AAPT UNITED STATES PHYSICS TEAM
AIP 2006

Preliminary Exam
Open Response Questions

4 Questions, 60 minutes

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  #1 – P. 1/3

- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Each of the four questions is worth 25 points. The questions are not necessarily of the same difficulty. Good luck!
- In order to maintain exam security, do not communicate any information about the problems or the solutions on this exam (or on the multiple-choice exam) until after February 13th.

Possibly Useful Information

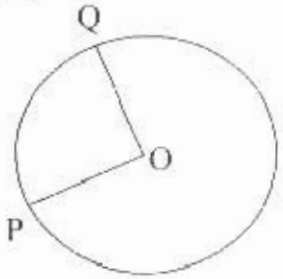
Gravitational field at the Earth's surface	$g = 9.8\ \text{N/kg}$
Newton's gravitational constant	$G = 6.67 \times 10^{-11}\ \text{N-m}^2/\text{kg}^2$
Average Earth-Sun Distance	1 Astronomical Unit (A.U.) $= 1.5 \times 10^{11}$ m
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x \rvert << 1$

Moment of Inertia about Center of Mass – Uniform Object
(will not be provided on the second screening exam)

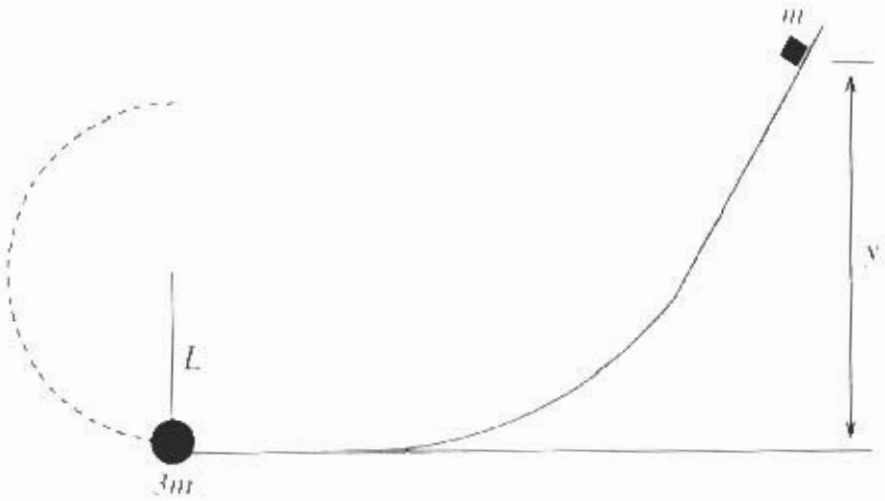
Disk	$^1/_2\ MR^2$
Sphere	$^2/_5\ MR^2$
Rod	$^1/_{12}\ ML^2$

AAPT UNITED STATES PHYSICS TEAM
AIP 2006

1. A car starts at point P from rest, with zero initial acceleration, but with a constant rate of change of tangential acceleration of 2 m/s^3. The car is constrained to travel on a circular track of unknown radius. When the car reaches point Q, the angle between the acceleration and velocity vectors is 30°. The center of the track is point O. What is the angle (in radians) POQ?



2. A small hard block of mass $3m$ is suspended from a thread of length L. A second block of mass m is located on an incline, originally at rest, a height y above the level of the large mass. When the smaller block is released it slides, without friction, down a ramp, and then collides elastically with the larger block. The large block swings around so that the tension in the string just barely drops to zero at the top of the loop. The small block slides back up the ramp, rising to a maximum vertical height h.



a) Find h in terms of y.

b) Find L in terms of y.

3. A uniform thin rod is suspended from the ceiling by two identical ideal vertical springs attached to the ends of the rod. Two experiments are performed:

   Expt. 1: From equilibrium, both ends of the rod are pulled down by an equal, small amount. When released, the rod oscillates with period T_1.

   Expt. 2: From equilibrium, one end of the rod is pulled down by a small amount, and the other end is pushed up by an equal amount. When released, the rod oscillates with a period T_2.

   Find the numerical value of the ratio T_1 / T_2.

4. Consider a spherical asteroid of radius R_a and mass M_a in a circular orbit of radius $r >> R_a$ around a spherical planet of radius R_p and mass M_p. The asteroid is "tide-locked;" that is, its rotation is such that the same point on its surface is always facing the planet. Let the gravitational constant be G. If the asteroid is too close to the planet, it may be torn apart by tidal forces. Ignore any oblation due to tidal effects.

   a) Consider a particle of mass m sitting on the surface of the asteroid closest to the planet. Find the normal (contact) force from the surface of the asteroid acting on the particle.

   b) Approximate your answer to part (a) to first order in $\frac{R_a}{r}$.

   c) Assume that the asteroid and the planet have the same mass density. What is the smallest value of r such that the particle remains on the surface, in terms of R_p and/or R_a only? This value is known as the "rigid Roche limit."