AAPT | UNITED STATES PHYSICS TEAM
AIP | 2000

2000 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. It may not be used in graphing mode. Calculators may not be shared. You may not use any tables, books, or collections of formulas. You may use a ruler or straight edge.
- Questions with the same point value are not necessarily of the same difficulty.
- Good luck!

Possibly Useful Information

Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 9.0 \times 10^{9}$ N·m²/C²
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.0 \times 10^{8}$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}$ (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/mol·K
Elementary charge	$e = 1.6 \times 10^{-19}$ C
1 electron volt	1 eV $= 1.6 \times 10^{-19}$ J
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s $= 4.14 \times 10^{-15}$ eV·s
Electron mass	$m = 9.1 \times 10^{-31}$ kg $= 0.51$MeV/c^2
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x\rvert << 1$

AAPT | UNITED STATES PHYSICS TEAM
AIP | 2000

2000 Semi-Final Exam

Part A

A1. An empty 4.2-g balloon is filled with air to a pressure of 2.0 atm at the ambient temperature of 20.0° C. When filled it is spherical with a radius of 15.0 cm. The mean molecular mass of air is 28.8 g/mole and the atmospheric pressure is 1.01×10^5 Pa.

(15) a. The filled balloon is weighed by placing it on a scale. What is the scale reading?

(5) b. The filled balloon is now gently heated until its radius increases by 10%. What is the scale reading now?

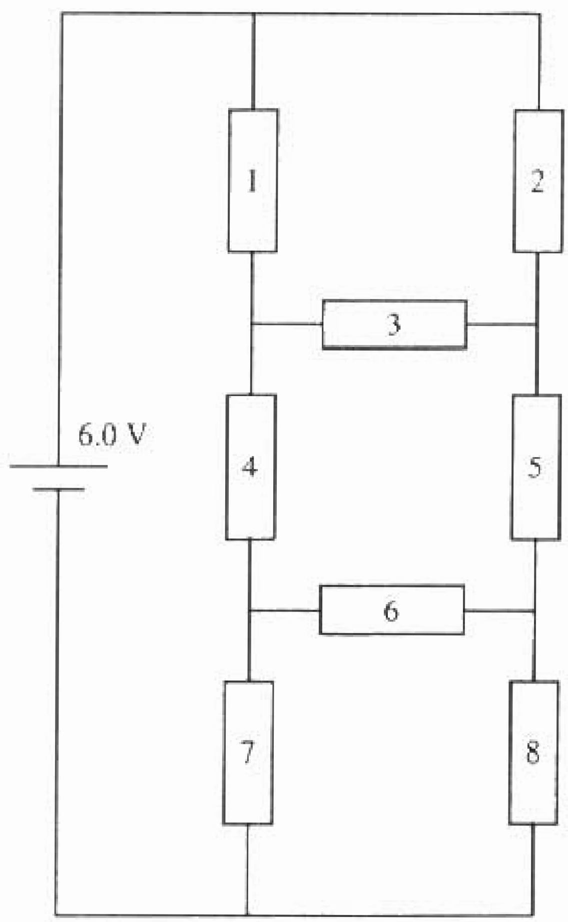
A2. The circuit diagrammed at the right consists of a 6.0–V power supply and eight 12.0-Ω resistors, labeled 1 – 8.

(5) a. What is the equivalent resistance of the circuit?

(5) b. What is the current in each resistor? Please use I_j to denote the current in resistor j.

(5) c. Resistors 1, 5, and 7 are replaced with 4.0–μF capacitors. What is the charge on each capacitor after the circuit has been connected for a very long time? Please use Q_j to denote the charge on the capacitor in location j.

(5) d. The capacitor in location 5 is replaced with a second 6.0–V power supply (positive end closest to resistor 3 and negative end closest to resistor 6). What is the current through each resistor after the circuit has been connected for a very long time? Please use I_j to denote the current in resistor j. In this part 2, 3, 4, 6, and 8 denote 12.0-Ω resistors, 1 and 7 denote 4.0–μF capacitors, and 5 denotes a 6.0–V power supply.



A3. A relativistic particle decays into two photons. One of the photons is traveling to the right along the x-axis with frequency ν_1. The second photon is traveling to the left with frequency ν_2, where $\nu_1 > \nu_2$. Express your answers to the following in terms of given quantities and known constants.

(5) a. Find the velocity V of the original particle.

(5) b. Find the rest mass of the original particle.

(5) c. What are the frequencies of the photons in the rest frame of the original particle? What are their directions of travel in this rest frame?

(5) d. You are given the following formula:

$$p'_x = F_1 p_x + F_2 \frac{E_\gamma}{c}$$

where p'_x is the observed x-component of the momentum of either photon, whereas E_γ and p_x are the energy and x-component of the momentum of the photon in the rest frame of the particle. Using your results determine the functions F_1 and F_2 in terms of V/c, where V is the speed of the original particle.

A4. Two movable wedges of mass M meet the horizontal plane smoothly. They are free to slide without friction on the plane. See accompanying diagram. A coin of mass m is released from rest at the height h and slides down the left hand wedge. Assume that the coin always slides without friction. Express your answers to the following in terms of given quantities and known constants.

(10) a. What is the maximum height the coin will rise on the right-hand wedge?

(10) b. For what minimum mass M will the coin again contact the left-hand wedge?

AAPT | UNITED STATES PHYSICS TEAM

AIP | 2000

2000 Semi-Final Exam

Part B

B1. A hollow spherical shell of radius b and negligible thickness is constructed out of insulating material with total charge $-Q$. A second similar shell has radius c where $c > 2b$ and total charge $+Q$. Each shell has its charge distributed uniformly over its surface. Assume the electrostatic potential is zero at infinity. As shown at the right, the symbols I, II, and III may be used to denote the regions inside the smaller shell, between the shells, and outside the larger shell, respectively. Express your answers to the following in terms of given quantities and known constants. You may use either spherical or Cartesian coordinates.

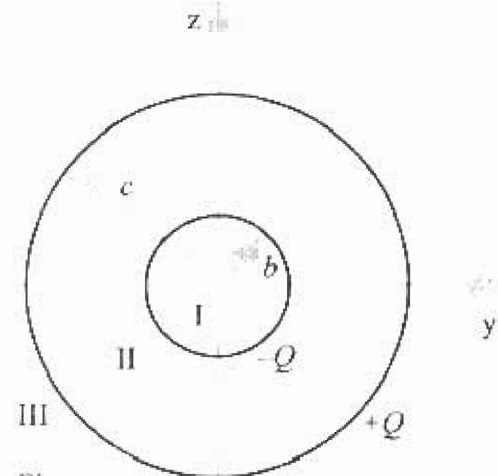


Figure a.

The spherical shells are concentric as shown in Figure a. at the right.

(10) a. Find the electrostatic field in regions I, II, and III as a function of position.

(10) b. Find the electrostatic potential in regions I, II, and III as a function of position.

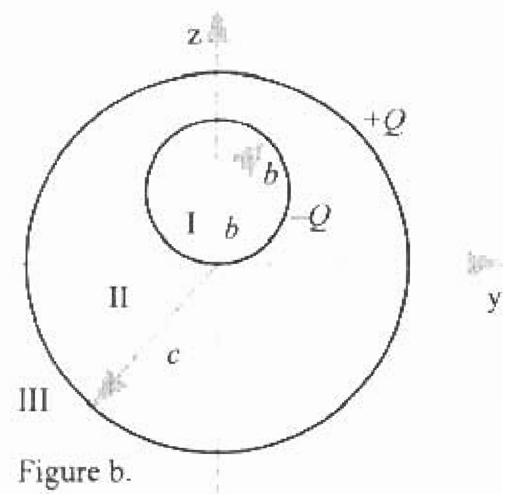


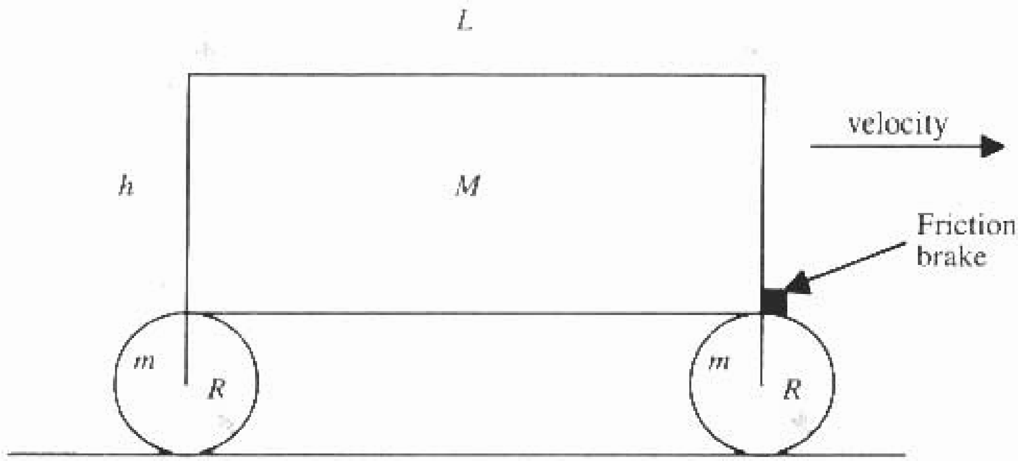
Figure b.

The spherical shells are not concentric as shown in Figure b at the right. The inner shell is shifted in the positive z direction a distance b equal to its radius.

(10) c. Find the electrostatic field in regions I, II, and III as a function of position.

(10) d. Find the electrostatic potential in regions I, II, and III as a function of position.

B2. We shall model a car as a uniform rectangular block of length L and height h with mass M that has struts of length R that rest on the frictionless axles of two wheels of radius R, one front wheel and one rear wheel. Each wheel has a mass m and a uniform mass density. The driver can exert a friction force at the top of the front wheel to slow down the car. See the diagram below.



Assume that the two identical wheels have a coefficient of static friction μ with the ground. The driver applies a braking force that causes the vehicle to decelerate with a deceleration of magnitude a.

Express your answers to the following in terms of given quantities and known constants.

(4) a. Find f_R the frictional force of the ground on the rear wheel.
(4) b. Find F_{CR} the horizontal component of the force exerted by the car on the axle of the rear wheel.
(4) c. Find f_F the frictional force of the ground on the front wheel.
(4) d. Find f_B the force of the brake that is applied to the front wheel.
(4) e. Find F_{CF} the horizontal component of the force exerted by the car on the axle of the front wheel.
(8) f. Find n_F and n_R the normal forces exerted by the ground on the front and rear wheels.
(4) g. For each wheel, find the maximum possible deceleration of the car before that wheel starts to slip.

If $L = 2.50$ m, $h = 1.50$ m, $R = 0.300$ m, $M = 6000$ kg, $m = 200$ kg, $\mu = 0.600$,

(4) h. Numerically evaluate the magnitude of the maximum possible deceleration of the car in terms of the acceleration of gravity, g.
(4) i. Numerically evaluate the ratio of the normal force of the ground on the front wheel to that on the rear wheel at this maximum deceleration.