AAPT UNITED STATES PHYSICS TEAM
AIP 2001

2001 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. It may not be used in graphing mode. Calculators may not be shared. You may not use any tables, books, or collections of formulas. You may use a ruler or straight edge.
- Questions with the same point value are not necessarily of the same difficulty.
- Good luck!

Possibly Useful Information

Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 9.0 \times 10^{9}\ \mathrm{N \cdot m^2/C^2}$
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.0 \times 10^{8}$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}\ (\mathrm{mol})^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/(mol·K)
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}\ \mathrm{J/(s \cdot m^2 \cdot K^4)}$
Elementary charge	$e = 1.6 \times 10^{-19}$ C
1 electron volt	$1\ \mathrm{eV} = 1.6 \times 10^{-19}$ J
Planck's constant	$h = 6.63 \times 10^{-34}\ \mathrm{J \cdot s} = 4.14 \times 10^{-15}\ \mathrm{eV \cdot s}$
Electron mass	$m = 9.1 \times 10^{-31}\ \mathrm{kg} = 0.51\ \mathrm{MeV/c^2}$
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x \rvert << 1$

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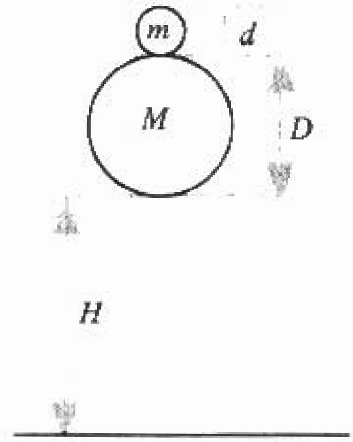
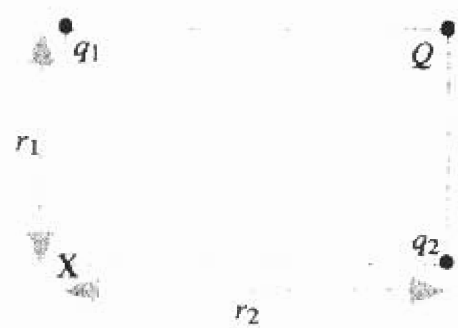
Part A

A1. (25 points) A small ball of diameter d and mass m rests on top of a large ball of diameter D and mass M ($M > m$) which is initially held so that its bottom is a height H above the floor. Please see the diagram to the right. The balls are released from rest. Assume all collisions are perfectly elastic and occur along a vertical line. How high does the small ball rise above its initial position after its first bounce? Express your answer in terms of the quantities given in the diagram.



A2. Point charges q_1, q_2, and Q are located at three corners of an r_1 by r_2 rectangle as shown in the diagram to the right.



(10) a. Write a general expression for the electrostatic field at the point marked X in the diagram. Define any quantities you introduce.

(5) b. For what values of q_1 and q_2 (expressed in terms of r_1, r_2, and Q) does the field at X vanish?

(5) c. Write a general expression for the electrostatic potential at X and evaluate it for the charges found in Part b. Assume the potential vanishes an infinite distance from the charges.

(5) d. How much total work is done by the electric force as a fourth charge q_4 moves from infinity to the point marked X? Assume q_1 and q_2 are the charges found in Part b.

A3. A plane wave traveling in the positive z-direction can be described by the following wavefunction: $\psi(z,t) = A\sin(kz - \omega t + \phi)$, where A (the amplitude) and ϕ are constants, $k = \dfrac{2\pi}{\lambda}$, and $\omega = 2\pi f$. The factor $(kz - \omega t + \phi)$ is called the phase of the wave.

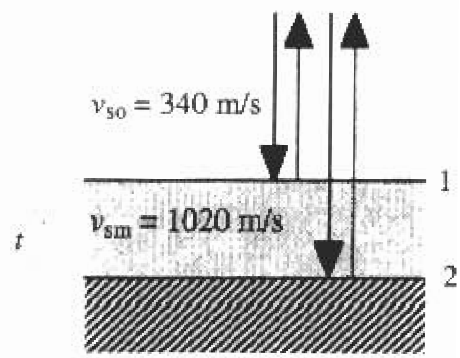
(5) a. Two sources, S_1 and S_2, produce identical plane waves traveling in the same direction. The sources operate in phase and the waves have the same amplitude, frequency, and wavelength. Source S_1 is located a distance z_1 from point P and source S_2 is a distance z_2 from point P. What is the wavefunction of the resultant wave at point P? The following relation may be useful:

$$\sin\alpha + \sin\beta = 2\cos\left[\tfrac{1}{2}(\alpha - \beta)\right]\sin\left[\tfrac{1}{2}(\alpha + \beta)\right].$$

(4) b. For what values of $\Delta z = | z_1 - z_2 |$ is the amplitude of the resultant traveling wave zero?

(4) c. For what values of $\Delta z = | z_1 - z_2 |$ is the amplitude of the resultant traveling wave maximum?

A plane sound wave is normally incident on a coated surface as shown to the right. At surface 1, the wave is both transmitted and reflected. The reflected and transmitted waves have equal amplitudes. All of the wave reaching surface 2 is reflected. Assume the wave does not shift phase on reflection at either surface. In air, the wave has frequency f, wavelength λ, and speed $v_{so} = 340$ m/s. In the material of thickness $t = 0.200$ m, the wave has speed $v_{sm} = 1020$ m/s.



(4) d. In terms of f and λ (the frequency and wavelength in air), what are the frequency and wavelength of the sound in the material?

(8) e. List all frequencies smaller than 10,000 Hz for which the reflected sound intensity is a minimum?

A4. (25 points) The solar radius is 6.96×10^8 m. The sun has a surface temperature of 5.80×10^3 K and is 1.50×10^{11} m from the earth. Assume the sun is a perfect absorber/emitter. Suppose this thermal energy comes from the following set of nuclear reactions:

$${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + e^+ + \nu$$

$${}^1_1\text{H} + {}^1_1\text{H} \rightarrow {}^2_1\text{H} + e^+ + \nu$$

$${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$$

$${}^1_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + \gamma$$

$${}^3_2\text{He} + {}^3_2\text{He} \rightarrow {}^4_2\text{He} + {}^1_1\text{H} + {}^1_1\text{H}$$

where e^+ represents a positron, ν represents a neutrino, and γ represents a gamma ray. This set of reactions releases a thermal energy of 26.2 MeV. Assume that the sun's power production and temperature at any depth does not change with time. Find the intensity (number per second per square meter) of solar neutrinos at the location of the Earth and the number of solar neutrinos in the adult human body. Assume that the adult human body has a volume of 8.00×10^{-2} m^3 and that neutrinos travel at the speed of light.

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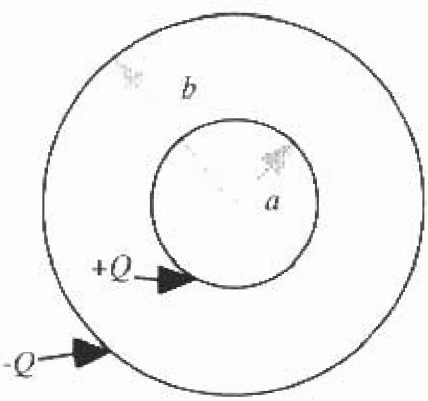
Part B

B1. A spherical capacitor consists of two concentric conducting spherical shells separated by air. The inner shell has radius a and charge $+Q$. The outer shell has radius b and charge $-Q$.

(5) a. What is the field in the region between the shells ($b > r > a$)?

(5) b. What is the potential difference between the shells?



(5) c. What is the capacitance of the system?

(5) d. How much energy is stored in the field between the shells?

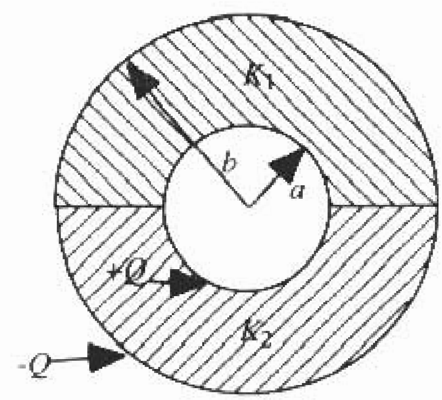
After the capacitor has acquired a charge Q, it is disconnected from the battery and a linear, homogenous, isotropic insulating material with dielectric constant K is inserted to completely fill the volume between the shells without discharging them. The effect of such a dielectric is to multiply the vacuum permittivity by K.

Wherever possible, express your answers to the following parts in terms of your answers to Parts (a), (b), (c), and (d).

(10) e. Find the field, potential difference, capacitance, and stored energy in this case.

(5) f. What is the total surface charge induced on the inner surface ($r = a$) of the dielectric?

(10) g. Find the field, potential difference, capacitance, stored energy, charge on the capacitor, and total surface charge induced on the inner surface ($r = a$) of the dielectric if the capacitor is left connected to the battery as the dielectric material is inserted.



(5) h. Find the capacitance if, instead of a single dielectric, two different linear, homogeneous, isotropic dielectrics are inserted between the shells. Dielectric 1 fills the top half and has dielectric constant K_1. Dielectric 2 fills the bottom half and has dielectric constant K_2.

B2. (Cylindrical coordinates are used in this problem and $\hat{u}_r$, $\hat{u}_\theta$, and $\hat{u}_z$ represent unit vectors in the r, θ, and z directions, respectively. The coordinates form a right-handed orthogonal system, e.g., $\hat{u}_r \times \hat{u}_\theta = \hat{u}_z$.) A particle with charge q and mass m is constrained to move in the xy-plane with velocity that can be expressed as $\vec{v} = v_r\hat{u}_r + r\omega\hat{u}_\theta$. The particle is initially at rest and a distance r_o from the origin. Two forces act on the particle. One is due to the presence of a uniform magnetic field given by $\vec{B} = B_o\,\hat{u}_z$. The other force and its corresponding potential energy are:

$$\vec{F} = \frac{-2k}{r^3}\hat{u}_r \qquad \text{and} \qquad U(r) = -\frac{k}{r^2},$$

where k is a constant and r is the distance from the origin. The particle executes a trajectory that periodically varies between its initial radius r_o and a minimum radius, r_{min}. Express your answers in terms of q, m, v_r, r, ω, B_o, k, r_o, r_{min}, and the unit vectors.

(5) a. What is the magnetic force acting on the particle?

(5) b. How much work is done by this magnetic force?

(8) c. Use the work-energy theorem to derive an equation relating the variables v_r, r, and ω to the initial parameters

(7) d. What is the net torque acting on the particle?

(5) e. What is the angular momentum of the particle in terms q, B_o, r, and r_o up to an arbitrary constant vector?

(5) f. What is the particle's angular velocity ω in terms of m, q, B_o, r, and r_o?

(5) g. Using your results from (c) and (f), find r_{min}, the particle's closest approach to the origin.

(5) h. Find the range of B_o for which the particle does not reach the origin.

(5) i. Find ω for large magnetic fields B_o at each of the limiting radii $r = r_o$ and $r = r_{min}$. Write your answer to lowest nonvanishing order in $\frac{1}{B_o}$.