Mark Scheme September/October 2006

1. energy change = mgh (1)

power generated = m/t x g x h

$826 \times 10^6 = m/t \times 9.81 \times 25$

$m/t = 3.37 \times 10^6$ kg^3/s (1)

50% efficiency → 6.74×10^6 kg^3/s (1)

volume flow = mass flow/density

= 6740 m^3/s (1)

per pipe =6740 ÷ 7 = 963 m^3/s (1)

speed of flow = volume rate of flow/cross sectional area of pipe (1)

= $963/(\pi \times 3.7^2)$

= 22.4 m/s (1)

2. There will be no change in the load on the supports. (1)

Using Archimedes Principle, the barge floats in the water by having an upthrust on it equal to the weight of water displaced. So as the barge floats by, it is indistinguishable weight-wise from the water that it has displaced. (1)

3. Energy of a photon $= 6.6 \times 10^{-34} \times 3 \times 10^{10}$

$= 19.8 \times 10^{-24}$ J (1)

$$number\ of\ photons, N = \frac{11}{19.8 \times 10^{-24}} = 5.5 \times 10^{23} s^{-1}$$

$$area\ at\ earth\ radius\ which\ receives\ these\ photons = 10^{-4} \times 4\pi \times (10^{13})^2$$

$$= 4\pi \times 10^{22} m^2$$

$$dish\ area = \pi \times 40^2 m^2$$

$$No\ of\ photons\ received\ by\ the\ dish\ per\ second = \frac{\pi 40^2}{4\pi 10^{22}} \times 5.5 \times 10^{23} = 22{,}000$$

(3)

4. Extra length is given by $2\pi(r + \Delta r) - 2\pi r = 2\pi\Delta r$

For Δr=1m, extra length =6.3 m, independent of r. (3)

The earth's circumference would increase by 0.63m (1)

Surface area of earth is given by $A = 4\pi r^2$

(i) algebraically

$$A + \Delta A = 4\pi (r + \Delta r)^2$$
$$A + \Delta A = 4\pi (r^2 + 2r\Delta r + (\Delta r)^2)$$
$$\Delta A \approx 8\pi r \Delta r$$
$$\frac{\Delta A}{A} = 2\frac{\Delta r}{r}$$

or (ii) differentiate

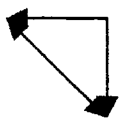
$$\frac{dA}{dr} = 4\pi r$$
$$\frac{dA}{A} = 2\frac{dr}{r}$$

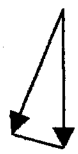
result $\frac{dA}{A} = 4x10^{-5}\%$ (3)

5. (a) tangential arrows of equal length 

magnitude of change of velocity is 2v

acceleration $= = \frac{2v}{T/2} = \frac{2v}{\pi r / v} = \frac{2}{\pi}\frac{v^2}{r}$ (3)

(b)  $\Delta v = \sqrt{2}v$ (2)

(c) 

$$(\Delta v)^2 = 2v^2 - 2v^2 \cos\theta$$
$$(\Delta v)^2 = 2v^2(1 - \cos\theta)$$

(3)

(d) $\frac{\Delta t}{T} = \frac{\theta}{2\pi}$ and $T = \frac{2\pi r}{v}$ (2)

(e)

$$a = \frac{\Delta v}{\Delta t} = \sqrt{2}v\frac{(1-\cos\theta)^{\frac{1}{2}}}{r\theta/v}$$
$$= \sqrt{2}\frac{v^2}{r}\frac{(1-\cos\theta)^{\frac{1}{2}}}{\theta}$$
$$= \sqrt{2}\frac{v^2}{r}\frac{(1-1+\theta^2/2)^{\frac{1}{2}}}{\theta}$$
$$= \sqrt{2}\frac{v^2}{r}\frac{\theta}{\sqrt{2}}\frac{1}{\theta} = \frac{v^2}{r}$$

(4)

6. a) R α $V^{1/3}$ so that the ratio $\frac{R_{final}}{R_{initial}} = \sqrt[3]{\frac{1}{10^{15}}} = \frac{1}{10^5}$ (3)

b) $\Omega_{initial} R^2_{initial} = \Omega_{final} R^2_{final}$

and $\Omega_{final} = 3.6 \times 10^4$ radians/second

and T=0.17 ms (3)

c) R^2 decreases by a factor 10^{10}

and so B increases by 10^{10} to a final field strength of 10^8 T (2)

d) Assume that the star remains spherical so that R describes its radius.

Equate gravitational field strength with the centripetal acceleration at the equator.

$$\frac{v^2_{\max}}{R} = \frac{GM}{R^2} \quad with \ v_{\max} = \frac{2\pi r}{T_{\min}} \Rightarrow T_{\min} = \frac{2\pi}{\sqrt{G}} \frac{R^{\frac{3}{2}}}{M^{\frac{1}{2}}}$$

which evaluates to give T_{min}=0.46 seconds (much longer period than the answer to part (b)) (4)

e) The initial BE is that for the two stars summed together. The factor relating the initial and final stars is the mass; the masses add whereas the radii do not.

$$BE_{initial} = k_1 \frac{GM^2_{initial}}{R_{initial}} \times 2 \quad with \ R_{initial} M^{\frac{1}{3}}_{initial} = k_2$$

giving

$$BE_{initial} = k_1 \frac{GM^{2+\frac{1}{3}}_{initial}}{k_2} \times 2$$

after collision

$$BE_{final} = k_1 \frac{GM^{2+\frac{1}{3}}_{final}}{k_2}$$

with $M_{final} = 2 \times M_{initial}$

hence

$$\frac{BE_{final}}{BE_{initial}} = \frac{M^{\frac{7}{3}}_{final}}{2M^{\frac{7}{3}}_{initial}}$$

$$= \frac{1}{2} \times 2^{\frac{7}{3}} = 2.52$$ (4)