AAPT | UNITED STATES PHYSICS TEAM

AIP | 2005

2005 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 - 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's, or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty
- Do not discuss the contents of this exam with anyone until after the submission deadline.
- Good luck!

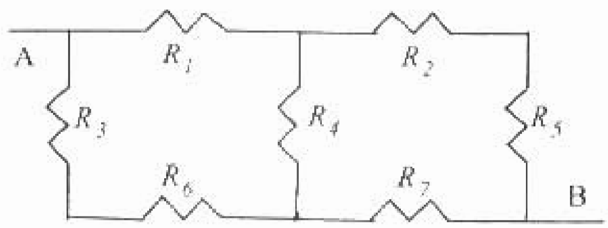
Possibly Useful Information

Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 8.99 \times 10^9$ N·m²/C²
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.00 \times 10^8$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}$ (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/(mol·K)
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J/(s·m²·K⁴)
Elementary charge	$e = 1.602 \times 10^{-19}$ C
1 electron volt	1 eV = 1.602×10^{-19} J
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s = 4.14×10^{-15} eV·s
Electron mass	$m = 9.109 \times 10^{-31}$ kg = 0.511 MeV/c²
Binomial expansion	$(1+x)^n \approx 1 + nx$ for $\lvert x\rvert << 1$
Small angle approximations	$\sin\theta \approx \theta$
	$\cos\theta \approx 1 - {}^1/_2\,\theta^2$

AAPT | UNITED STATES PHYSICS TEAM

AIP | 2005

Semi-Final Exam
Part A



A1. Each of the seven resistors in the network above has the same resistance R.

(10) a. If an ideal battery of emf $\mathcal{E}$ and no internal resistance is connected across the points labeled A and B, what is the current I_i that flows in the j^{th} resistor R_j (for each of the seven resistors)?

(5) b. What equivalent resistor could be used to replace the network so that the same current flowed from the battery?

(10) c. If the resistor labeled R_4 was removed and replaced by a capacitor with capacitance C, what would the charge be on the capacitor after the circuit had been connected to the battery for a very long time?

A2. (25) A sample of water of mass 0.0360 kg and temperature 100°C is added to an enclosed container of volume 2.00 m^3 that contains air at a pressure of 1.00×10^5 Pa and a temperature of 500 K. Assume both air and steam can be treated as ideal gases and that all of the water vaporizes. What is the final temperature and pressure inside the container?

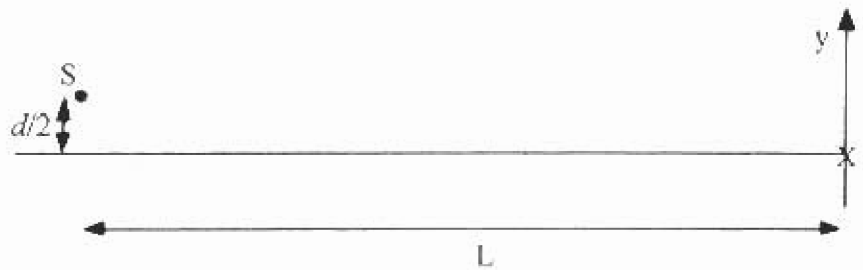
Possibly useful information:

Molar Heat Capacities	Air	Steam
C_V (at constant volume)	20.8 J/(mol•K)	27.9 J/(mol•K)
C_P (at constant pressure)	29.1 J/(mol•K)	36.2 J/(mol•K)

L_V (Latent heat of vaporization) = 2.26×10^6 J/kg

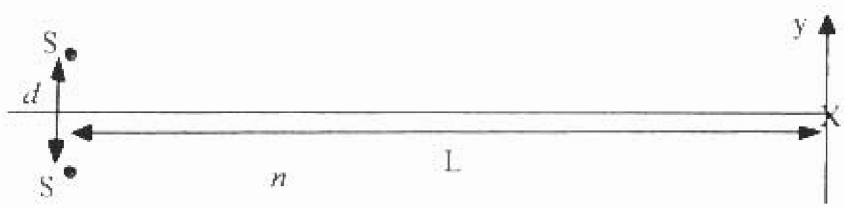
Molar mass of water = 18.0 g/mol

A3. A plane wave traveling in the positive z-direction can be described by the following wavefunction: $\psi(z,t) = A\sin(kz - \omega t + \phi)$, where A (the amplitude) and ϕ are constants, $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$, f is the frequency, and λ is the wavelength. The factor $(kz - \omega t + \phi)$ is called the phase of the wave. The following relation may be useful:
$\sin\alpha + \sin\beta = 2\cos\left[\frac{1}{2}(\alpha - \beta)\right]\sin\left[\frac{1}{2}(\alpha + \beta)\right]$.



A point source S of monochromatic electromagnetic radiation of wavelength λ is located a distance $d/2$ above a perfectly reflecting surface represented by the solid horizontal line in the diagram above. An interference pattern is formed on the screen a distance L from the source, $L >> d$. The point $y = 0$, at the intersection of the screen and the reflecting surface, represented by an X in the diagram, is an interference minimum. Point X is on the surface of the material.

(10) a. The first bright band is located at distance y_b above point X. Find y_b.



The reflecting surface is replaced by a material that does not reflect but transmits without loss of amplitude. The material has index of refraction n. One source is placed a distance $d/2$ above the air-material interface. The other source is embedded in the material a distance $d/2$ below the interface. The two sources are in phase, coherent, emit electromagnetic radiation of the same monochromatic wavelength λ in vacuum, and are both located a distance L from a screen, $L >> d$. Assume that the wave amplitude A due to each source individually is the same at point X and that the wave from the lower source travels entirely in the material.

(8) b. What is the resultant wavefunction at point X as a function of time?

(7) c. Let I_0 denote the maximum intensity reaching point X, which occurs when n = 1. Derive an expression for the time averaged intensity of the electromagnetic radiation at point X as a function of n, I_0, L, and λ.

A4. Just like electrostatic fields, gravitational fields obey Gauss's Law. One of the arguments for dark matter in the universe is the observation that the velocity of stars orbiting far from the center of the galaxy is approximately constant. This requires a spherically symmetric mass distribution which the visible stars do not have. Let the mass density of the galaxy be $\rho(r) = Ar^n$ where A is a constant, r is the distance from the center of galaxy, and the power n is as yet undetermined.

(5) a. Show that the total mass M enclosed by a sphere of radius R is $M(R) = CR^{n+3}$ and find the constant C in terms of given variables and known constants.

You may use $M(R) = CR^{n+3}$, with C unspecified, for the remaining parts of this problem.

(5) b. Find the magnitude of the gravitational force on a star of mass m which has a circular orbit of radius R about the center of this galaxy.

(5) c. Find the orbital velocity v of this star.

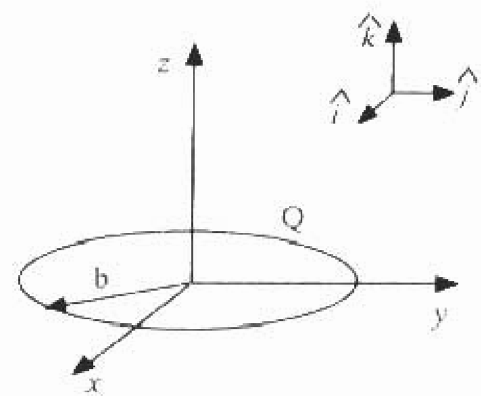
(5) d. What power n is required by the observation that v is constant?

(5) e. If the mass density were $\rho(r) \approx 0$ for $r > R$, a reasonable assumption based on the observed visible star distribution, what would be the orbital velocity of a star with $r > R$? Let M_0 = the total mass of the galaxy.

AAPT | UNITED STATES PHYSICS TEAM

AIP | 2005

Semi-Final Exam
Part B



B1. A length of wire is formed into a ring of radius b and total charge Q that is distributed uniformly around the ring. The ring is in the x-y plane and centered on the origin as shown in the accompanying diagram. $\hat{i}$, $\hat{j}$, and $\hat{k}$ are unit vectors in the x, y, and z directions respectively. Assume the potential vanishes at infinity.

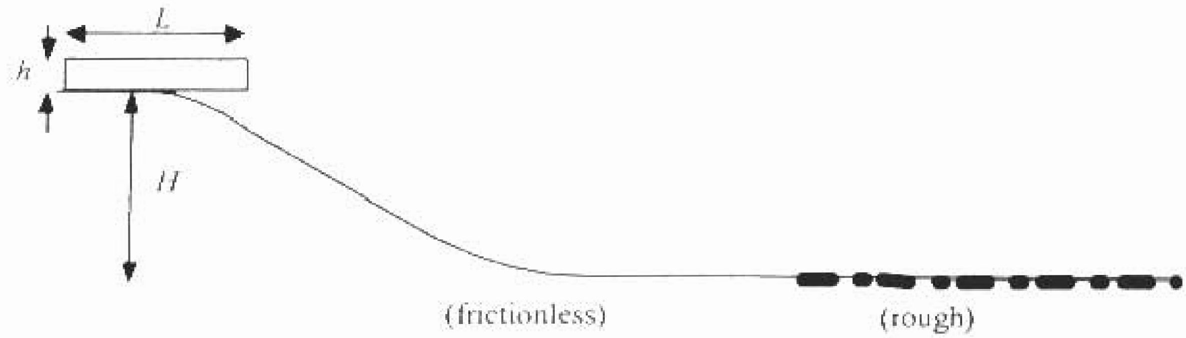
(7) a. Find the field at any point on the z axis.
(5) b. Find the potential at any point on the z axis.

A point charge with mass m and charge $-q$ is placed at $z = z_0$ where $z_0 << b$ and released from rest at $t = 0$. Ignore any effects due to radiation.

(10) c. Find the potential energy of the system as a function of z, the position of the charge. Expand this potential energy function about the origin for $|z| \le z_0$. Keep only terms to lowest non-zero order in z.
(5) d. Find the speed of the charge as it passes through the origin.
(8) e. Find the velocity of the charge as a function of time.

The point charge is removed and the ring is set into rotation about the z-axis with angular velocity $\vec{\omega} = \omega\hat{k}$. Ignore any effects due to radiation.

(10) f. Find the magnetic field due to the rotating ring at any point along the z-axis.
(5) g. A second point charge with mass m and charge $-q$ is placed at $z = z_0$ where $z_0 << b$ and released from rest. What is the magnetic force on the charge as it passes through the origin?



B2. A block with uniform density, length L, and height h, $h \ll L$, starts from rest at the top of a hill of height H as shown in the accompanying diagram. The hill is frictionless and there is a flat frictionless surface at the bottom of the hill that is longer than L. Then, there is a flat horizontal surface that has friction. The coefficient of friction between the block and the flat surface with friction is μ. The leading edge of the block comes into contact with the surface with friction when the rest of the block is on a flat frictionless surface.

Express your answers to part (a) in terms of μ, L, and g.

a. If the hill has a certain critical height, H_{crit} the block comes to a stop at the instant that the entire block is on the surface with friction. (This is the instant at which the leading edge has traveled a distance L across the surface with friction).

(6) i. Find the critical height H_{crit}

(8) ii. If the hill has the critical height found in part (i), what is the time from the instant that the leading edge of the block encounters friction until the block comes to a stop?

Express your answers to parts (b) and (c) in terms of H, μ, L, and g.

b. If the hill has a height $H < H_{crit}$:

(6) i. Find the distance that the leading edge of the block travels across the surface with friction before it comes to a stop.

(6) ii. Find the time from the instant that the leading edge of the block encounters friction until the block comes to a stop.

c. If the hill has a height $H > H_{crit}$:

(8) i. Find the distance that the leading edge of the block travels across the surface with friction before it comes to a stop.

(16) ii. Find the time from the instant that the leading edge of the block encounters friction until the block comes to a stop.