United States Physics Team

Entia non multiplicanda sunt praeter necessitatem

1998 Exam 2

INSTRUCTIONS

1. DO NOT OPEN THIS TEST UNTIL YOU ARE GIVEN THE SIGNAL TO BEGIN.
2. Work Part A first. You have 90 minutes to complete all four problems.
3. After you have completed Part A, you may take a break.
4. Then work Part B. You have 90 minutes to complete both problems.
5. Show all work, as partial credit may be earned.
6. Begin each problem on a fresh sheet of paper. In the upper right-hand corner of *each page*, write your name (last name first), the problem number, and the page number / total number of pages for that particular problem, for example:

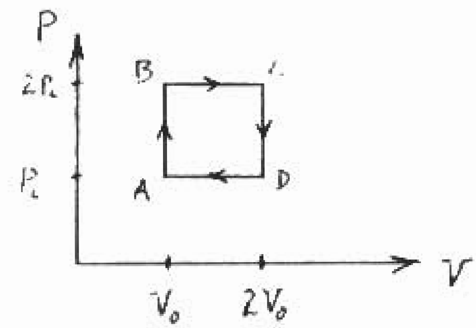
   Einstein, Albert
   A2 - 3/4 for "Problem A2, page 3 of 4"

7. A hand-held calculator may be used. Its memory must be cleared of data and programs. You may not use any books, notes, tables, or collections of formulas.
8. Best wishes for an outstanding exam!

Possibly useful information:

Gravitational field at Earth's surface: $g = 9.8$ N/kg
Newton's gravitational constant: $G = 6.67\times10^{-11}$ N-m^2/kg^2
Coulomb's constant: $k = 1/4\pi\epsilon_0 = 9\times10^9$ N-m^2/C^2
Biot-Savart constant: $k_m = \mu_0/4\pi = 10^{-7}$ N-s^2/C^2
Speed of light in vacuum: $c = 3\times10^8$ m/s
Boltzmann's constant: $k_B = 1.38\times10^{-23}$ J/K
Avogadro's number: $N_A = 6.02\times10^{23}$ mol^{-1}
Ideal gas constant: $R = N_A k_B = 8.31$ J / mol-K
Elementary charge: $e = 1.6\times10^{-19}$ C
1 electron volt: 1 eV $= 1.6\times10^{-19}$ J
Planck's constant: $h = 6.63\times10^{-34}$ J-s $= 4.14\times10^{-15}$ eV-s
Electron mass: $m = 9.1\times10^{-31}$ kg $= \frac{1}{2}$ MeV/c^2
Binomial expansion: $(1+z)^n \approx 1 + nz$ whenever $|z| << 1$
$\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\sin\theta \approx \theta$ whenever $|\theta| << 1$.

A1. (25 points) Two moles of monatomic ideal gas are taken through the reversible cycle shown on the PV diagram below. The cycle forms a rectangle, starting from State A at (V_0, P_0), to State B at (V_0, $2P_0$), to State C at ($2V_0$, $2P_0$), to State D at ($2V_0$, P_0), and back to State A. Express all answers to the following questions in terms of V_0, P_0, and the ideal gas constant R.



(a. 5 points) Calculate the net work done by the gas during the cycle.
(b. 5) Calculate the net heat transferred between the gas and the surroundings during one cycle.
(c. 10) Calculate cycle's thermodynamic efficiency when operating as an engine.
(d. 5) Calculate the change in entropy of the gas as the system goes from State A to State D.

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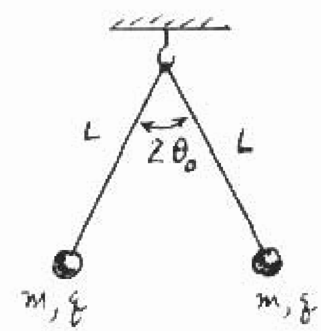
A2. (25 points) Two uniformly charged balls each have mass m and carry electric charge q. Each ball is tied to a string of length L and negligible mass. The other ends of the strings are tied to a hook in the ceiling. When the balls are in equilibrium the strings stand apart with the angle $2\theta_o$ between them.



(a. 7) Find an expression for θ_o in terms of the given quantities and known constants. Neglect the gravitational force which the balls exert on each other.

(b. 6) Now let both balls be displaced an additional angle $\delta << 1$. Find the magnitude of the new electric force F' which one ball exerts on the other one. In particular, show that $F' = F_o(1 - 2\,\delta\cot\theta_o)$ where F_o is the electric force's magnitude for the situation of part (a). Along with the binomial expansion, the following approximations may be useful:

$\sin(\theta_o + \delta) \approx \sin\theta_o + \delta\cos\theta_o$,
$\cos(\theta_o + \delta) \approx \cos\theta_o - \delta\sin\theta_o$.

(c. 10) When the balls are released from the angle $\theta_o + \delta$, each ball undergoes an oscillation about its equilibrium position (assume that the balls move only in the original plane of the two strings). Find an expression for the period T of this oscillation in terms of given quantities and known constants. Neglect air resistance.

(d. 2) Evaluate T/T_o for $\theta_o = 45$ degrees, where T_o is the period of a simple pendulum of length L.

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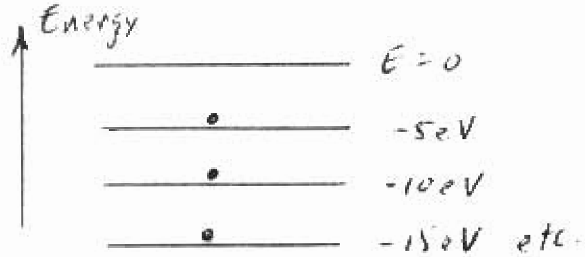
A3. (25) A certain metal has various energy levels for the electrons in its atoms. Assuming the energy of a free electron at rest is zero, the first five energy levels of the electrons in the metal's atoms are as follows:

State 1:	-5 eV
State 2:	10 eV
State 3:	-15 eV
State 4:	-30 eV
State 5:	-50 eV.



(a, 7) In this part of the problem we will see what the wave model of light predicts for the ability of light to free the electrons from the metal surface. Suppose the intensity of light irradiating the metal is 0.01 W/m^2 and the density of surface electrons is 5×10^{19} electrons per square meter. Calculate the minimum time required to liberate an electron from the metal surface, assuming that this energy falling on the metal can be absorbed by the electrons, so that they will be ejected as soon as they acquire sufficient energy.

(b, 5) Unfortunately for the wave model's result suggested by part (a), experiments show that, even when the intensity of the light is *very* low, some electrons are liberated in less than 10^{-9} s, *provided* the frequency of the light is above a specific minimum value. Calculate this minimum frequency for the metal described above.

(c, 8) Suppose the metal surface is irradiated with photons corresponding to the frequency $f = 4.5\times10^{15}$ Hz. Calculate the kinetic energies of the electrons that originally occupied States 1 through 5. (If you like, you may leave your answers in units of electron volts.)

(d, 5) The electrons liberated from the metal by the photons of part (c) form an electric current. What minimum stopping voltage would be required to halt the motion of every liberated electron, and thus bring the current to zero?

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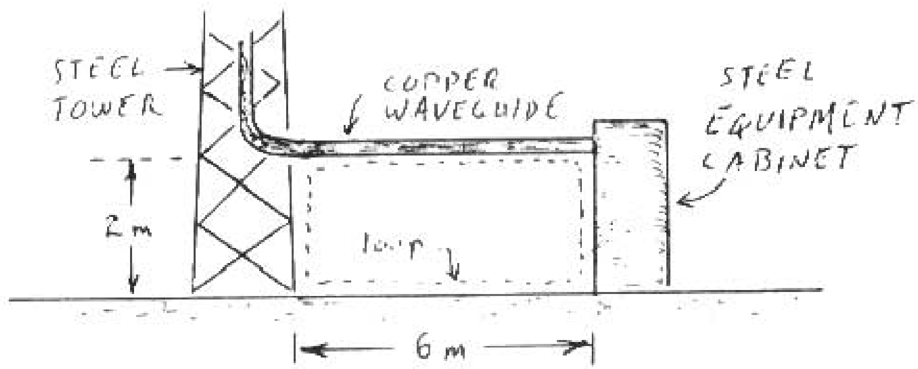
A4. (25) This problem describes a real situation once faced by Federal Aviation Administration engineers. In Florida, where there are frequent thunderstorms, the FAA experienced a large number of communications equipment failures. Suspecting lightning strikes, a power recording monitor was installed at one Florida site. After carefully studying the problem, the engineers concluded that the failures were the result of inductive coupling of energy into the communications system. They determined that a conducting loop (with dimensions of about 2 meters by 6 meters) was formed by the steel tower, the copper microwave waveguide, the steel equipment cabinet, and the ground (see sketch). Using typical figures for the rise time of electric current in a lightning bolt, one can estimate that significant voltages would be induced in this loop, even by a lighting strike several kilometers away



Let us model the process as follows:

(a, 8) Begin with the magnetic field produced by a straight, infinite line of steady current I. Use either the Biot-Savart law or Amperé's law to obtain an expression for the magnetic field at the distance r from the current.

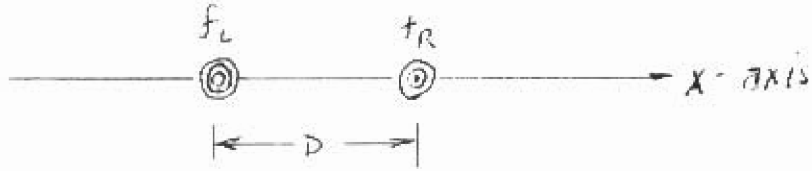
(b, 10) Although the result of part (a) holds rigorously only for a *steady* current, let us use it to estimate the time-dependent magnetic field produced by a lightning bolt. Let us model the lightning bolt as a straight, vertical, infinite line of current that rises linearly from zero to 1×10^6 A in 5×10^{-5} s. Let the lightning bolt strike one kilometer from the tower. Find the *emf* induced in the conducting loop while the lightning current increases. Describe any other approximations you make besides the ones we have already mentioned.

(c, 7) Suppose the conducting loop described above has a resistance of 50 Ohms. Inside the cabinet there is a solid state device that can survive a maximum current of ½A. Could this device be damaged by the lightning strike described in part (b)?

(This problem adapted from the *Structures and Interactions in Nature* model curriculum of the Introductory University Physics Project. Special thanks to Paul Griffith of the FAA.)

B1. (40) Two sirens located on the x-axis are separated by a certain distance D. As heard by an observer at rest, f_L denotes the frequency emitted by the left-hand siren, and f_R denotes the frequency emitted by the right-hand siren. You are moving with constant velocity at speed v_o along the x-axis, and record the following observations:

When you are on the *right* side of both sirens, you hear a beat frequency of 1.01 Hz.
When you are on the *left* of both sirens, you hear a beat frequency of 0.99 Hz.
When you are *between* the sirens, the beat frequency is zero.



(a, 5) Determine in which direction you are moving along the x-axis.
(b, 10) Evaluate your speed v_o as a fraction of the speed of sound.
(c, 5) Determine which frequency is greater, f_L or f_R.
(d, 10) Determine the numerical values of f_L and f_R.
(e, 10) Suppose you stop moving, and the siren that emits sound of frequency f_L is taken aboard a helicopter. The helicopter hovers with zero velocity above you. The siren (assumed to be battery-powered) is dropped from the helicopter. In the case of negligible air resistance, sketch a graph of frequency as a function of time, that describes the frequency you hear from this falling siren. [If you did not obtain a value for f_L in part (d), use 50 Hz.] Take the speed of sound to be 343 m/s. The siren falls for 30 seconds before striking the ground beside you.

B2. (40) *Electron-Nucleus Interactions*

Consider an electron that starts from rest, essentially infinitely far from a completely ionized, extremely massive nucleus containing Z protons. The electron is captured by this nucleus. In the capture process, one photon is emitted. After being captured, the electron possesses angular momentum of magnitude L as it orbits the nucleus. In parts (a) and (b) let us suppose the electron orbits classically in an elliptical orbit whose perigee (minimum distance from the nucleus) is r_{min} and whose apogee (maximum distance from the nucleus) is $r_{max} = \beta r_{min}$. For numerical values use:

$L = 1.4\times10^{-33}$ kg-m^2/s
$\beta = 2$
$r_{max} = 2.5\times10^{-10}$ m

(a, 15) Find the value of Z.
(b, 10) What is the wavelength of the single photon that was emitted during the capture process?

(c, 15) Now let us consider another heavy but completely ionized nucleus of mass M, where $M = 121$ GeV/c^2. Let an electron be accelerated such that, as observed in the reference frame where the nucleus is originally at rest, the electron has 1 GeV of kinetic energy right before it impacts the nucleus. Upon entering the nucleus, the electron reacts with one of the protons in an inverse beta decay,

$$(\text{electron}) + (\text{proton}) \rightarrow (\text{neutron}) + (\text{neutrino}).$$

The neutrino immediately escapes the nucleus. Let us approximate the mass M' of the transformed nucleus as equal to M, since $|M' - M| << M$. The mass of the neutrino is very small; experimentally it cannot be above about 5 eV/c^2, so let us assume that the neutrino's mass is exactly zero. Find the neutrino's energy in the case where it is emitted in the direction exactly opposite the direction of the incoming electron (that is, the angle between the incoming electron's momentum and the outgoing neutrino's momentum is 180 degrees). Describe any other approximations you make in addition to those already stated.

Here again are the numerical values for this problem:
$M = 121$ GeV/$c^2 \approx M'$
neutrino mass = 0
electron mass $m \approx ½$ MeV/c^2
incoming electron's kinetic energy: 1 GeV
Note: 1 MeV = 10^6 eV, and 1 GeV = 10^9 eV