United States Physics Team

Entia non multiplicanda sunt praeter necessitatem

1997 Creative Response Portion of Exam 1

4 Questions, 60 Minutes

INSTRUCTIONS

DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

Show all work, as partial credit may be earned.

Communicate! The grader will not attempt to read your mind.

A hand-held calculator may be used. Its memory must be cleared of data and programs.
Calculators may not be shared.

Possibly useful approximations:

$(1 + x)^n \approx 1 + nx$ for $|x| << 1$
$\cos\theta \approx 1 - \theta^2/2!$ for $\theta << 1$
$\sin\theta \approx \theta$ for $\theta << 1$

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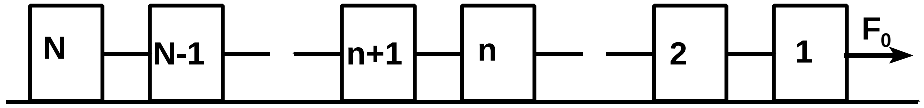
1. A collection of N identical blocks, each of mass m, are connected by unstretchable ropes of negligible mass. The blocks are on a horizontal surface. An external force F_o acts on Block 1, pulling horizontally to the right. What is the tension in the rope connecting Block n to Block $n+1$, if:

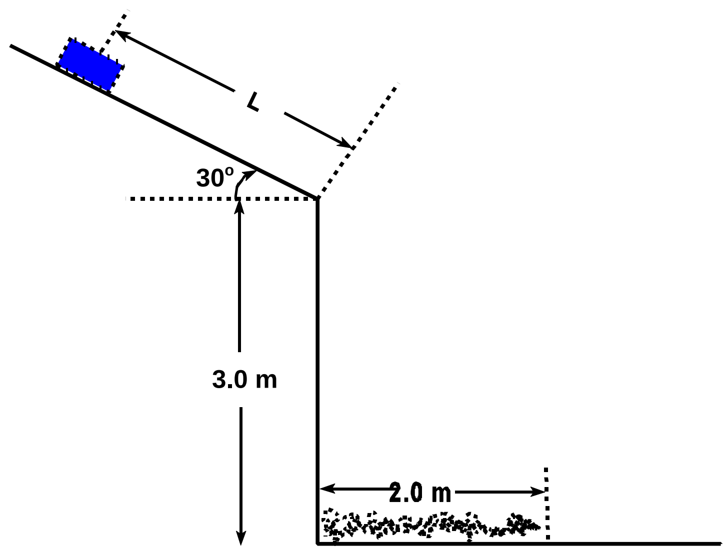
(a, 8) There is no friction between the blocks and the surface?

(b,12) The coefficient of kinetic friction between each block and the surface is μ_k? (F_o is large enough to accelerate the blocks despite the friction.)

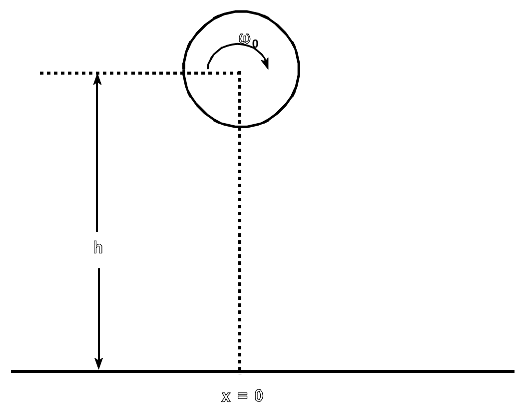


2. **(a, 10)** A box of nails begins sliding from rest on the roof a house. The roof makes an angle of 30° above the horizontal. The coefficient of sliding friction is μ_k = ¼. The box slides off the edge of the roof with a speed of 3.5 m/s. What is the distance L the box slides on the roof before falling off? Use g = 9.8 m/s^2 (which also = 9.8 N/kg).

(b, 10) The flower bed extends from the side of the house and is 2.0 m wide. The edge of the roof is 3.0 m above the ground. Does the box of nails land in the flowers? Justify your answer with a calculation that shows where the box lands. Neglect air resistance.



3. A basketball of mass M, radius R, and moment of inertia I about its center of mass (CM) is set spinning with angular velocity ω_o about a horizontal axis through its CM. The CM is originally at rest and located at the height h above the floor. The basketball is dropped while spinning, and subsequently collides with the floor. Neglect air resistance.



(a, 3) Let K_1 be the basketball's kinetic energy just before it collides with the floor. Write K_1 in terms of the given parameters and any needed constants.

(b, 17) Immediately after the first bounce, the basketball is no longer spinning, and its kinetic energy is βK_1, where $\beta < 1$ is a known factor. What are the horizontal and vertical components of the basketball's velocity immediately after the first bounce?

4. Consider the gravitational force F on a planet due to the Sun when that force includes a small perturbation Γ giving a departure from Newton's law of gravitation,

$$F = (1 + \Gamma)\,(GMm/R^2)$$

where M is the Sun's mass, m is the planet's mass, G is Newton's constant, R is the distance between the center of the Sun and the center of the planet, and we take Γ to be a constant $\ll 1$. (For example, general relativity provides such a perturbation.) Approximate the orbit as circular.

(a, 7) Show that the planet's period is $T \approx T_o\,(1 - \tfrac{1}{2}\Gamma)$, where T_o would be the planet's period if there were no perturbation.

(b, 7) In one revolution, a planet moving under the purely Newtonian force would travel through the angle 2π. But with the perturbation, in the same amount of time the planet travels through an additional angle δ. Calculate δ in terms of Γ.

(c, 2) The general theory of relativity contributes the perturbation $\Gamma = 6v^2/c^2$ to Newton's law of gravitation, where v is the speed of the planet relative to the Sun, and c is the speed of light. The precession angle δ of part (b) may be approximated as $\delta \approx (6\pi/c^2)(GM/R)$. Demonstrate this claim that $\delta \approx (6\pi/c^2)(GM/R)$, by using your result of part (b) and the given perturbation.

(d, 4) Using the result $\delta \approx (6\pi/c^2)(GM/R)$, and using the data below, calculate the numerical value of δ for the planet Mercury. Express your answer in seconds of arc per century. This is our simple model's prediction of general relativity's contribution to the precession of Mercury's orbit.

Data: Period of Mercury's orbit = 88 days
Radius of Mercury's orbit $= 5.8\times10^{10}$ m
$G = 6.7\times10^{-11}$ Nm^2/kg^2
Mass of Sun $= 2.0\times10^{30}$ kg
$c = 3.0\times10^8$ m/s
3600'' = 1 degree.

[This problem adapted from A.P. French, *Newtonian Mechanics*, Norton & Co., NY, 1971.]