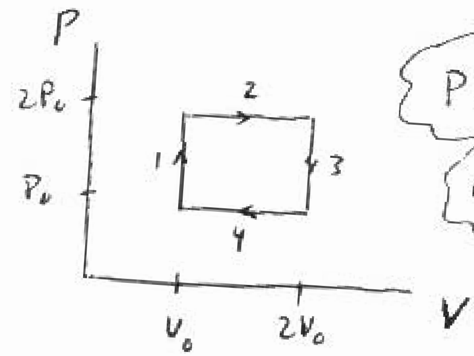
Solutions - 1998 Exam 2 US Physics Team

A1 (a) $W = \int P\,dV = P_0 V_0$




$PV = nRT$

$C_V = \frac{3}{2} R$

(b) $Q = \cancel{\Delta U}^{\,0} + W = P_0 V_0$

(c) $e = \dfrac{W}{Q_{in}}$ where

$Q_{in}$ = heat put <u>into</u> system in cycle (all <u>positive</u> heat)

<u>Segments 1 and 3:</u> $V = \text{const}$

$$Q = \Delta U + \cancel{W}^{\,0} = n C_V \Delta T = \frac{C_V}{R} \Delta(PV) = \frac{3}{2} V \Delta P$$

segment $Q_1 = \frac{3}{2} V_0 (2P_0 - P_0) = \frac{3}{2} P_0 V_0 > 0$ $\boxed{\checkmark}$

$Q_3 = \frac{3}{2}(2V_0)(P_0 - 2P_0) < 0$

2 and 4: $P = \text{const.}$

$$Q = \Delta U + W = \frac{3}{2} V \Delta P + P \Delta V = \frac{5}{2} P \Delta V$$

$Q_2 = \frac{5}{2}(2P_0)(2V_0 - V_0) = 5 P_0 V_0$ $\boxed{\checkmark}$

$Q_4 = \frac{5}{2} P_0 (V_0 - 2V_0) < 0$

$\therefore Q_{in} = Q_1 + Q_2 = \frac{13}{2} P_0 V_0$

and $e = \dfrac{2}{13}$

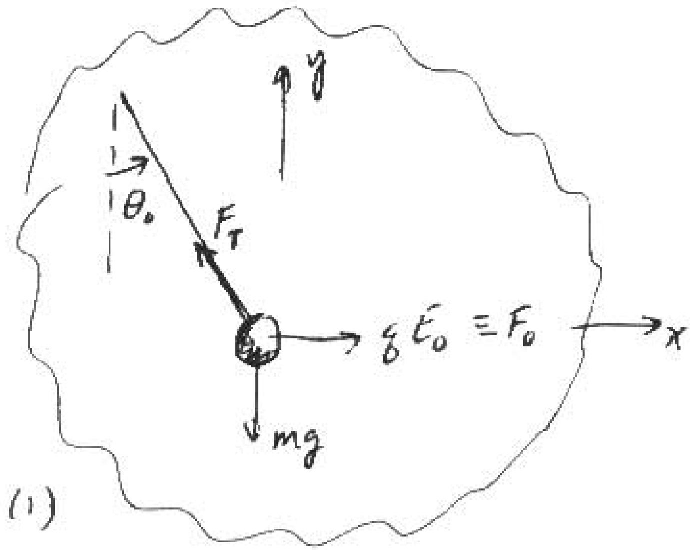
A1 (d) $$\Delta S = \int_A^D \frac{dQ}{T} = \int_A^D \frac{dU + P\,dV}{T} = \int_A^D \left[ n c_v \frac{dT}{T} + nR \frac{dV}{V} \right]$$

$$= n c_v \ln \frac{T_D}{T_A} + nR \ln \frac{V_D}{V_A}$$

$$\frac{T_D}{T_A} = 2 = \frac{V_D}{V_A}$$

$$= n(c_v + R) \ln 2 = 5R \ln 2$$

$\boxed{A2}$ (a) $$\vec{F} = m\cancel{\vec{a}}^{\,0}$$




| $x$ | $y$ |
|---|---|
| $qE_0 - F_T \sin\theta_0 = 0$ | $F_T \cos\theta_0 - mg = 0$ |

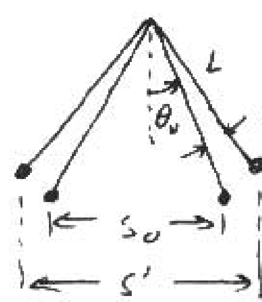
$$\frac{qE_0}{mg} = \tan\theta_0 \quad \cdots (1)$$

(b) $$s_0 = 2L \sin\theta_0$$

$$s' = 2L \sin(\theta_0 + \delta)$$

$$\simeq 2L(\sin\theta_0 + \delta\cos\theta_0)$$

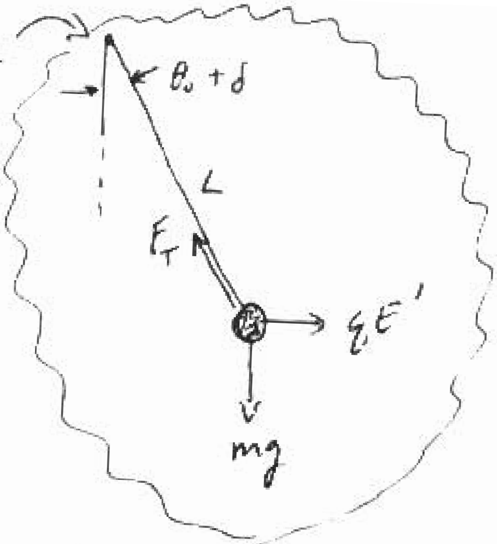
$$= s_0(1 + \delta\cot\theta_0)$$




$$F' = qE' = \frac{kq^2}{s'^2} = \frac{kq^2}{s_0^2}(1 + \delta\cot\theta_0)^{-2}$$

$$\simeq F_0(1 - 2\delta\cot\theta_0)$$

A 2 (c) Apply $\vec{\tau} = I\vec{\alpha}$ about hook




$$q E' L \cos(\theta_0 + \delta) - mg L \sin(\theta_0 + \delta) = m L^2 \ddot{\delta}$$

$$q E_0 (1 - 2\delta \cot\theta_0)(\cos\theta_0 - \delta \sin\theta_0) - mg(\sin\theta_0 + \delta \cos\theta_0) = m L \ddot{\delta} \quad \ldots(2)$$

Use Eq.(1), $q E_0 \cos\theta_0 = mg \sin\theta_0$. To $O(\delta)$, Eq. (2) becomes

$$\ddot{\delta} + \frac{g}{L}\left(\frac{1}{\cos\theta_0} + 2\cos\theta_0\right)\delta = 0$$

$$\omega^2 = \frac{g}{L}\left(\frac{1}{\cos\theta_0} + 2\cos\theta_0\right)$$

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}\left(\frac{1}{\cos\theta_0} + 2\cos\theta_0\right)^{-\frac{1}{2}}$$

(d) at $\theta_0 = 45^\circ$, $\cos\theta_0 = \frac{1}{\sqrt{2}} = \sin\theta_0$ $\quad T_0 = 2\pi\sqrt{\frac{L}{g}}$

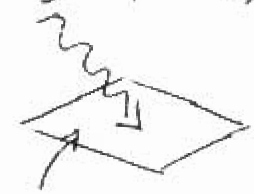
$$T = \frac{T_0}{\sqrt{\sqrt{2} + \frac{2}{\sqrt{2}}}}$$

$$T = \frac{T_0}{\sqrt{2\sqrt{2}}}$$

$\boxed{A3}$

(a) min. energy required is 5eV

$I = 10^{-2}\ W/m^2$



Area with 1 surface electron

$$I A \Delta t = 5eV$$

$$\Delta t = \frac{5\,(1.6 \times 10^{-19}\ J)}{(2\times 10^{-20}\ m^2)(10^{-2}\ \frac{J}{s\text{-}m^2})}$$

$$= 4 \times 10^{3}\ s \cong 1.1\ hr.$$

$$A = (5\times 10^{19}/m^2)^{-1}$$

$$= 2\times 10^{-20}\ m^2$$

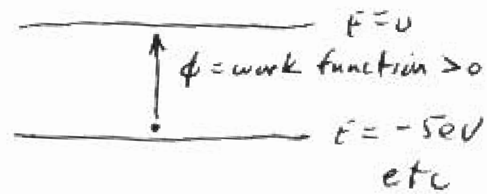
(b) $hf = 5eV \Rightarrow f = \dfrac{5eV}{4.14\times 10^{-15}\ eV\text{-}s} = 1.21\times 10^{15}\ Hz$

(c) $hf = K + \phi$ (cons. of Energy)

incoming photon energy

K.E. left over

what it costs to liberate electron




$$K = hf - \phi = (4.14\times 10^{-15}\ eV\text{-}s)(4.5\times 10^{15}\ Hz) - \phi$$

$$= 18.6\,eV - \phi$$

State 1 : $K_1 = 18.6eV - 5eV = 13.6\,eV$

2 : $K_2 =$ 8.6 eV

3 : $K_3 =$ 3.6 eV

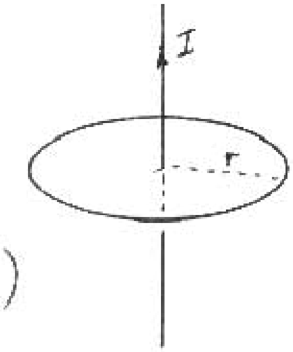
4 : —

5 : —

K.E. cannot be < 0

(d) $eV_{stop} = 13.6\ eV \Rightarrow V_{stop} = 13.6$ Volts

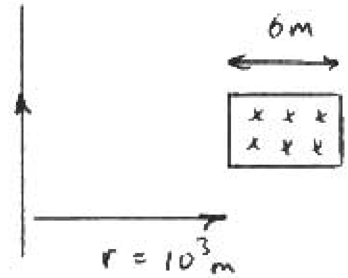
$\boxed{A4}$ (a) $\oint \vec{B}\cdot d\vec{l} = \mu_0 I$




$$B = \frac{\mu_0 I}{2\pi r} \quad \text{or} \quad \frac{2k_m I}{r}$$

$$\left( k_m \equiv \frac{\mu_0}{4\pi} \right)$$

(b) Neglect variation of B with r over area of loop




in magnitude, $\mathcal{E} = \frac{d\Phi_B}{dt}$

$$= \frac{2k_m}{r}\frac{dI}{dt}$$

$$= \frac{(12\ m^2)(2\times10^{-7}\ Ns^2/C^2)(10^6\ A)}{(10^3\ m)(5\times10^{-5}\ s)}$$

$$= 48\ V$$

(c) $I = \frac{\mathcal{E}}{R} = \frac{48V}{50\Omega} = .96\ A$ yes the device may be damaged

B1

(a) The beat frequency is "blue shifted" on the . and "red shifted" on the L - hence moving

(b) The beats are a "source" of frequency $|f_L - f_R| \equiv \Delta$

when on L: $\Delta f_L' = \Delta f_0 \left(1 - \frac{v}{c}\right)$ $c$ = speed of sound

moving away

using given data,

$$.99 \text{ Hz} = \Delta f_0 (1 - v/c) \quad \text{---}(1)$$

moving towards

when on R: $\Delta f_R' = \Delta f_0 \left(1 + \frac{v}{c}\right)$

$$1.01 \text{ Hz} = \Delta f_0 (1 + v/c) \quad \text{---}(2)$$

(1) and (2) $\Rightarrow$

$$\beta \equiv \frac{1.01}{.99} = \frac{1 + v/c}{1 - v/c} \quad \text{---}(3)$$

$$\Rightarrow v/c = .01 \quad \text{---}(4)$$

(c) In between, $f_R$ is red-shifted:

$$f_R' = f_R - \varepsilon_R \quad \text{for some } \varepsilon_R > 0$$

In between, $f_L$ is blue-shifted:

$$f_L' = f_L + \varepsilon_R \quad \text{for some } \varepsilon_L > 0$$

In between, $f_L' = f_R'$

$$f_L + \varepsilon_R = f_R - \varepsilon_R$$

$$\Rightarrow f_R = f_L + \varepsilon_R + \varepsilon_R > f_L$$

so $f_R > f_L$

B1 (d) In between, $f_L' = f_L\left(1 + \frac{v}{c}\right)$ $f_R' = f_R\left(1 - \frac{v}{c}\right)$

blue shifted red shifted

and $f_L' = f_R'$, so $f_L\left(1 + v/c\right) = f_R\left(1 - v/c\right)$

i.e., $\frac{f_R}{f_L} = \frac{1 + v/c}{1 - v/c} = \beta$ [from (3)] ... (5)

Also, from either (1) or (2), and using (4) [$v/c = .01$]

it follows that $f_R - f_L = 1\text{ Hz}$ ... (6)

Solving (5) and (6) gives $f_R = 50.5\text{ Hz}$, $f_L = 49.5\text{ Hz}$

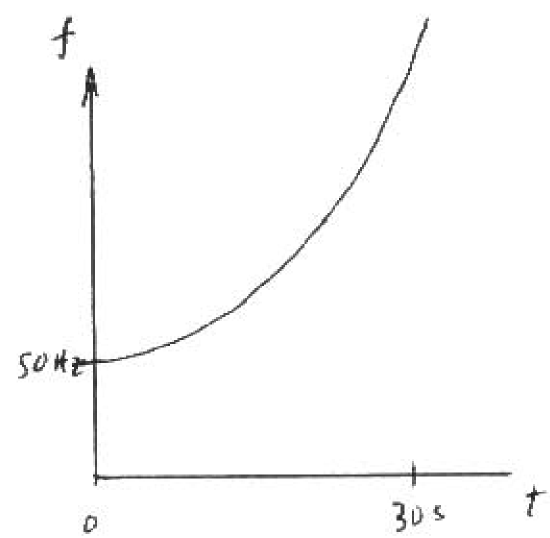
(e). Source moving: $v = -gt$

$$f' = f_0\left(1 + \frac{v}{c}\right)^{-1}$$

$$= f_0\left(1 - \frac{gt}{c}\right)$$

$g = 9.8\text{ m/s}^2$ $f_0 = 50\text{ Hz}$
$c = 343\text{ m/s}$

$$f' = \frac{50\text{ Hz}}{1 - \frac{t}{35\text{ s}}}$$



B2

$$E = \frac{1}{2} m\dot{r}^2 + \frac{L^2}{2mr^2} - \frac{kZe^2}{r}$$ for orbiting electron

(a) At apogee (2) and perigee (1), $\dot{r} = 0$

$$E_1 = E_2 \qquad L = \text{const.}$$

$$\frac{L^2}{2mr_1^2} - \frac{kZe^2}{r_1} = \frac{L^2}{2mr_2} - \frac{kZe^2}{r_2}$$ where $r_2 = \beta r_1$

$$\frac{L^2}{2m}\left(\frac{1}{r_2^2} - \frac{1}{r_1^2}\right) - kZe^2\left(\frac{1}{r_2} - \frac{1}{r_1}\right) = 0$$

$$\frac{L^2}{2mr_1^2}\underbrace{\left(\frac{1}{\beta^2} - 1\right)}_{\cancel{\left(\frac{1}{\beta}-1\right)}\left(\frac{1}{\beta}+1\right)} - \frac{kZe^2}{r_1}\cancel{\left(\frac{1}{\beta} - 1\right)} = 0$$

$$\frac{L^2}{2mr_1^2}\left(\frac{1}{\beta} + 1\right) = \frac{kZe^2}{r_1}$$

$$Z = \frac{L^2\left(\frac{1}{\beta} + 1\right)}{2mr_1 ke^2} = 28$$

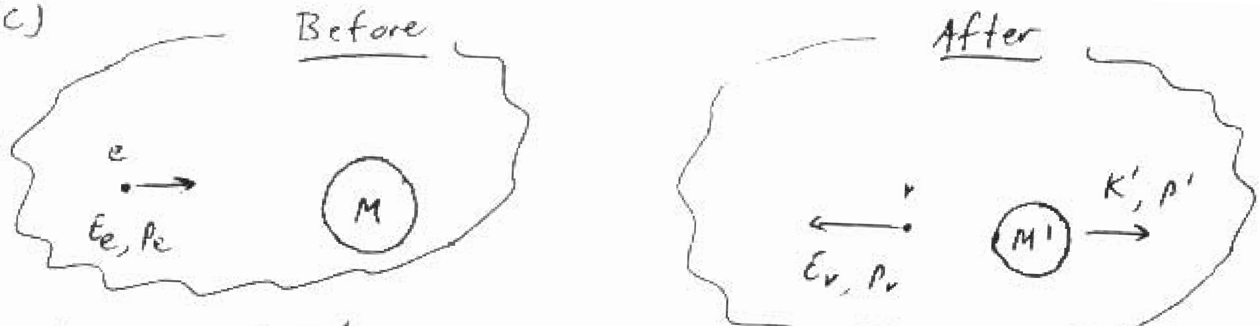
(b) $$E_{final} + hf = \cancel{E_{initial}}^{\,0}$$ "initial" and "final" refer to electron's energy

$$hf = -E_f$$

$$= -\frac{L^2}{2mr_2^2} + \frac{kZe^2}{r_2} = 8.6 \times 10^{-18}\ \text{J}$$

$$\lambda = \frac{hc}{E}$$
$$= 2.3 \times 10^{-8}\ \text{m}$$

B2 (c)




$M \simeq M' = 121 \text{ GeV}/c^2$

$E_e = K_e + m_e c^2 = 1 \text{ GeV} + \frac{1}{2}\text{MeV} \simeq 1 \text{GeV}$ neglect electron mass

$M'c^2 \gg K_e \Rightarrow$ M' kinetic energy $(K') \simeq \frac{p'^2}{2m}$

$\therefore$ The electron is relativistic, the nucleus non-relativistic

[ Need we consider electron's electrical potential energy?

Estimate P.E. at surface of nucleus (order of magnitude)

$$U = \frac{Zke^2}{r} \qquad \left\{ \begin{array}{l} Z \sim 50 \\ r \sim 10^{-14} \text{ m} \end{array} \right\}$$

$$\simeq \frac{(50)(9\times10^9)(1.6\times10^{-19})\, e}{10^{-14}\text{ m}}$$

$$= 7.2\times10^5 \text{ eV} < 1 \text{ MeV} \ll K_e$$ – may ignore P.E. ]

Ok, here we go: (units: ignore c's – E in eV, p in $\frac{eV}{c}$, m in $\frac{eV}{c^2}$

<u>cons. of Energy:</u>

$$K_e + M = E_\nu + M' + K'$$

$M \simeq M'$

$$K_e \simeq E_\nu + K' \qquad \cdots (1)$$

$$\simeq E_\nu + \frac{p'^2}{2M'}$$

cons. of momentum:

$$p_e = p' - p_\nu \qquad \text{(one-dimensional)}$$

$$\Rightarrow \quad p' = p_e + p_\nu \qquad \text{--- (2)}$$

(2) into (1) $\Rightarrow$

$$K_e = E_\nu + \frac{(p_e + p_\nu)^2}{2M}$$

$$K_e = E_\nu + \frac{1}{2M}\left(p_e^2 + p_\nu^2 + 2 p_e p_\nu\right) \qquad \text{--- (3)}$$

For neutrino,

$$E_\nu^2 - p_\nu^2 = \cancel{m_\nu^2} \to 0$$

$$p_\nu = E_\nu$$

For electron

$$E_e^2 - p_e^2 = m_e^2$$

$$(K_e + m_e)^2 - p_e^2 = m_e^2 \qquad m_e \ll K_e$$

$$K_e \approx p_e$$

(3) becomes

$$K_e = E_\nu + \frac{1}{2M}\left(K_e^2 + E_\nu^2 + 2 K_e E_\nu\right)$$

numerically,

$$1\text{ GeV} = E_\nu + \frac{1}{242\text{ GeV}}\left(1\text{ GeV}^2 + E_\nu^2 + (2\text{ GeV})E_\nu\right)$$

neglect $\frac{1}{242}$

$$1\text{ GeV} \approx \frac{E_\nu^2}{242\text{ GeV}} + E_\nu\left(1 + \frac{1}{121}\right) \qquad \text{neglect } \frac{1}{121}$$

$$E_\nu^2 + 242\, E_\nu - 242\text{ GeV} = 0$$

$$E_\nu = \left[-121 \pm \tfrac{1}{2}\sqrt{(242)^2 + 4(242)}\,\right]\text{GeV} = -121\text{ GeV} + 121.996\text{ GeV}$$

(+ root) $\approx 1$ GeV (a little less than 1 GeV)

keeping 1/242 and 1/121 gives .98

(11)