AAPT UNITED STATES PHYSICS TEAM
AIP 2003

2003 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. It may not be used in graphing mode. Calculators may not be shared. You may not use any tables, books, or collections of formulas. You may use a ruler or straight edge.
- Questions with the same point value are not necessarily of the same difficulty.
- Good luck!

Possibly Useful Information

Gravitational field at the Earth's surface	g = 9.8 N/kg
Newton's gravitational constant	G = 6.67 x 10^{-11} N·m^2/kg^2
Coulomb's constant	$k = 1/4\pi\varepsilon_o$ = 9.0 x 10^9 N·m^2/C^2
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	c = 3.0 x 10^8 m/s
Boltzmann's constant	k_B = 1.38 x 10^{-23} J/K
Avogadro's number	N_A = 6.02 x 10^{23} (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B$ = 8.31 J/(mol·K)
Stefan-Boltzmann constant	σ = 5.67 x 10^{-8} J/(s·m^2·K^4)
Elementary charge	e = 1.6 x 10^{-19} C
1 electron volt	1 eV = 1.6 x 10^{-19} J
Planck's constant	h = 6.63 x 10^{-34} J·s = 4.14 x 10^{-15} eV·s
Electron mass	m = 9.1 x 10^{-31} kg = 0.51MeV/c^2
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\|x\| << 1$
Small angle approximations	$\sin\theta \approx \theta$
	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$

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Part A

A1. A satellite of mass m is in a circular orbit of radius r around a star of mass M.

(5) a. What is the magnitude of the satellite's momentum p? Express your answer in terms of given quantities and constants.

In Parts b, c, and d you should express the impulse vector $\vec{I}$ in terms of the satellite's original momentum vector $\vec{p}$.

(5) b. We wish to give the satellite a single impulse of minimum possible magnitude, causing it to escape from the star. What impulse $\vec{I}_e$ is necessary?

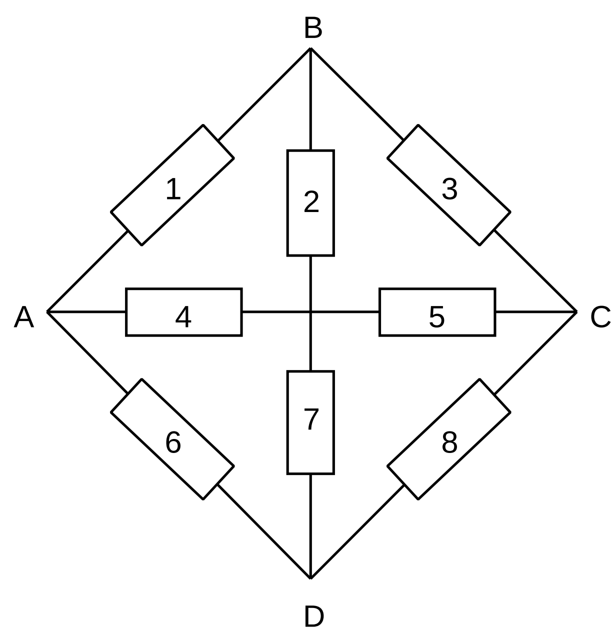
(5) c. We wish to give the satellite a single impulse of minimum possible magnitude, causing it to crash into the star. What impulse $\vec{I}_{C0}$ is necessary if you neglect the radius of the star?

(10) d. Suppose that the star has a radius R. Now what impulse $\vec{I}_{CR}$ is necessary to crash the satellite into the star? Your answer should also involve the ratio r/R.

A2. All of the numbered boxes represent resistors. Each one has resistance R.

(10) a. What is the equivalent resistance between points A and C?

(15) b. What is the equivalent resistance between points A and B?



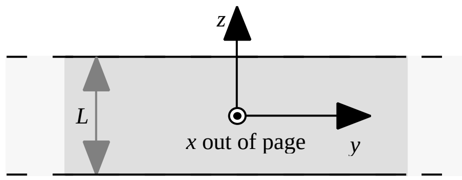


Figure a

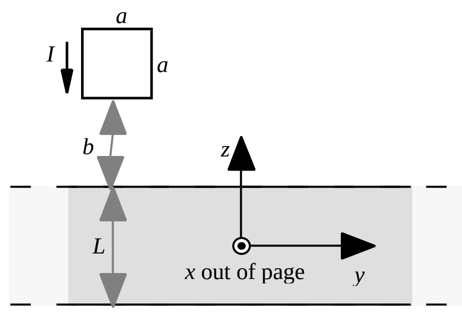


Figure b

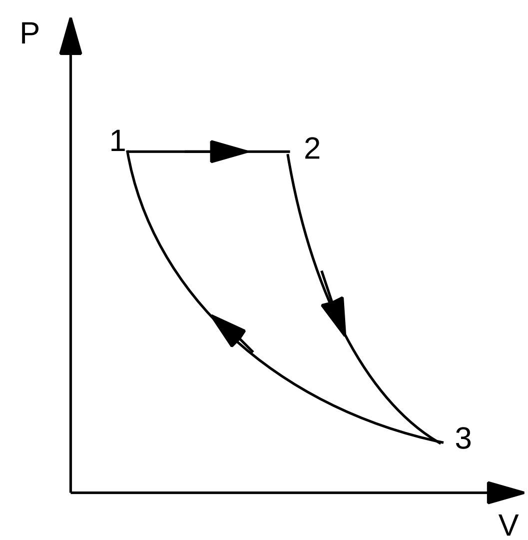
A3. A conducting slab has infinite extent in the x and y directions and thickness L in the z direction as shown in Figure a above. The slab is centered at $z = 0$ and carries a uniform current density $\vec{J} = J\hat{i}$ where $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x, y, and z directions, respectively.

(10) a. Find the magnetic field $\vec{B}$ at all points.

(10) b. A square wire loop of side a is placed a distance b above the slab (as shown in Figure b above). The loop has unit normal vector $\hat{n} = \cos\theta\hat{i} + \sin\theta\hat{j}$ and applied current I. (Figure b depicts the special case $\theta = 0$.) What is the net force and the net torque on the loop as a function of θ?

(5) c. The applied current I is now removed from the loop described above and the current density in the slab $\vec{J} = J\hat{i}$ is reduced to zero over time T. The wire used to construct the loop has resistance per unit length S. How much charge flows through each cross section of the loop wire due to the reduction in current density?

A4. An ideal gas has a heat capacity per unit mole at constant pressure of $C_P = 41.9$ J/(mol·K) and a heat capacity per unit mole at constant volume of $C_V = 33.5$ J/(mol·K). An 0.100-mole sample of this gas goes through the accompanying cycle. Process 1–>2 is isobaric (constant pressure), 2–>3 is adiabatic (no heat enters or leaves), and 3–>1 is isothermal (constant temperature).



$T_1 = 300$ K $\quad$ $T_2 = 400$ K

$V_1 = 0.0118$ m^3 $\quad$ $V_2 = 0.0157$ m^3

(15) a. Find the work done by the gas during each process ($W_{1\to2}$, $W_{2\to3}$, and $W_{3\to1}$).

(10) b. What is the efficiency of this cycle?

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Part B

B1. (10) a. In classical mechanics, the kinetic energy K, momentum p, and mass m of a particle are related by $K = \frac{p^2}{2m}$. Derive the relationship between the total energy E, momentum p, and mass m of a particle in special relativity?

Hint: Express E^2 in terms of p^2. In addition to m, the expression may also include c. Relativistic expressions for the total energy E and momentum p of a particle of mass m moving at speed v are

$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}} \qquad\qquad p = \frac{mv}{\sqrt{1-(v/c)^2}}$$

Particle A (mass $1000 \text{ MeV} / c^2$) is at rest at the center of a spherical gamma detector, which completely surrounds it except for a small hole. We accelerate particle B (mass $500 \text{ MeV} / c^2$) to a total energy of 700 MeV, sending it through the hole towards particle A. When the particles collide, a single, excited particle of unknown mass, C*, is produced. C* quickly decays to its unexcited state, C (mass $1300 \text{ MeV} / c^2$), emitting a single gamma ray.

(10) b. Find the mass of C*.

(15) c. If the gamma is emitted directly away from the hole, what energy does the detector record?

(15) d. The experiment is repeated, and the detector records an energy of 300 MeV. At what angles did particle C and the gamma emerge? Assume that C* decays at the center of the detector. Let an angle of 0 be directly away from the hole, and an angle of π be directly towards the hole.

B2. A spherical container of radius R is filled with a light, incompressible fluid carrying uniform charge density ρ. Let a point inside the container be described by $\vec{r} = r\hat{r} = x\hat{i} + y\hat{j} + z\hat{k}$ where the origin is at the center of the container and $\hat{r}$, $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the radial, x, y, and z directions, respectively.

(10) a. Find the electrostatic field inside the container.

(15) b. A small amount of fluid is replaced with a small, uncharged ball with mass density δ which is much, much greater than the fluid density. There is no gravitational field present. Ignore energy loss due to drag, and ignore any magnetic fields created by the moving fluid. Suppose that at time $t = 0$ the ball is released from rest at $\vec{r}_0 = x_0\hat{i}$, $x_0 < R$. What is $\vec{r}(t)$?

(5) c. Repeat Part b, if there is a uniform gravitational field $\vec{g} = g\hat{i}$. Assume that g is small enough so that the ball does not hit the wall of the container.

d. The external gravitational field is removed, and the ball is again placed at $\vec{r}_0 = x_0\hat{i}$. Now, however, we start the ball with some nonzero velocity $\vec{v}_0 = v_0\hat{j}$.

(10) i. What is the ball's trajectory (before it hits the wall, if it does)? That is, what is the time-independent relationship between the ball's x and y coordinates? Give the name of this trajectory, e.g. straight line, parabola, spiral…

(5) ii. Is it possible for the ball to avoid the wall of the container? If so, what condition(s), if any, are required of x_0 and v_0?

(5) iii. If you answered "yes", is the ball's orbit periodic when it avoids the wall, and if so, what is the period? If you answered "no", what is the maximum time that can elapse before it hits the wall?