will pick up angular velocity faster and will end up with a larger speed at the time when the ball stops slipping.

15. A uniform rigid rod of mass  $M$  and length  $2L$  is attached to a massless rod of length  $L$ , which is fixed at one end to the ceiling and free to rotate in a vertical plane. The massive rod is connected to the free end of the massless rod. Suppose an impulse  $J$  is applied horizontally to the bottom of the massive rod. Determine the relationship between the magnitudes of the angular velocity  $\omega$  of the massless rod and  $\Omega$  of the massive rod immediately after the impulse is applied. The moment of inertia of a uniform rod of length  $d$  and mass  $m$  about its center of mass is given by  $I = \frac{1}{12}md^2$ .



- (A)  $\Omega = \frac{3}{4}\omega$       (B)  $\Omega = \frac{2}{3}\omega$       (C)  $\Omega = \frac{4}{3}\omega$       (D)  $\Omega = \frac{1}{12}\omega$       **E**  $\Omega = \frac{3}{2}\omega$

We need to find the instantaneous center of rotation just after the impulse. Let  $v$  be the velocity of the center of mass. Let  $x$  be the distance from the center of mass of the massive rod to the center of rotation, hence  $v = \Omega x$ . Conservation of linear and angular momenta gives us  $J = M\Omega x$  and  $J(L+x) = I\Omega$ , where  $I = \frac{1}{3}ML^2 + Mx^2$ . From these we can solve that  $x = \frac{L}{3}$ . To get the relationship between  $\theta$  and  $\phi$ , we use the following figure, which gives  $\sin \phi = -\frac{L}{2/3L} \sin \theta$ . Just after the impulse the angles are infinitesimal and so equal to their sines. Hence  $\Omega = \frac{3}{2}\omega$ .



16. Two soap bubbles of radii  $R_1 = 1$  cm and  $R_2 = 2$  cm conjoin together in the air, such that a narrow bridge forms between them. Assuming the system starts in equilibrium, the bubbles are extremely thin, and that air can flow freely between the bubbles through the bridge, describe the evolution and final state of the bubbles.

- A** The smaller bubble will shrink and the larger bubble will grow.  
 (B) The larger bubble will shrink and the smaller bubble will grow.  
 (C) The bubbles will maintain their sizes.  
 (D) Air will oscillate between the two bubbles.  
 (E) Both bubbles will simultaneously shrink.

Since  $\Delta P_2 > \Delta P_1$ , the pressure inside the smaller bubble is greater than the pressure inside the larger bubble. As a result, air will flow from the smaller bubble to the larger bubble.

The smaller bubble will shrink and eventually collapse, while the larger bubble will grow.

17. A particle of mass  $m$  moves in the  $xy$  plane with potential energy

$$U(x, y) = -k \frac{x^2 + y^2}{2}.$$

The closest point to the origin ( $x = 0$ ,  $y = 0$ ) during its motion was at a distance  $d$ , and the particle's speed at that point was  $v \neq 0$ . Which of the following statements is true regarding the path of the particle after a long time  $t$  ( $t \gg d/v$ )?

- (A) The particle's trajectory will be circular.  
**B** The particle's trajectory will be asymptotic to a straight line pointing away from the origin.  
 (C) The particle will spiral outwards away from the origin.  
 (D) The particle will travel on a parabolic trajectory.  
 (E) The particle will spiral inwards towards the origin.

Unlikely the spring potential, this one has "-" sign. Corresponding force points radially outwards, with magnitude proportional to distance from the origin. Under such a force, the particles will be driven out to infinity.

Note that the force provides no torque about the origin. Conservation of angular momentum means that the particles' tangential velocity component will be the same as it was at time 0, while the radial component will grow as the particle moves farther away. This means that the direction of the particle's velocity will approach "radially outwards". Of the answer choices, that's only consistent with moving in a straight line away from the origin.

For an exact solution, let a particle's position be  $(q_1, q_2)$  (i.e.  $q_1 = x, q_2 = y$ ). The equations of motion can be written as  $\ddot{q}_j = \omega^2 q_j$ ,  $j = 1, 2$ , where  $\omega^2 = \frac{k}{m}$ . This gives  $q_j(t) = a_j e^{\omega t} + b_j e^{-\omega t}$ . In the long term, the positive exponential term will dominate, and the particle's trajectory will be asymptotic to the line through the origin with slope  $a_2/a_1$ . Note that the positive exponential term cannot be equal to 0, since the particle had a point of closest approach to the origin.

18. A particle of mass  $m$  moves in the  $xy$  plane with potential energy

$$U(x, y) = kxy/2.$$

If the particle begins at the origin, then it is possible to displace it slightly in some direction, so that the particle subsequently oscillates periodically. What is the period of this motion?

- (A)  $2\pi\sqrt{m/4k}$       (B)  $2\pi\sqrt{m/2k}$       (C)  $2\pi\sqrt{m/k}$       **D**  $2\pi\sqrt{2m/k}$       (E)  $2\pi\sqrt{4m/k}$

The shape of the potential function is a saddle, and the direction that allows simple harmonic motion is displacement along the line  $y = x$ . (All other directions cause the particle to be deflected away, towards the line  $y = -x$ .) Suppose the particle is displaced by a distance  $\ell$  along this line. Then  $x = y = \ell/\sqrt{2}$ , so the potential energy is  $k\ell^2/4$ . In other words, the potential energy is just the same as a spring with spring constant  $k/2$ , so the angular frequency is  $\sqrt{k/2m}$ .

19. Near the ground, wind speed can be modeled as proportional to height above the ground. (This is a reasonable assumption for small heights.) A wind turbine converts a constant fraction of the available

kinetic energy into electricity. The conditions are such that when operating at 10 m above the ground, the turbine delivers 15 kW of power. How much power would the same windmill deliver if it were operating at 20 m above the ground?

- (A) 15 kW      (B) 21 kW      (C) 30 kW      (D) 60 kW      **E** 120 kW

By assumption, at 20 m, the wind speed will be doubled; we're asked to find how the power of the turbine scales with wind speed. The energy that's available to be extracted over a small time interval is the product of the mass of air that flows past the turbine, and the mass density of the kinetic energy of the air. The mass of air scales as  $v$ , and the kinetic energy per unit mass scales as  $v^2$ , so the total scales as  $v^3$ .

Alternatively, you can use dimensional analysis; the power ( $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$ ) has to be a function of air density ( $\text{kg} \cdot \text{m}^{-3}$ ), air speed ( $\text{m} \cdot \text{s}^{-1}$ ), and some relevant turbine dimension (m). Looking at the time component, power has to scale with the cube of air speed.

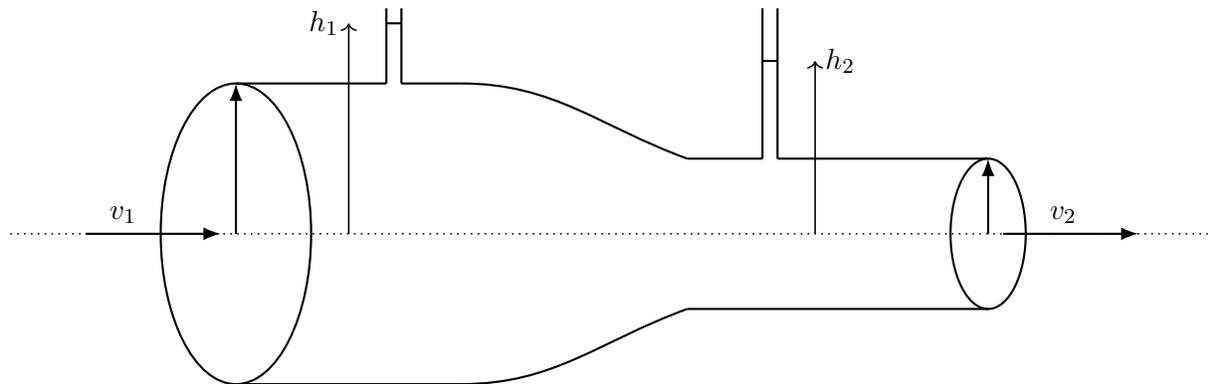
20. The International Space Station orbits the Earth in a circular orbit 400 km above the surface, and a full revolution takes 93 minutes. An astronaut on a space walk neglects safety precautions and tosses away a spanner at a speed of 1 m/s directly towards the Earth. You may assume that the Earth is a sphere of uniform density. At which of the following five times will the spanner be closest to the astronaut?
- (A) After 139.5 minutes.      (B) After 131.5 minutes.      **C** After 93 minutes.  
(D) After 46.5 minutes.      (E) After 1 minute.

The spanner will, of course, also orbit the Earth, in a slightly different orbit from that of the ISS. The total energy of the spanner is increased by this tossing maneuver, so its orbital period will be very slightly longer than that of the ISS. After exactly one orbital period of the ISS, the spanner will be very slightly behind the ISS on its trajectory.

At other points along the orbit, the spanner and the astronaut will be farther apart. Because the spanner was tossed inward, its initial angular velocity will be slightly higher than that of the astronaut, i.e. it will pull ahead in its orbit. At the point where the spanner's and the ISS's trajectories cross for the first time (approximately halfway around the orbit), the spanner's angular position will be maximally ahead of the ISS's. Over the second half of the orbit, the ISS's angular position is catching up to that of the spanner, and it very nearly catches up by the time they pass through the point where the astronaut first tossed it.

**The following information applies to problems 21 and 22**

Water flows through a pipe with a radius of 5 cm at a velocity of 10 cm/s before entering a narrower section of pipe with a radius of 2.5 cm.



21. What is the difference in the speed of water between the two pipes?

- (A) 20 cm/s      **B** 30 cm/s      (C) 40 cm/s      (D) 50 cm/s      (E) 60 cm/s

Use  $A_1V_1 = A_2V_2$  to find that  $V_2$  is 40 cm/s and remember that difference is  $V_2 - V_1$  to get 30 cm/s.

22. To measure this difference, two graduated cylinders are connected to the top of the pipe (one in the broad section and the other in the narrowed section). Water then flows up each pipe and the height the water reaches is measured. Estimate the difference in height between the two cylinders.

- (A) 6.3 mm      **B** 7.5 mm      (C) 8.2 mm      (D) 12.2 mm      (E) 12.7 mm

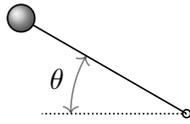
Use that the difference in pressure is equal to the  $\rho g \Delta h$  and Bernoulli's equation ( $\Delta p = 1/2\rho(v_2^2 - v_1^2)$ ) to get  $\Delta h = 7.5$  mm.

23. A student conducted an experiment to determine the spring constant of a spring using a ruler and two different weighing scales. The measured elongation of the spring was 1.5 cm, and the smallest division on the ruler was 1 mm. The mass of the attached weight was measured using two different scales in the school laboratory, yielding values of 198 g and 210 g. The student also found that the local acceleration due to gravity in her city is given as  $(9.806 \pm 0.001)$  m/s<sup>2</sup>. Calculate the percent error in measuring the spring constant.

- (A) 2 %      **B** 4 %      (C) 8 %      (D) 11 %      (E) 14 %

$$\sigma_l = \frac{0.5}{15} = 0.033, \sigma_m = \frac{6}{204} = 0.029, \sigma_g = \frac{0.001}{9.8} \approx 10^{-4}, \sigma_k = \sqrt{0.033^2 + 0.029^2 + 10^{-8}} \approx 0.044$$

24. A massive bead is attached to the end of a massless rigid rod of length  $L$ . The other end of the rod is attached to an ideal pivot, which allows it to rotate frictionlessly in any direction. The rod is initially at angle  $\theta$  to the horizontal, and there is no gravitational force.

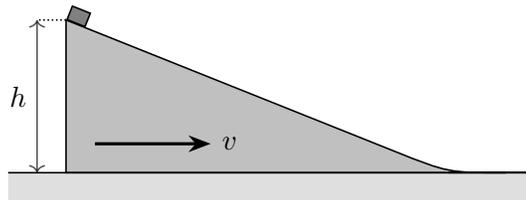


Next, the bead receives an impulse directly into the page, giving it a speed  $v$ . How long does it take for the bead to return to its original position?

- A**  $2\pi L/v$       (B)  $(2\pi L/v) \sin \theta$       (C)  $(2\pi L/v) \cos \theta$       (D)  $(2\pi L/v) \cos^2 \theta$       (E)  $(2\pi L/v) \cos^2(2\theta)$

No matter what  $\theta$  is, the bead always moves in a circle of radius  $L$ . The total distance it has to travel is  $2\pi L$ , so the time is  $2\pi L/v$ .

25. A puck of mass  $m$  can slide on a frictionless inclined plane (prism). The prism has a much greater mass compared to the puck and is itself sliding without friction on a horizontal surface. The velocity of the prism is  $v = \sqrt{2gh}$ , where  $g$  is the acceleration due to gravity and  $h$  is the height from which the puck starts sliding on the prism. The transition from the prism to the horizontal surface is smooth. The puck starts from rest relative to the prism. Find the final velocity of the puck once it begins sliding on the horizontal surface.



- (A)  $\frac{v}{2}$       (B)  $\frac{v}{\sqrt{2}}$       (C)  $v$       (D)  $\sqrt{2}v$       **E**  $2v$

In the given frame, the conservation of mechanical energy should include the mass of the prism. A quick solution is in the prism frame, where the puck eventually travels with the velocity  $v$ .