AAPT UNITED STATES PHYSICS TEAM
AIP 2006

2006 Semi-Final Exam

INSTRUCTIONS
DO NOT OPEN THIS TEST UNTIL YOU ARE TOLD TO BEGIN

- Work Part A first. You have 90 minutes to complete all four problems.
- After you have completed Part A, you may take a break.
- Then work Part B. You have 90 minutes to complete both problems.
- Show all your work. Partial credit will be given.
- Start each question on a new sheet of paper. Be sure to put your name in the upper right-hand corner of each page, along with the question number and the page number/total pages for this problem. For example,

  Doe, Jamie
  A1 – 1/3
- A hand-held calculator may be used. Its memory must be cleared of data and programs. You may use only the basic functions found on a simple scientific calculator. Calculators may not be shared. Cell phones, PDA's, or cameras may not be used during the exam or while the exam papers are present. You may not use any tables, books, or collections of formulas.
- Questions with the same point value are not necessarily of the same difficulty.
- Do not discuss the contents of this exam with anyone until after March 20th.
- Good luck!

Possibly Useful Information - (Use for both part A and for part B)

Gravitational field at the Earth's surface	$g = 9.8$ N/kg
Newton's gravitational constant	$G = 6.67 \times 10^{-11}$ N·m²/kg²
Coulomb's constant	$k = 1/4\pi\varepsilon_o = 8.99 \times 10^{9}$ N·m²/C²
Biot-Savart constant	$k_m = \mu_o/4\pi = 10^{-7}$ T·m/A
Speed of light in a vacuum	$c = 3.00 \times 10^{8}$ m/s
Boltzmann's constant	$k_B = 1.38 \times 10^{-23}$ J/K
Avogadro's number	$N_A = 6.02 \times 10^{23}$ (mol)$^{-1}$
Ideal gas constant	$R = N_A k_B = 8.31$ J/(mol·K)
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ J/(s·m²·K⁴)
Elementary charge	$e = 1.602 \times 10^{-19}$ C
1 electron volt	1 eV = 1.602×10^{-19} J
Planck's constant	$h = 6.63 \times 10^{-34}$ J·s = 4.14×10^{-15} eV·s
Electron mass	$m = 9.109 \times 10^{-31}$ kg = 0.511 MeV/c²
Binomial expansion	$(1 + x)^n \approx 1 + nx$ for $\lvert x\rvert << 1$
Small angle approximations	$\sin\theta \approx \theta$
	$\cos\theta \approx 1 - \frac{1}{2}\theta^2$

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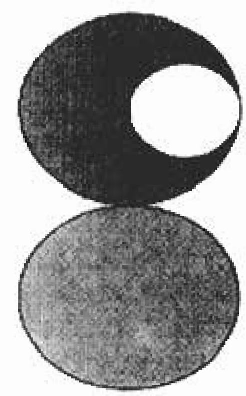
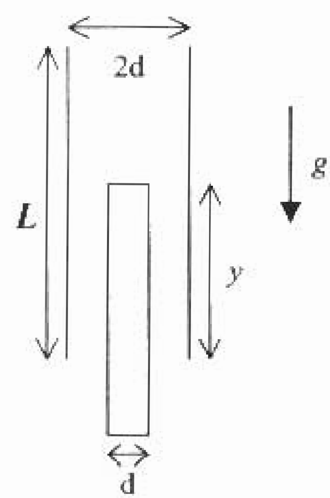
Part A

A1. (25) An otherwise uniform disk of radius R has a circular hole of radius $R/2$ cut out so that the hole touches both the center and the edge of the disk. The disk has a mass M after the hole is cut out. The disk is placed above a wheel so that when the lower wheel rotates, the disk will rotate with a constant angular velocity. The disk is constrained so that it can only move in the up and down direction or rotate freely. If the angular velocity of the disk exceeds a certain value, ω_{max}, the disk will bounce up and down on the lower wheel. Find the total kinetic energy of the disk when the angular velocity is ω_{max}. Express your answer in terms of M, R, and g (but not in terms of ω_{max}).



A2. A parallel plate capacitor is constructed from two vertical square conducting surfaces with sides of length L separated horizontally by a distance $2d$. Assume that $d \ll L$, so that fringing effects can be neglected. The plates are charged to Q and $-Q$, and then electrically isolated. A square conducting slab, also with sides of length L, but with thickness d, is then inserted vertically between the two plates without making contact with either plate. Assume that the slab moves only in the vertical direction. Give your answers to each part only in terms of the variables listed above and any appropriate fundamental constants.



(7) a. Find an expression for the capacitance C of the system assuming that the movable slab extends inward between the plates by a distance y.

(9) b. Find an expression for the mass of the slab m given that the equilibrium location (where the force of gravity on the slab is balanced by the attraction into the capacitor) for the slab is $y = L/2$.

(9) c. The slab is perturbed vertically *slightly* from its equilibrium position so that it undergoes simple harmonic motion. Find an expression for the frequency of the motion of the slab.

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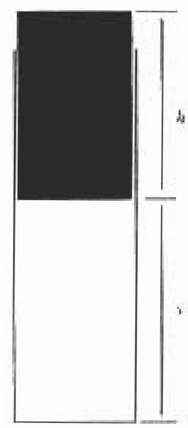

A3. A hollow vertical cylinder contains a solid cylindrical piston with length h. The cylindrical piston is originally held in place by a retaining pin so that a fixed amount of gas at temperature T_0 and pressure p_0 is trapped beneath the piston. T_0 and p_0 are also the temperature and pressure of the gas outside the cylinder. The bottom of the piston is originally a height y_0 above the bottom of the cylinder.

When the retaining pin is removed, the piston falls under the influence of gravity, compressing the gas so that the piston drops to $y_1 = y_0 / 8$ before bouncing back up. This process occurs so quickly that no heat enters or leaves the gas.

The piston bounces up and down, but eventually comes to rest at a height y_2 when the gas in the cylinder is once again in thermal equilibrium with the surroundings. The specific heat capacity at constant volume of the gas in the cylinder is $C_V = \frac{3}{2}R$.

(12) a. Find the density of the piston in terms of p_0, g, and h.

(13) b. Find the numerical value of the ratio y_2 / y_0

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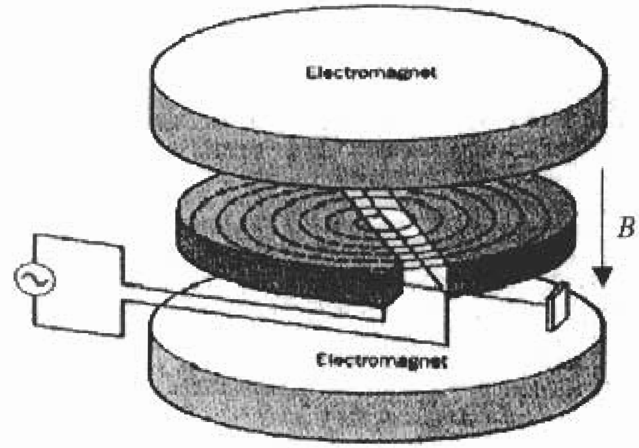
A4. A cyclotron is a particle accelerator consisting of dipole magnets that create two semi-circular regions of magnetic field B, pointing uniformly down (toward the bottom of the page). These regions are called "Dees" due to their D shape. The two Dees are placed back-to-back, but with a small separation. Consider a small cyclotron of radius R. An AC potential $V = V_o \sin \omega t$, that is switched on at $t = 0$, is applied across the Dee electrodes. The particles are accelerated whenever they cross the gap between the electrodes. The gap is small enough that any variation in $V(t)$ during the time it takes a particle to cross the gap may be neglected.



(Source: Wikne, Jon. Oslo Cyclotron Laboratory. 1994. 14 Feb. 2006. <http://lynx.uio.no/cycdescr.html>.)

The cyclotron is filled with a variety of ions of different masses, but each has the same charge q. All ions start at the center of the cyclotron and are emitted when they reach the edge of the cyclotron. An ion will make many laps around the cyclotron before being emitted. Ignore relativistic effects in parts (a) – (c).

In terms of ω, q, V_o, R, and B,

(5) a. What is the maximum possible momentum of an emitted ion?

(7) b. Of all of the possible ions that satisfy part (a), what is the kinetic energy of the ion emitted first?

(6) c. At what time t after turning on the AC potential are the ions from part (b) emitted?

(7) d. Such a device won't work well with electrons, as relativistic effects can't be ignored for energies above one hundred thousand electron-volts. One possible correction is to create a magnetic field that is not constant, but varies with distance from the center of the cyclotron. This enhanced device is called an isochronous cyclotron. Find an expression for the necessary magnetic field B in terms of r, ω, m (the mass of the electron), and q (the charge on the electron). Relativistic effects must be considered.

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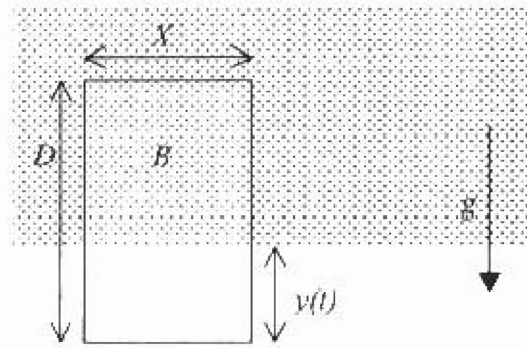
Part B

B1. Consider a wire loop of width X and height D. The loop is held at rest vertically. There is a uniform magnetic field B passing through the loop. However, the magnetic field abruptly decreases to zero everywhere below the location of the bottom edge of the loop at $t = 0$. The magnetic field is perpendicular to the plane of the loop. The mass of the loop is m. At time $t = 0$, the loop is released from rest so that it begins to fall out of the magnetic field. Let $y(t)$ be the distance that the bottom edge has traveled out of the magnetic field at t seconds after the loop has been released. $y(0) = 0$.



a. Suppose that the loop has resistance R but negligible inductance. Assume that D is large enough that the loop very nearly reaches terminal velocity before the loop entirely falls out of the magnetic field. Express your answers to part (a) in terms of X, D, B, m, g, R, and t.

(5) i Find the terminal velocity, v_{term}, of the loop.

Answers to part (a) ii-iv may include the symbol v_{term} in addition to any of the symbols listed above.

(7) ii. Derive an expression for velocity as a function of time that is valid until the loop is entirely out of the magnetic field.

(3) iii. Derive an expression for the rate at which thermal energy is dissipated due to the resistance of the loop.

(5) iv. Determine the total amount of thermal energy dissipated from $t = 0$ until the time that the top edge of the loop just starts to leave the magnetic field.

b. Suppose instead that the loop is made of a superconducting material so that it has negligible resistance. In this case, we cannot neglect the self-inductance of the loop L. Assume that D is large enough that the loop never falls entirely out of the magnetic field. Express your answers to part (b) in terms of X, D, B, m, g, L, and t.

(12) i. Prove mathematically that the loop oscillates up and down in simple harmonic motion and determine the period of the oscillation.

(6) ii. Derive an expression for $y(t)$.

(4) iii. Determine the maximum kinetic energy of the loop.

(5) iv. Determine the maximum energy stored in the magnetic field of the loop.

(3) v. At what times is the energy stored in the magnetic field at its maximum value?

(Based on a problem by W. M. Saslow, *Am. J. Phys.* **55**, 986 (1987).)

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B2. In this problem, we present a simple model of seasonal temperature variation. We ignore temperature and illumination changes between day and night, considering only daily average values. We also ignore all forms of heat transfer other than radiation, including any heat transfer between areas of the surface of the Earth. Finally, we ignore the effects of the atmosphere and clouds.

We therefore consider a patch of ground with heat capacity **per unit area** c and emissivity ε, illuminated by light of varying intensity I. The patch therefore has a varying temperature T.

(8) a. Develop an equation relating T, $\frac{dT}{dt}$, I, c, ε, and the Stefan-Boltzmann constant σ.

Figure 1 shows the yearly variation in average daily sunlight intensity on Earth for various latitudes. The variation is not sinusoidal, but for temperate latitudes, it may be approximated as a sinusoid. In particular, Figure 2 shows the variation in College Park, Maryland (at 39° N latitude), together with a good sinusoidal approximation. Therefore, we assume that the intensity has the form $I = I_0 + I_1 \sin(\omega t)$. If the annual variation in temperature is small compared to the absolute temperature, we will likewise have $T = T_0 + T_1 \sin(\omega t - \phi)$, where by hypothesis $T_1 << T_0$.

(14) b. Show that your previous result now implies an equation of the form

$$A + B\sin(\omega t - \phi) + C\cos(\omega t - \phi) = I_0 + I_1 \sin(\omega t) \qquad (1)$$

and provide the expressions for A, B, and C in terms of T_o, T_l, c, ε, σ, and ω.

(9) c. Using equation (1), find expressions for A, B, and C in terms of I_o, I_l, and ϕ.

(7) d. Combine your results to obtain a simple relationship between the relative amplitude of intensity fluctuations $\frac{I_1}{I_0}$, the relative amplitude of temperature fluctuations $\frac{T_1}{T_0}$, and the phase shift ϕ.

(9) e. Plot on graph paper the predicted difference between summer and winter temperatures in Kelvin vs. the number of months between the summer solstice (day of maximum sunlight intensity) and the hottest day of the year. Use the values for I_o and I_l given for College Park in Figure 2, and assume a mean temperature of 283 K. Show work indicating the method of your calculations for the numbers plotted on the graph.

(3) f. Actual temperature data for College Park is shown in Figure 3. Does the model provide a reasonable fit to the data? Briefly justify your answer. (The summer solstice occurs on June 21.)

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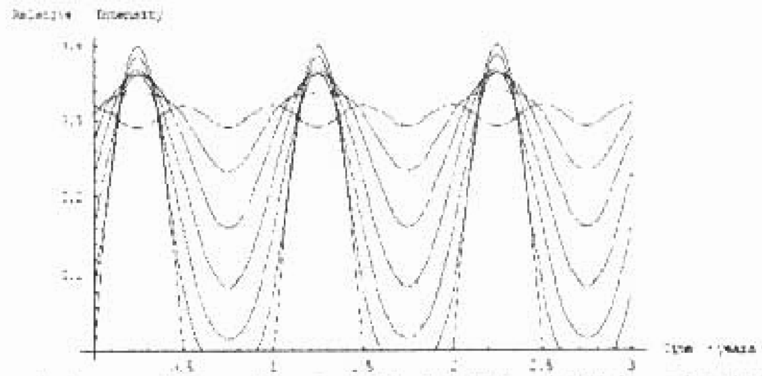

Figure 1: Average daily sun intensity on Earth (23.5° axis tilt) for various latitudes
The vertical scale is normalized so that the intensity when the sun is overhead is 1.

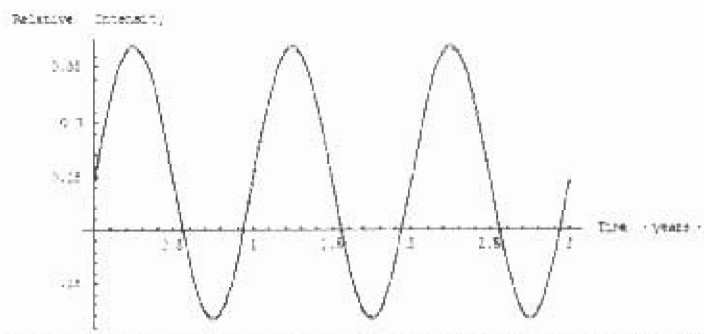

Figure 2: Average daily sun intensity in College Park, with sinusoidal fit
The fit has equation $0.244 - 0.126\sin(2\pi t)$.

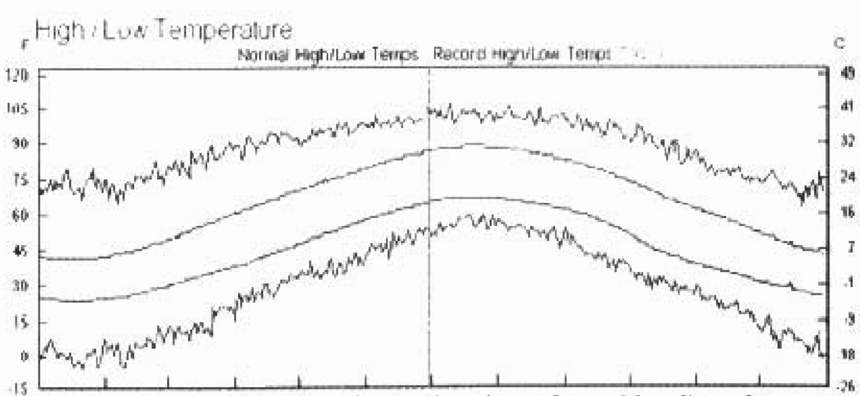

Figure 3: Actual average temperature data at Baltimore-Washington International Airport.
The vertical line is June 30.
Source: Weather Underground. http://www.wunderground.com/.