BPhO
British Physics Olympiad

British Physics Olympiad 2017-18

A2 Challenge

September/October 2017

Instructions

Time: 1 hour.

Questions: Answer ALL questions

Marks: Total of **50 marks**.

Instructions: You are allowed any standard exam board data/formula sheet.

Calculators: Any standard calculator may be used.

Solutions: These questions are about problem solving. Draw diagrams in order to understand the questions. You must write down the questions in terms of symbols and equations; then try calculating quantities in order to work quickly towards a solution. In these questions you will need to explain your reasoning by showing clear working. Even if you cannot complete the question, show how you have started your thinking, with ideas and, generally, by drawing a diagram.

Clarity: Solutions must be written legibly and set out properly with a "narrative" which links one step to the next (and, so, therefore, hence, but, using equ 5, etc.).

Important Constants

Constant	Symbol	Value
Speed of light in free space	c	$3.00 \times 10^{8}\,\mathrm{m\,s^{-1}}$
Elementary charge	e	$1.60 \times 10^{-19}\,\mathrm{C}$
Acceleration of free fall at Earth's surface	g	$9.81\,\mathrm{m\,s^{-2}}$
Planck's constant	h	$6.63 \times 10^{-34}\,\mathrm{J\,s}$
Gravitational constant	G	$6.67 \times 10^{-11}\,\mathrm{m^{3}\,kg^{-1}\,s^{-2}}$

Qu 1. This question explores some ideas about distributions of forces.

a) (i) State TWO conditions for the mechanical equilibrium of an extended body.

(ii) Sketch a vector diagram to demonstrate that the three equal forces P, Q, R, acting on an object, in **Figure 1a**, satisfy the equilibrium condition for the vector sum of forces. ($\alpha = \beta = \gamma = 120°$).

(iii) The same forces acting on a different object in **Figure 1b** clearly have the same vector-sum as in **Figure 1a**, but they are not in equilibrium. Use the diagram to explain *qualitatively* why these forces are not in equilibrium.

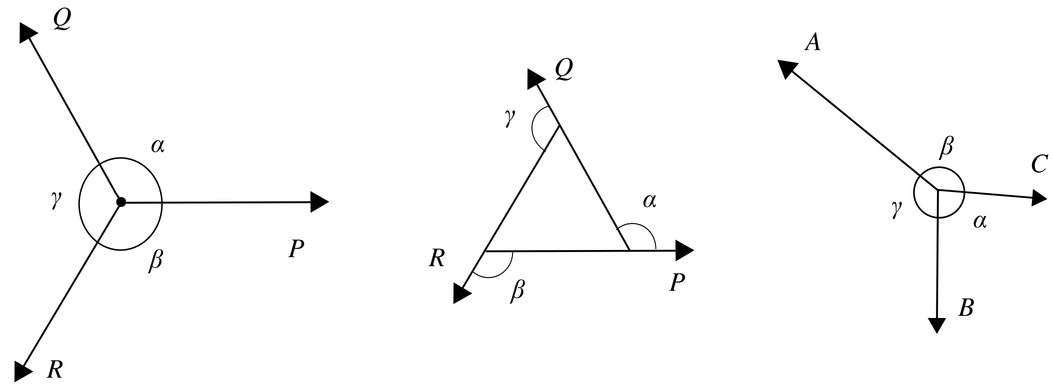


Figure 1: **(a)** **(b)** **(c)**

(iv) Extend the explanation you have given to justify the rule that *three co-planar forces in equilibrium must be concurrent* i.e. the lines all intersect at a single point.

(v) *Lami's Theorem* relates any three coplanar forces which are in equilibrium, as shown in **Figure 1c**. (Notice that now α, β, γ may take differing values, according to the magnitudes of A, B, C.) Use the diagram to prove the result of Lami's Theorem, which is usually expressed as:

$$\frac{A}{\sin\alpha} = \frac{B}{\sin\beta} = \frac{C}{\sin\gamma}$$

[7 marks]

b) (i) Calculate the force exerted by the atmosphere on one side of a pane of glass measuring $20\,\mathrm{cm} \times 20\,\mathrm{cm}$. (Atmospheric pressure may be taken as 100 kPa.)

(ii) Why is this force not normally noticed?

(iii) Why is it extremely difficult to separate two such panes of glass, which may be assumed to be perfectly flat, when they are stacked on top of each other?

(iv) A student suggests that it might be easier to separate the panes by putting them vertical and sliding them apart. Comment on the feasibility of this idea. (Coefficient of friction for clean glass-to-glass contact is ≈ 0.95.)

(v) Comment on why a glass merchant will pack multiple panes of glass with sheets of newspaper between them. **[5 marks]**

c) Tubes and cylinders with pressurised contents are subject to stresses, which are different along their length and around their circumference.

(i) **Figure 2** shows a length L of a thin walled tube of diameter D and wall thickness t, with $D \gg t$, sustaining a pressure p.

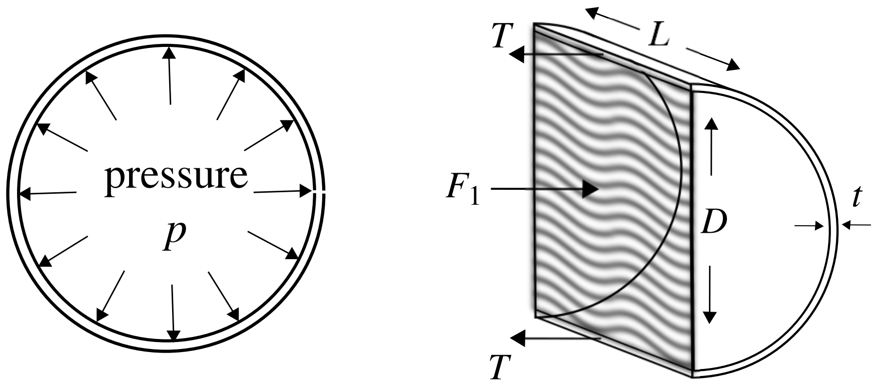


Figure 2

By considering the force F_1 acting on the shaded plane due to the internal pressure, determine the value of the force T on the sectioned wall of the pipe, and hence the lateral stress σ_{H} in the wall, often known as the *hoop stress*.

(ii) Apply a similar method to **Figure 3** to find the force F_2 (not the same as F_1 in **Figure 2**) on the end of the same pipe, and hence the longitudinal stress σ_{A} in the wall represented by the many small arrows, often known as the *axial stress*.

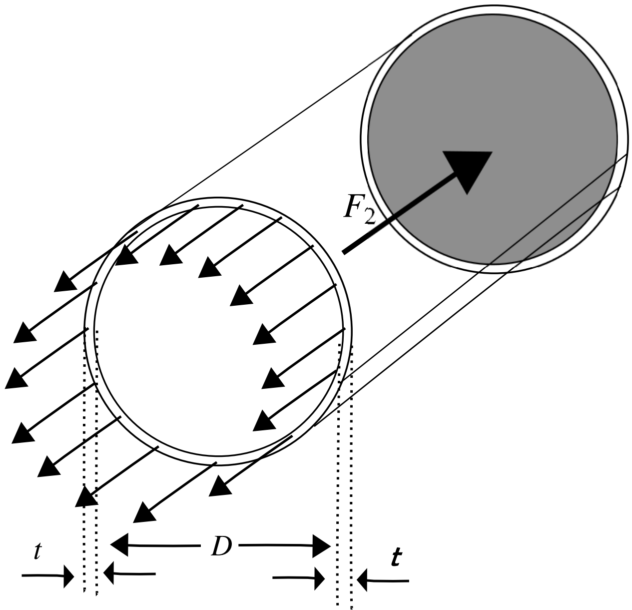


Figure 3

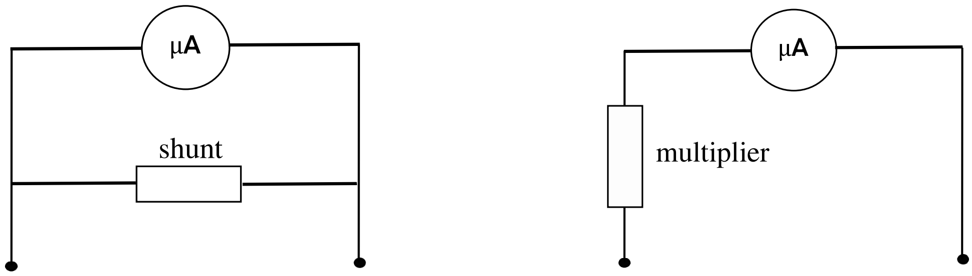
(iii) Find the ratio of the hoop stress to the axial stress in the tube, $\sigma_{\mathrm{H}} : \sigma_{\mathrm{A}}$.

(iv) If several thin steel bands were to be used to reinforce a closed cylinder containing a pressurised gas, in which direction would they be attached to the outside of the cylinder?

[6 marks]

Qu 2. This question explores the properties of some resistor networks.

a) Analogue voltmeters and ammeters consist of a galvanometer (a very sensitive ammeter that has a resistance of $1.00\,\mathrm{k\Omega}$ and is marked μA in **Figure 4** below) together with a resistor, as shown, which adapts it to the range of current or voltage required. Such a galvanometer requires a current of $10.0\,\mu\mathrm{A}$ to give a maximum reading on its scale (known as full scale deflection).



(a) Use of a shunt to scale an ammeter. **(b)** Use of a multiplier to scale a voltmeter.

Figure 4: Shunts and multipliers for ammeters and voltmeters.

(i) Calculate the pd across the galvanometer which would give full scale deflection.
(ii) The arrangement in **Figure 4a** converts the galvanometer into an ammeter. What value for the resistance of the shunt would give the ammeter a full scale deflection for a current of $10.0\,\mathrm{A}$?
(iii) The arrangement in **Figure 4b** converts the galvanometer into an voltmeter. What value for the resistance of the multiplier would give the voltmeter a full scale deflection for a potential difference of $10.00\,\mathrm{V}$?

[5 marks]

b) **Figure 5** shows a hollow regular octahedron, ABCDEF, constructed of twelve identical wires each of resistance $10\,\Omega$.

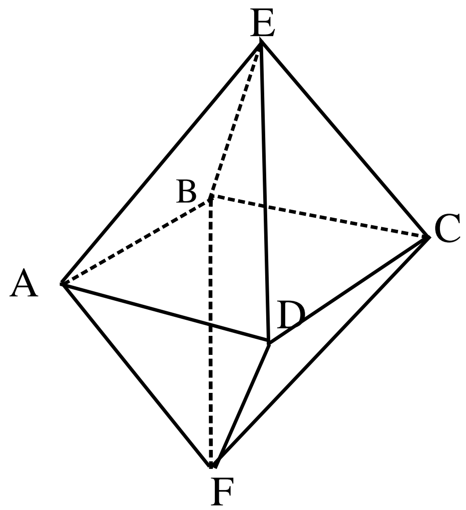


Figure 5

By drawing one or more equivalent diagrams, determine the resistance between the vertices E, F of the octahedron. **[4 marks]**

Qu 3. This question shows how we can understand the principles underlying familiar macroscopic properties using simple microscopic models.

a) **Figure 6** shows a simplified model of a crystal, with atoms represented by spheres and the bonds between them represented by springs, of spring constant k. Each bond is of length a, and a stress σ is applied to the front and rear faces of the model such that a force F is applied to each bond going from front to rear. The stress causes each bond in the front to rear direction to extend by an amount x. (It should be noted that this is a highly simplified model: the primitive cubic arrangement, although common for compounds, is very unusual for elements.)

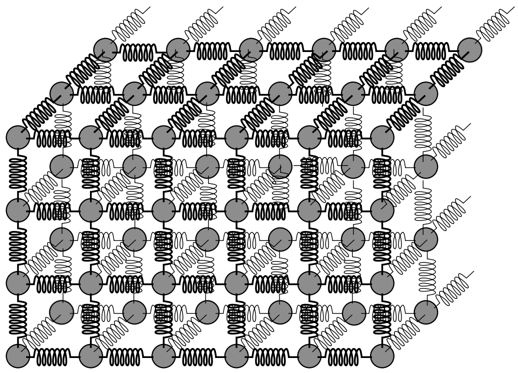


Figure 6

The coordination number of the atoms in a solid is equal to the number of bonds acting on each atom within the body of the solid (i.e. not those on the surface).

(i) What is the coordination number for the arrangement shown in **Figure 6**?
(ii) What area on the front face of the model is associated with each atom?
(iii) Derive an expression for the stress, σ, in terms of F and a.
(iv) Find the strain in the front-to-rear bonds.
(v) Hence derive an expression for the Young Modulus E of the material in terms of k and a.

[5 marks]

b) A more realistic model, applicable to many elements, is to consider what is known as a close-packed layer, as shown in **Figure 7**.

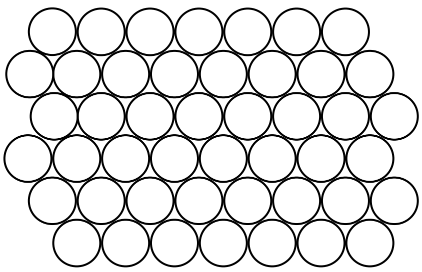


Figure 7

(i) What is the co-ordination number for atoms in the middle of this single layer?
(ii) Now consider placing more identical atoms in the dimples on the top of this layer to form a new close-packed layer. How many atoms in the new layer are in contact with any given atom in the layer beneath?
(iii) Hence, what is the co-ordination number of atoms in the middle of a structure consisting of many close-packed layers stacked on top of each other?
(iv) When this structure is heated it is often erroneously stated that it melts because the bonds are weakened. If all the bonds remained intact but were of lower spring constant, what change would actually be observed in the macroscopic behaviour of the material? Give a brief reason for your answer.

(v) In fact melting occurs when a small proportion or the bonds rupture and are therefore able to re-form with different partners, thus enabling relative motion of the atoms. Use the data below to estimate the proportion of bonds which typically break when simple solids melt.

Specific latent heats for some simple elements:

Element	$\mathbf{SLH(fusion)}/\mathrm{kJ\,kg^{-1}}$	$\mathbf{SLH(vaporisation)}/\mathrm{kJ\,kg^{-1}}$
argon	29.5	161
helium	3.45	20.7
hydrogen (H_2)	59.5	445
krypton	16.3	108
neon	16.8	84.8

[8 marks]

Qu 4. This question explores some consequences of the interference of waves.

a) **Figure 8** shows even, vertical monochromatic illumination of an air-wedge (of exaggerated inclination) formed between two microscope slides separated by a tiny piece of foil. An observer views the reflection with his eye directly above.

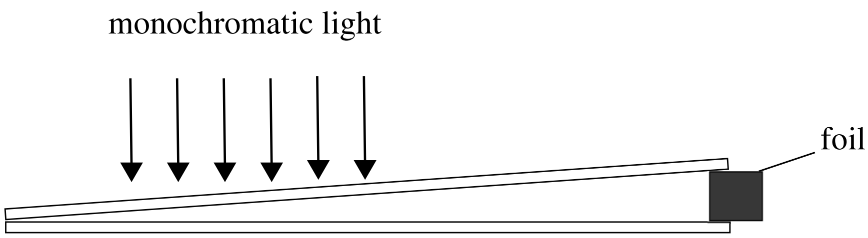


Figure 8: Two glass slides enclosing a thin air wedge.

When the light reflects from the surfaces of the air wedge, (the lower surface of the top glass slide and the top surface of the lower slide) as in **Figure 9**, it undergoes a phase inversion (π radians phase change) at the horizontal surface, while at the inclined surface there is no phase change.

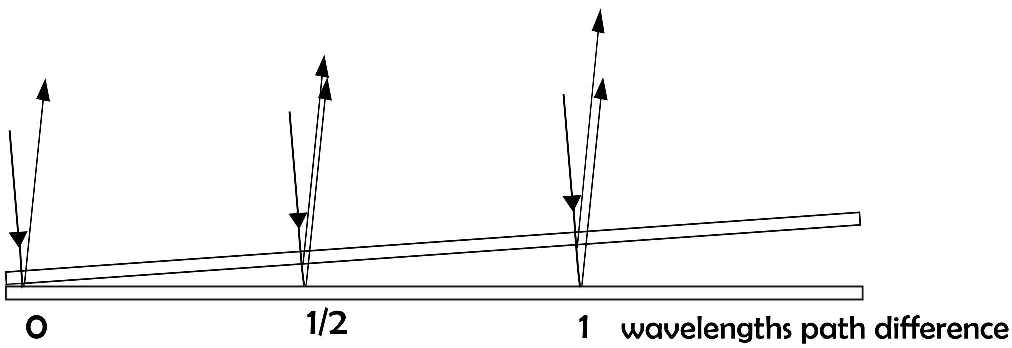


Figure 9: The very left hand end of the slides of Figure 8.

(i) For each of the three positions indicated, state whether an observer above the system will see bright or dark.

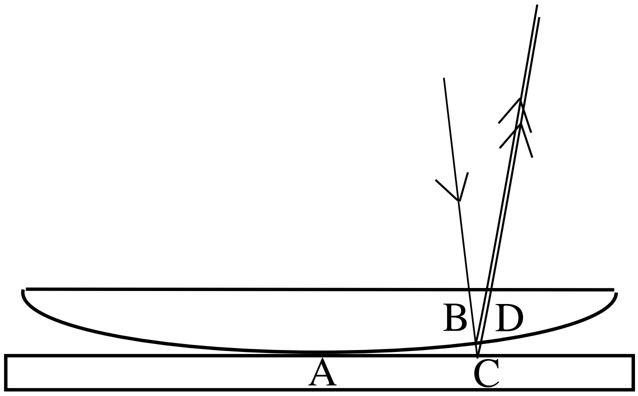
(ii) Hence sketch what the observer will see looking down on the system.

(iii) Describe what the observer will see when the air-wedge is illuminated with white light instead of monochromatic light.

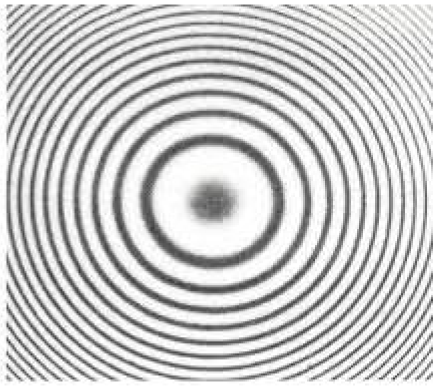
(iv) Why are there no pure spectral colours (i.e. ROYGBIV) visible in this arrangement?

[5 marks]

b) A different system, illustrated in **Figure 10a**, based on the same principle, produces an interference pattern called Newton's Rings, as illustrated in **Figure 10b**.



(a) Thin lens on an optically flat glass plate.



(b) Newton's Rings interference pattern.

Figure 10: Setup and illustration of Newon's Rings.

(i) Apply the ideas used in part (a) above to explain why the spacing between rings decreases with increasing distance from the centre of the ring pattern.

(ii) Counting the central disc as the zero order of interference, what is the path difference associated with the n^{th} order dark ring? (i.e. an approximation for the path BCD in **Figure 10a**)

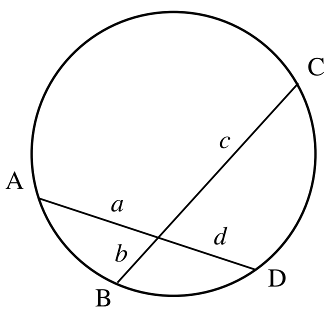
(iii) Hence show that the radius, r , of the n^{th} dark ring is given by

$$r \approx \sqrt{R\lambda n}$$

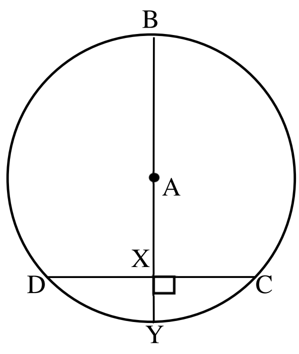
where $R(\gg r)$ is the radius of curvature of the lower side of the lens.

Hint: you may find it useful to use the property of intersecting chords that in **Figure 11a** $ad = bc$ and so, in **Figure 11b**, $\mathbf{DX^2} = \mathbf{BX}.\mathbf{XY} \approx 2R \times \mathbf{XY}$.

[5 marks]



(a)



(b)

Figure 11

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