AAPT | UNITED STATES PHYSICS TEAM

AIP | 2004

2004 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's, or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- Do not discuss the contents of this exam with anyone until after the submission deadline.
- Good luck!

Possibly Useful Information

Gravitational field at the Earth's surface	g = 9.8 N/kg
Newton's gravitational constant	G = 6.67 x 10^{-11} N·m^2/kg^2
Coulomb's constant	k = $1/4\pi\varepsilon_o$ = 8.99 x 10^9 N·m^2/C^2
Biot-Savart constant	k_m = $\mu_o/4\pi$ = 10^{-7} T·m/A
Speed of light in a vacuum	c = 3.00 x 10^8 m/s
Boltzmann's constant	k_B = 1.38 x 10^{-23} J/K
Avogadro's number	N_A = 6.02 x 10^{23} (mol)$^{-1}$
Ideal gas constant	R = $N_A k_B$ = 8.31 J/(mol·K)
Stefan-Boltzmann constant	σ = 5.67 x 10^{-8} J/(s·m^2·K^4)
Elementary charge	e = 1.602 x 10^{-19} C
1 electron volt	1 eV = 1.602 x 10^{-19} J
Planck's constant	h = 6.63 x 10^{-34} J·s = 4.14 x 10^{-15} eV·s
Electron mass	m = 9.109 x 10^{-31} kg = 0.511 MeV/c^2
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x\rvert << 1$
Small angle approximations	$\sin\theta \approx \theta$
	$\cos\theta \approx 1 - {}^1/_2\,\theta^2$

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Part A

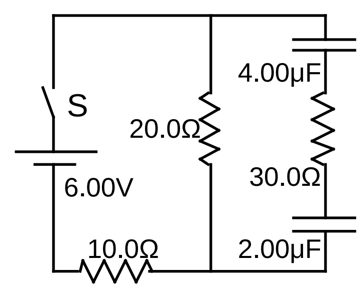
A1. The capacitors in the circuit diagrammed to the right are initially uncharged.

(10) a. What is the current in each resistor immediately after the switch labeled S is closed?

After a very long time, the capacitors are fully charged.

(5) b. What is the current in each resistor at this time?

(10) c. What is the charge on each capacitor?



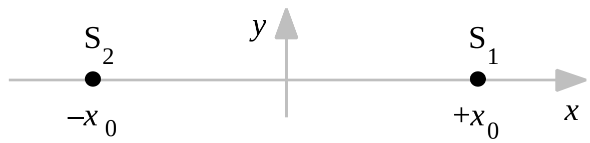
A2. A hot air balloon with average density 1.20 kg/m^3 is in equilibrium at a height of 1.00 km above sea level. A gust of wind pushes the balloon to a height of 1.10 km above sea level and releases it at rest.

(20) a. How long does it take for the balloon to return to its equilibrium position?

(5) b. How fast is it going when it passes through equilibrium?

Assume that: The atmospheric density is a linear function of height.
The balloon volume and average density are constant.
Variations in the gravitational field with height are negligible.
The density of air at sea level is 1.29 kg/m^3.
Drag is negligible.

A3. Sound sources S_1 and S_2 are located at $+x_0$ and $-x_0$, respectively. The sources produce tones with the same phase and amplitude. The frequency can be varied from 175 Hz to 625 Hz, but both speakers have the same frequency. Use 340 m/s for the velocity of sound in air and $x_0 = 0.850$ m.

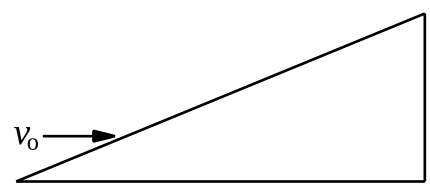


(8) a. For which frequency or frequencies in the above range is minimum sound intensity heard at all points along the x-axis with $x > x_0$.

(10) b. Next consider the region on the x-axis between the two sound sources S_1 and S_2. Assuming that the amplitude A does not decrease with distance from the source, write an equation that gives the resultant sound wavefunction in the form $\Psi(x,t) = 2A\psi(x)\phi(t)$, where ψ and ϕ are functions of x and t, respectively. The equation may also depend on the wavelength λ and frequency f of the sound waves and should be valid for all points on the x-axis with $-x_0 < x < +x_0$.

(7) c. For the frequency or frequencies found in Part (a), at which points between $x = -0.85$ m and $x = +0.85$ m does minimum sound intensity occur?

A4. A ball is thrown so that it hits a stationary ramp and then bounces up the ramp. Immediately before the first collision between the ball and the ramp, the ball has a velocity v_o that is horizontal as shown in the accompanying diagram. Assume that all collisions are perfectly elastic, that there is no friction between the ramp and the ball, and that air resistance is negligible. At the instant that the ball makes its N+1st collision with the ramp, the ball is moving perpendicular to the ramp. Express your answers in terms of N, v_o, and g.



(20) a. Find the angle that the ramp makes with the horizontal.

(5) b. What is the distance, measured along the ramp, between the point at which the first collision occurs and the point at which the N+1st collision occurs?

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Part B

B1. (20) a. In 1913, Niels Bohr first derived a formula for the energy levels of the hydrogen atom. He postulated that the single electron orbited a very massive proton under the influence of the electrostatic force and that the circular electron orbits had quantized angular momentum

$$L = n\hbar = n\frac{h}{2\pi}$$

where $n = 1, 2, 3, \ldots$ and h is Planck's constant. These postulates gave rise to discrete values of the electron's total energy E_n. Derive these Bohr Model energy levels. Express your answer in terms of Coulomb's constant k, the electron mass m, the electron charge e, $\hbar$, and n. If done correctly, your answer should evaluate to

$$E_n = -\frac{2.18\times 10^{-18}\text{ J}}{n^2} = -\frac{13.6\text{ eV}}{n^2}.$$

b. In a hypothetical modern day experiment, electrons are accelerated from rest through a potential V into a cloud of cold atomic hydrogen (Figure 1-1). A series of plates with aligned holes select a beam of scattered electrons moving perpendicular to the plates. Immediately beyond the final plate, the electrons enter a uniform magnetic field B perpendicular to the beam; they curve and strike a piece of film mounted on the final plate.

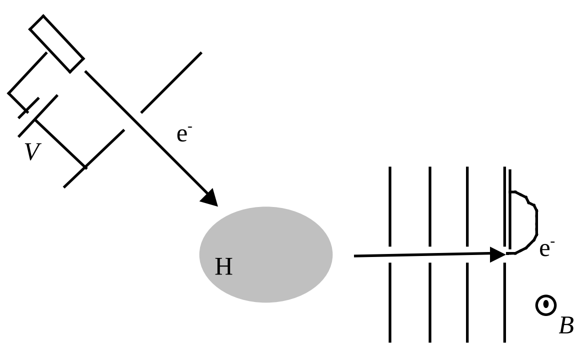


Figure 1-1

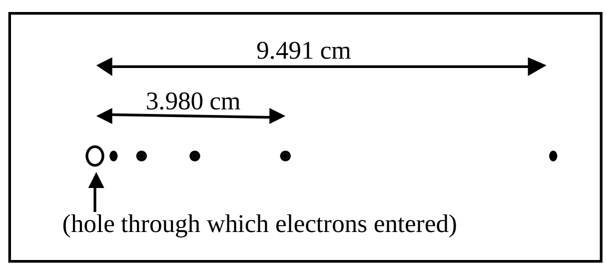


Figure 1-2

When the film is developed, a series of spots is observed (Figure 1-2). The distances between the hole and the two most distant spots are measured. You may assume that the film is large enough to have intercepted all of the electrons, i.e. that there are no spots farther from the hole than those shown. The number of spots shown is not necessarily accurate.

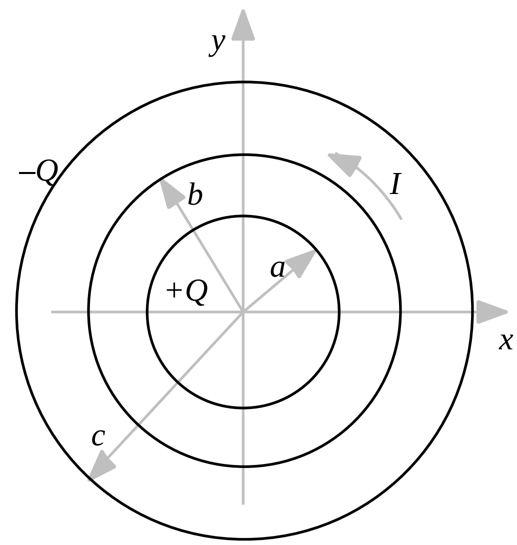
Make the approximation that the mass of the hydrogen atom is much larger than the mass of the electron. Assume that each electron scatters off only one atom, which is initially in the ground state (lowest energy state) and has negligible thermal velocity.

Determine:
(10) i. The magnitude of the magnetic field B.
(10) ii. The magnitude of the accelerating potential V.
(10) iii. The total number of spots on the film.

B2. A very long solenoid has radius b, n turns per unit length, and carries current I as shown in the accompanying cross-sectional view. The solenoid is firmly fixed in place and cannot rotate.

(5) a. What is the magnetic field $\vec{B}$ inside the solenoid, far from its ends?

(5) b. What is the magnetic energy density u_B stored in the solenoid?



cross-sectional view

There are two long cylindrical shells of length h and mass m coaxial to the solenoid – one, inside the solenoid at a radius a, carries a charge $+Q$, uniformly distributed over its surface; the other, outside the solenoid at radius c, carries charge $-Q$. $h >> c$. Use r to represent the distance from the cylinders' axis. The cylinders are free to rotate about their axes.

(5) c. What is the electric field $\vec{E}$ between the cylindrical shells, far from their ends?

(5) d. What is the electric energy density u_E stored between the shells?

The current in the solenoid is gradually reduced from I to zero at a constant rate, $\Delta I/\Delta t$. The cylindrical shells begin to rotate. Assume the magnetic field generated by the rotating cylindrical shells is negligible.

(15) e. Calculate the gain in angular momentum of each cylinder about its axis when the current in the solenoid is reduced from I to zero.

(15) f. In addition to storing energy, fields can also store linear momentum and angular momentum. The density of <u>linear</u> momentum stored in electromagnetic fields is

$$\varepsilon_0 \vec{E} \times \vec{B}.$$

Calculate the change in <u>angular</u> momentum of the electromagnetic fields about the cylinders' axis as the current in the solenoid is reduced from I to zero.

{*Suggested by a problem in Introduction to Electrodynamics, 3rd Ed., by David J. Griffiths (Prentice Hall, Upper Saddle River, New Jersey, 1999), page 359.*}