Solutions of equation (5) are:

$$n_{1,2} = -\frac{B}{2 \cdot A} \pm \sqrt{\frac{B^2}{4 \cdot A^2} - \frac{C}{A}} \qquad (6).$$

Equation (5) has only one physical correct solution, if...

I) A = 0 (i.e., the coefficient of n^2 in equation (5) vanishes)

In this case the following relationships exists:

$$r_1 - r_2 = d \qquad (7),$$

$$n = \frac{f \cdot d}{f \cdot d + r_1 \cdot r_2} > 1 \qquad (8).$$

II) B = 0 (i.e. the coefficient of n in equation (5) vanishes)

In this case the equation has a positive and a negative solution. Only the positve solution makes sense from the physical point of view. It is:

$$f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2 = 0 \qquad (9),$$

$$n^2 = -\frac{C}{A} = -\frac{d}{(r_2 - r_1 + d)} > 1 \qquad (10),$$

III) $B^2 = 4\ AC$

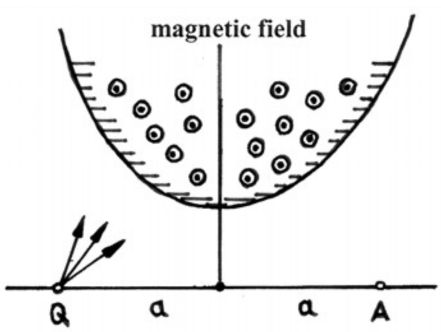
In this case two identical real solutions exist. It is:

$$\left[f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2\right]^2 = 4 \cdot (r_2 - r_1 + d) \cdot f^2 \cdot d \qquad (11),$$

$$n = -\frac{B}{2 \cdot A} = \frac{f \cdot (r_2 - r_1) + 2 \cdot f \cdot d + r_1 \cdot r_2}{2 \cdot f (r_2 - r_1 + d)} > 1 \qquad (12).$$

Theoretical problem 3: “Ions in a magnetic field”

A beam of positive ions (charge +e) of the same and constant mass *m* spread from point Q in different directions in the plane of paper (see figure[2]). The ions were accelerated by a voltage *U*. They are deflected in a uniform magnetic field *B* that is perpendicular to the plane of paper. The boundaries of the magnetic field are made in a way that the initially diverging ions are focussed in point A ($\overline{QA} = 2 \cdot a$). The trajectories of the ions are symmetric to the middle perpendicular on $\overline{QA}$.



[2] Remark: This illustrative figure was <u>not</u> part of the original problem formulation.

Among different possible boundaries of magnetic fields a specific type shall be considered in which a contiguous magnetic field acts around the middle perpendicular and in which the points Q and A are in the field free area.

a) Describe the radius curvature R of the particle path in the magnetic field as a function of the voltage U and the induction B.

b) Describe the characteristic properties of the particle paths in the setup mentioned above.

c) Obtain the boundaries of the magnetic field boundaries by geometrical constructions for the cases $R < a$, $R = a$ and $R > 0$.

d) Describe the general equation for the boundaries of the magnetic field.

Solution of problem 3:

a) The kinetic energy of the ion after acceleration by a voltage U is:

$$\tfrac{1}{2} mv^2 = eU \qquad (1).$$

From equation (1) the velocity of the ions is calculated:

$$v = \sqrt{\frac{2 \cdot e \cdot U}{m}} \qquad (2).$$

On a moving ion (charge e and velocity v) in a homogenous magnetic field B acts a Lorentz force F. Under the given conditions the velocity is always perpendicular to the magnetic field. Therefore, the paths of the ions are circular with Radius R. Lorentz force and centrifugal force are of the same amount:

$$e \cdot v \cdot B = \frac{m \cdot v^2}{R} \qquad (3).$$

From equation (3) the radius of the ion path is calculated:

$$R = \frac{1}{B}\sqrt{\frac{2 \cdot m \cdot U}{e}} \qquad (4).$$

b) All ions of mass m travel on circular paths of radius $R = v \cdot m / e \cdot B$ inside the magnetic field. Leaving the magnetic field they fly in a straight line along the last tangent. The centres of curvature of the ion paths lie on the middle perpendicular on $\overline{\mathrm{QA}}$ since the magnetic field is assumed to be symmetric to the middle perpendicular on $\overline{\mathrm{QA}}$. The paths of the focussed ions are above $\overline{\mathrm{QA}}$ due to the direction of the magnetic field.